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NON-EQUIDISTANT NEW INFORMATION OPTIMUM GM(1,1) MODEL AND ITS APPLICATION

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Abstract: Grey system is a theory which studies poor information specially, and it possesses wide suitability. Applying the principle in which new information should be used fully and modeling method of grey system, a non-equidistant new information optimum GM(1,1) model is put forward in which the nth component modified by optimization is taken as the initialization and the relation error to least is taken as objective function. As the background value is an important factor affecting the precision of grey system model, based on index characteristic of grey model and the definition of integral, the background value in non-equidistant GM(1,1) model is researched and the discrete function with non-homogeneous exponential law is used to fit the accumulated sequence and optimum formula is given. The formula of background value of new information GM(1,1) model can be used in non-equal interval & equal interval time series. Example validates the practicability and reliability of the proposed model.

Keywords: Initialization optimization; Background value; GM(1,1); New information; Non- equidistant; Grey system

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1. Introduction

Grey model as an important part in grey system theory has been widely used in many fields since Professor J.L. Deng proposed the grey system [1]. There are some grey models which have mainly GM (1,1), GM (1, N) and MGM (1, N). GM (1, 1) has an important role in engineering science such as data processing, data prediction, test on-line monitoring because of the research characteristics such as the small sample and the poor information, as well as the advantages which is simple and practical [1-9]. Most

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of grey system models are based on equidistant sequence, but the original data obtained from the actual work are mostly non-equidistant sequence. So that establishing non-equidistant sequence model has a certain practical and theoretical significance. Sequence spacing was regarded as a multiplier to establish the non-equidistance GM (1, 1) which supposed that there exists the linear relationship between data difference and time difference [2], but the result from this model can't be ensured to be consistent with the reality. Function transformation method was adopted to reduce the standard deviation coefficient to take the original sequence as new data sequence and estimate the model parameters, and then GM (1, 1) was set up [3], but there is the complicated calculation. In order to improve the accuracy of the fitting and the predict, a variety of methods constructing the background value were proposed and some non-equidistant GM(1,1) models were established [4-9]. The model improving the background value based on non-homogeneous GM(1,1) model was established, in which homogeneous exponent function is used for fitting one-time accumulated generating sequence to obtain the higher accuracy [4,5]. But according to the solution form of the albino differential equations in GM (1 , 1), the exponent form of one-time accumulated generating sequence is non-homogeneous, and only after accumulating and reducing it is homogeneous. There produce some error by using the homogeneous exponent function to fit. The optimal calculation formula for background value was deduced using non-homogeneous exponent function to fit one-time accumulated generating sequence and equidistant GM(1,1) model was established [6]. While the optimal calculation formula for background value was deduced using non-homogeneous exponent function and non-equidistant GM(1,1) model was established [7]. Non-equidistant GM(1,1) model was established using the coefficient λ in the optimized background value as $z^{(1)}(t_k)=\lambda x^{(1)}(t_k)+(1-\lambda)x^{(1)}(t_{k-1})$ where $\lambda\in[0,1]$ and the correction term $\beta=[\beta_1,\beta_2,\cdots,\beta_n]^{\mathbf{T}}$ in the initial condition [8]. But because the first component of the sequence $\mathbf{x}^{(1)}$ is taken as initial condition of grey differential equation in this model, it is inadequate for utilizing new information according to new information priority principle in the grey system. Equidistant new information GM(1,1) model regarded the nth component of the sequence $\mathbf{x}^{(1)}$ as initial condition of grey differential equation was established [9,10]. Multivariable equidistant new information MGM (1, n) model was built, where the nth component of the sequence $\mathbf{x}^{(0)}$ is regarded as initial condition of grey differential equation and the initial value and the coefficient

q of the background value are optimized where the form of the background value is $z_i^{(1)} = qx_i^{(1)}(k+1) + (1-q)x_i^{(1)}(k)$ where $(q \in [0,1])$ [10, 11]. Homogeneous exponent function fitting one-time accumulated generating sequence was used to establish GM (1, 1) with the nth component of $\mathbf{x}^{(0)}$ regarded as initial condition and the optimization initial value [12]. This paper absorbed the ideas constructing background value of non-equidistant GM (1,1) model in [7], adopted the optimization formula that non-homogeneous exponential function constructing the background value and the initial condition of grey differential equation as the nth component of $\mathbf{x}^{(0)}$, and then corrected them. Non-equidistant new information optimum GM(1,1) model based on the minimum relative error was established. This model with high precision as well as high adaptability has a better practical and theoretical significance. Example validates the practicability and reliability of the proposed model.

2. NON-EQUIDISTANT NEW INFORMATION OPTIMUM GM(1,1) MODEL

Definition 1: Supposed the sequence $X^{(0)} = [x^{(0)}(t_1), \cdots, x^{(0)}(t_m)]$, if $\Delta t_i = t_i - t_{i-1} \neq const$, where $i = 2, \cdots, m$, $X^{(0)}$ is called as non-equidistant sequence.

Definition 2: Supposed the sequence $X^{(1)} = [x^{(1)}(t_1), x^{(1)}(t_2), \cdots, x^{(1)}(t_n)]$, if $x^{(1)}(t_1) = x^{(0)}(t_1)$ and $x^{(1)}(t_{k+1}) = x^{(1)}(t_k) + x^{(0)}(t_{k+1}) \cdot \Delta t_{k+1}$, where $k = 1, \cdots, m-1$, $X^{(1)}$ is one-time accumulated generation of non-equidistant sequence.

Supposed the original data sequence $\mathbf{X}^{(0)} = [x^{(0)}(t_1), \cdots, ^{(0)}(t_m)]$ where $x^{(0)}(t_j)(j = 1, 2, \cdots m)$ is the observation value at t_j, m is the data number, and the sequence $[x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_m)]$ is non-equidistant, that is, the spacing $t_j - t_{j-1}$ is not constant.

In order to establish the model, firstly the original data is accumulated one time to generate a new sequence as:

$$\mathbf{X}^{(1)} = [x^{(1)}(t_1), x^{(1)}(t_2), \cdots, x^{(1)}(t_m)] \tag{1}$$

Where, $x^{(1)}(t_j)(j = 1, 2, \cdots, m)$ meets the conditions in the Definition 2, that is,

$$x^{(1)}(t_k) = \begin{cases} x^{(0)}(t_1) + \sum_{j=2}^{k} x^{(1)}(t_j)(t_j - t_{j-1}) & (k = 2, \cdots, m) \\ x^{(0)}(t_1) & (k = 1) \end{cases} \tag{2}$$

Accounting to one-time accumulated generation, a non-equidistant GM(1,1) model is established as a first-order grey differential equation as: $\frac{dx^{(1)}}{dt}+a\zeta^{(1)}=b$,where $\zeta^{(1)}$ is the background value. Its albino differential equation is as:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{3}$$

The differential form is as:

$$x^{(0)}(t_k)+az^{(1)}(t_k)=b \tag{4}$$

Where $z^{(1)}(t_k)=0.5(x^{(1)}(t_k)+x^{(1)}(t_{k-1}))$, $z^{(1)}(t_k)$ is called as the background value in non-equidistance GM (1,1), that is, the mean of accumulated generation sequence.

If $\hat{\mathbf{a}}=[a,b]^{\mathbf{T}}$ is the parameter in non-equidistance GM (1,1). The most least-squares estimation of the model is:

$$\hat{\mathbf{a}}=(\mathbf{B}^{\mathbf{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{Y} \tag{5}$$

Where

$$\mathbf{B}=\begin{bmatrix} -\frac{1}{2}(x^{(1)}(t_2)+x^{(1)}(t_1)) & 1 \\ -\frac{1}{2}(x^{(1)}(t_3)+x^{(1)}(t_2)) & 1 \\ \cdots & \cdots \\ -\frac{1}{2}(x^{(1)}(t_m)+x^{(1)}(t_{m-1})) & 1 \end{bmatrix}, \quad \mathbf{Y}=\begin{bmatrix} x^{(0)}(t_2) \\ x^{(0)}(t_3) \\ \cdots \\ x^{(0)}(t_m) \end{bmatrix}$$

The time response equation of Eq.(4) is:

$$\hat{x}^{(1)}(t_k)=\frac{\hat{b}}{\hat{a}}+(x^{(0)}(t_1)-\frac{\hat{b}}{\hat{a}})e^{-a(t_k-t_1)}, \quad (k=1,2,\cdots,m)$$

In order to choose the best initial conditions, the initial value is amended [12]. $x^{(0)}(t_1)$ instead of $x^{(0)}(t_1)+\beta$, the above equation becomes:

$$\hat{x}^{(1)}(t_k)=\frac{\hat{b}}{\hat{a}}+(x^{(0)}(t_1)+\beta-\frac{\hat{b}}{\hat{a}})e^{-a(t_k-t_1)}, (k=1,2,\cdots,m) \tag{6}$$

After restored, the fitting value of the original data is:

$$\hat{x}^{(0)}(t_k)=\begin{cases} x^{(0)}(t_1)+\beta & (k=1) \\ \dfrac{(x^{(0)}(t_1)+\beta-\frac{\hat{b}}{\hat{a}})(1-e^{a\Delta t_k})}{\Delta t_k} & (k=2,3,\cdots,m) \end{cases} \tag{7}$$

According to the new information priority principle of the grey system, after making full use of new information, the time response equation of Eq.(4) is:

$$\hat{x}^{(1)}(t_k)=\frac{\hat{b}}{\hat{a}}+(x^{(0)}(t_m)+\beta-\frac{\hat{b}}{\hat{a}})e^{-a(t_k-t_m)}, \quad (k=1,2,\cdots,m) \tag{8}$$

After restored, the fitting value of the original data is:

$$\hat{x}^{(0)}(t_k)=\begin{cases}\lim\limits_{\Delta t\to 0}\dfrac{x^{(1)}(t_1)-x^{(1)}(t_1-\Delta t)}{\Delta t} & (k=1)\\ \dfrac{(x^{(0)}(t_m)+\beta-\dfrac{\hat{b}}{\hat{a}})(1-e^{a\Delta t_k})e^{-a(t_k-t_m)}}{\Delta t_k} & (k=2,3,\cdots,m)\end{cases} \tag{9}$$

The absolute error of the fitting data:

$$q(t_k)=\hat{x}^{(0)}(t_k)-x^{(0)}(t_k) \tag{10}$$

The relative error of the fitting data（%）:

$$e(t_k)=\frac{\hat{x}^{(0)}(t_k)-x^{(0)}(t_k)}{x^{(0)}(t_k)}\cdot 100 \tag{11}$$

The mean of the relative error of the fitting data column:

$$f=\frac{1}{m}\sum_{k=1}^{m}\left|e_i(k)\right| \tag{12}$$

The optimization model based on the objective function for minimizing mean value of the relative error f and the variables as the amended initial values is obtained, the solution can be obtained by using the optimization function in Matlab.

3. OPTIMIZATION OF THE BACKGROUND VALUE IN NON-EQUIDISTANT NEW INFORMATION GM(1,1) MODEL

The essence of general mean generation of the background value is calculating approximately the area enclosed by $x^{(1)}(t)$ in $[t_k,t_{k+1}]$ and t-axis by using trapezoid formula. It is appropriate of the background value structured when the time interval is very small and the change in the sequence data is gentle, but when the sequence data change dramatically the background value constructed often make the model error greater and the model parameters of the background value in new information model is different from in other model by using the traditional method, which is not reasonable. Therefore, the construction of the background value in non-equidistant new information GM(1,1) model must be researched.

The albino differential equation as $\frac{dx^{(1)}}{dt}+ax^{(1)}=b$ is integrated on both sides in $[t_k,t_{k+1}]$ and simplified to obtain:

$$x^{(0)}(t_k)\Delta t_k+a\int_{t_{k-1}}^{t_k}x^{(1)}dt=b\Delta t_k \tag{13}$$

Compared with grey differential equation $x^{(0)}(t_k)+az^{(1)}(t_k)=b$ with Eq.(13),

$\int_{t_{k-1}}^{t_k} x^{(1)} dt$ as the background value is more responsive to the albino differential equation.

The solution from $x^{(0)}(t_k)\Delta t_k + a\int_{t_{k-1}}^{t_k} x^{(1)} dt = b\Delta t_k$ meet the non-homogeneous exponential form according on Eq.(8). Supposed $x^{(1)}(t_k) = Ge^{-a(t_k - t_m)} + C$, its regressive sequence is:

$$x^{(0)}(t_k) = \frac{x^{(1)}(t_k) - x^{(1)}(t_{k-1})}{\Delta t_k} = \frac{G(1-e^{a\Delta t_i})}{\Delta t_k} e^{-a(t_k - t_m)} = g_k e^{-a(t_k - t_m)} \tag{14}$$

Where

$$g_k = \frac{G(1-e^{a\Delta t_k})}{\Delta t_k} = \frac{G(1-(1+(a\Delta t_k)+\frac{(a\Delta t_k)^2}{2!}+\cdots)}{\Delta t_k},$$

When the values of a and Δt_k are smaller, the first two terms are taken after expanding $e^{a\Delta t_i}$ to obtain:

$$g_k = \frac{G(1-e^{a\Delta t_k})}{\Delta t_k} = \frac{G(-a\Delta t_k)}{\Delta t_k} = -Ga,\ \frac{x^{(0)}(t_k)}{x^{(0)}(t_{k-1})} = \frac{e^{-a(t_k - t_m)}}{e^{-a(t_{k-1} - t_m)}} = e^{-a\Delta t_k}$$

Then

$$a = -\frac{\ln x^{(0)}(t_k) - \ln x^{(0)}(t_{k-1})}{\Delta t_k}, (k = 2,3,\cdots,m) \tag{15}$$

That Eq.(15) substituting Eq.(14) can obtain:

$$g_k = \frac{x^{(0)}(t_k)}{e^{-a(t_k - t_m)}} = \frac{x^{(0)}(t_k)}{[x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_k - t_m}{\Delta t_k}}}, G = \frac{x^{(0)}(t_k)\Delta t_k [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_k - t_m}{\Delta t_k}}}{1-\frac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \tag{16}$$

According to the initial condition $x^{(1)}(t_m) = Ge^{-a(t_m - t_m)} + C = G + C$, the following equation can be obtained:

$$C = x^{(0)}(t_m) - G = x^{(0)}(t_m) - \frac{x^{(0)}(t_k)\Delta t_k [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_m - t_k}{\Delta t_k}}}{1-\frac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \tag{17}$$

That Eq.(15) and Eq.(17) substituting the calculation of background value $\int_{t_{k-1}}^{t_k} x^{(1)} dt$

can obtain:

$$z^{(1)}(t_k)=\int_{t_{k-1}}^{t_k} x^{(1)}dt=-\frac{\Delta t_k x^{(0)}(t_k)}{a}+C\Delta t_k$$
$$=\frac{(\Delta t_k)^2 x^{(0)}(t_k)}{\ln x^{(0)}(t_k)-\ln x^{(0)}(t_{k-1})}+x^{(0)}(t_m)\Delta t_k-\frac{x^{(0)}(t_k)(\Delta t_k)^2[x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_m-t_k}{\Delta t_k}}}{1-\dfrac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \quad (18)$$

The most least-squares estimation parameter in grey differential equation $x^{(0)}(t_k)\Delta t_k + a\int_{t_{k-1}}^{t_k} x^{(1)}dt = b\Delta t_k$ is:

$$\hat{\mathbf{a}}=(\mathbf{B}^{\mathbf{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{Y} \quad (19)$$

Where

$$\mathbf{B}=\begin{bmatrix}-z^{(1)}(t_2) & \Delta t_2\\ -z^{(1)}(t_2) & \Delta t_3\\ \cdots & \cdots\\ -z^{(1)}(t_m) & \Delta t_m\end{bmatrix}, \quad \mathbf{Y}=\begin{bmatrix}x^{(0)}(t_2)\Delta t_2\\ x^{(0)}(t_3)\Delta t_3\\ \cdots\\ x^{(0)}(t_m)\Delta t_m\end{bmatrix}$$

That Eq.(19) substituting Eq.(8-12) can obtain the optimization model of non-equidistant GM(1,1). After optimized some data such as the simulation value, the predicted value and the error can be obtain, and then the model is tested [1].

4. EXAMPLE

P. G. Foleiss researched that there is the influence of the temperature on fatigue strength under the long life symmetry cycle of many materials. Table 1 shows the experimental data of the change relation of Ti alloy fatigue strength along with temperature, which is a sequence of non-equidistant spacing. The data in [2, 3] were modeled by using the method proposed in this paper and we obtained the following result:

a= 9.2684e-004, b= 431.6988， $\beta=$-1562.707

Table 1 Change relation of Ti alloy fatigue strength along with temperature

T	100	130	170	210	240	270	310	340	380
σ_{-1}	560	557.54	536.10	516.10	505.60	486.1	467.4	453.8	436.4

The fitting value of the original data:

$\hat{x}^{(0)}(t_k)=$[561.0433, 553.2382, 535.5928, 516.1, 499.6141, 485.9136, 470.4155, 455.3889, 440.8644, 428.7565]

The absolute error of the fitting data:

$q(t_k) =$ [1.0433, -4.3018, 0.50718, -.482e-009, -5.9859, -0.18643, 3.0155, 1.5889, 4.4644]

The relative error of the fitting data（%）:

$e(t_k) =$ [0.1863, -0.77157, -0.094605, 8.6843e-010, -1.1839, -0.038351, 0.64517, 0.35014, 1.023]

The mean of the relative error of the fitting data is 0.47701%. If not optimizing $\beta = 0$, The mean of the relative error is 0.54572%.

After the original data was pre-processed by using $t = \frac{T-50}{50}$ and $X^{(0)} = \frac{\sigma_{-1} - 400}{50}$ in [2], the maximum relative error is 4.86% and the mean relative error is 3.19%. The model was established by using the function transformation method in [3] and the mean relative error is 0.6587%. Homogeneous exponent function fitting one-time accumulated generating sequence was used in [5] and it is 0.9765%. Thus, the example validates the adaptability and the scientific of the proposed model.

5. CONCLUSIONS

In this paper, applying the principle in which new information should be used fully and modeling method of grey system, a non-equidistant new information optimum GM(1,1) model was put forward in which the nth component modified by optimization is taken as the initialization and the relation error to least is taken as objective function. The MATLAB program of this model was written. As the background value is an important factor affecting the precision of grey system model, based on index characteristic of grey model and the definition of integral, the background value in non-equidistant GM(1,1) is researched and the discrete function with non-homogeneous exponential law is used to fit the accumulated sequence and optimum formula is given. The background value of new information GM(1,1) with can be used in non-equal interval & equal interval time series. The model proposed in this paper has the characteristic of high precision as well as high adaptability. Example validates the correctness and validity of the proposed model.There are important practical and theoretical significance and this model should be widely used.

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