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STANDARDIZED EXPONENTIATED GUMBEL ERROR INNOVATION DISTRIBUTION IN MODELLING VOLATILITY MODELS

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Abstract: Here we proposed three classes of volatility models using Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID). Important statistical and mathematical properties of this models have been discussed and derived. Hence, the parameters of the volatility models are discussed using the general log likelihood function of the SEGEID. Finally, the volatility models were obtained through partial derivative by adopting a method of numerical method BFGS.

Keywords: exponentiated Gumbel distribution; mathematical modelling; numerical method.

2010 AMS Subject Classification: 93A30.

1. INTRODUCTION

In the couple of years, numerous error innovation distributions have been proposed by different scholars to model different type of real-life data depending on the situation or nature of the data. The nature could be because of recession, political crisis, war, Covid-19 Pandemic to mention but a few which could result to outliers in the data set. Some of these error innovations early proposed are normal error innovation proposed by Engle [1] to model the United Kingdom inflation rate. Similarly, Bollerslev [2] introduced student-t error innovation distribution to address the limitation

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found in normal error innovation distribution for instance it is leptokurtic nature. Nelson [3] then introduced generalized Error Distribution which seems to be more fulfilled in term of satisfying the stationarity condition. It was discovered that all this above-mentioned error innovation distribution was still having serious limitation, which is the fact that they were all still having outliers and because of this, the skewed part of the distribution was introduced as skewed normal error innovation distribution by O'Hagan and Leonard [4], skewed student-t error innovation distribution by Hansen [5] and skewed Generalized Error Distribution by Theodossiou [6]. Among these, skewed student-t error innovation distribution outperformed others because it captures some of the outliers in the data set.

Due to the nature surrounding data in financial investment on daily bases, standardized exponentiated gumbel error innovation distribution proposed by Olayemi and Olubiyi [7] will be used to model some selected asymmetric volatility models in this paper. The selected volatility models are Exponential Generalized Autoregressive Conditional Heteroskedasticity EGARCH, Threshold Generalized Autoregressive Conditional Heteroskedasticity TGARCH and Glosten, Jagannathan and Runkle – Generalized Autoregressive Conditional Heteroscedasticity model GJR-GARCH.

2. MATERIALS AND METHODS

2.1 Exponentiated G distribution

Gupta et. al., [8] computed the PDF of the Exponentiated G distribution where a class of Exponentiated distribution was introduced. They specified the CDF and PDF as follow;

$$F(x) = G(x)^{\alpha}; \qquad (1)$$

where $u > 0$ is a shape parameter and $G(x)$ is the CDF of any baseline distribution. To obtain the PDF of the above CDF is by taking the differential equation with respect to x to give;

$$f(x) = uG(x)^{\alpha-1}g(x) \qquad (2)$$

where $u > 0$ is a shape parameter and $g(x) = \frac{dG(x)}{dx}$ (3)

2.1.1 Exponentiated Gumbel Distribution (EGD):

Nadarajah [9] applied this Exponentiated G distribution to transformed Gumbel Distribution into Exponentiated Gumbel Distribution (EGD). The CDF of Exponentiated Gumbel Distribution is given as:

$$F(x,\alpha,\mu,\sigma) = 1-\left[1-exp\left\{-exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]^{\alpha}, \alpha>0, -\infty<x<\infty, -\infty<u<\infty \quad (4)$$

and the density function (pdf) as:

$$f(x,\alpha,\mu,\sigma) = \frac{\alpha}{\sigma}\left[1-exp\left\{-exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]^{\alpha-1} exp\left(-\frac{x-\mu}{\sigma}\right) exp\left\{-exp\left(-\frac{x-\mu}{\sigma}\right)\right\} \quad (5)$$

where α is the shape parameter and σ is the scale parameter.

2.1.2 Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID) And Its Log Likelihood Function

The standardized exponentiated Gumbel Error Innovation Distribution was proposed by Olayemi and Olubiyi [7] as:

$$g\left(\varepsilon_t;\alpha,\sigma_t\right) = \frac{\alpha}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\left[1-\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\right]^{\alpha-1}\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right) \quad (6)$$

And the log likelihood as:

$$L(\theta) = \prod_{t=1}^{n} g(\varepsilon_t;\alpha,\sigma_t) = L(\varepsilon_t;\alpha,\sigma_t) = \prod_{t=1}^{n} g(\varepsilon_t;\theta) \quad (7)$$

$$\begin{aligned} LL\left(\varepsilon_t;\phi\right) = {} & n\log\left(\alpha\right) - \frac{n}{2}\log\left(\sigma_t^2\right) + \\ & (\alpha-1)\sum_{t=1}^{n}\log\left(1-\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\right) + \\ & \sum_{t=1}^{n}\log\left(\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right) + \sum_{t=1}^{n}\log\left(\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\right) - \frac{n}{2}\log\left(\sigma_t^2\right) \end{aligned} \quad (8)$$

where $\theta = (\alpha, \sigma_t)$, α is the shape parameter, σ_t^2 are the itemized volatility models with vector parameters

$\emptyset = (\omega, \theta_1, \theta_2, \dots, \beta_1, \beta_2, \dots, \gamma, \delta)$ respectively.

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3. RESULTS AND DISCUSSION

3.1 GJR-GARCH (1,1)

GJR-GARCH (1,1) model was proposed by Glosten et al., [10] as $\sigma_t^2 = \omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$.

Therefore, let σ_t^2 be GJR-GARCH (1,1) model given as $\sigma_t^2 = \omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$ (9)

Then, Substitute equation (9) into the equation (8) general form of the log-likelihood function of SEGEID and differentiate it partially with respect to the parameter below $\alpha, \omega, \theta_1, \gamma$ and β_1

$$L(\varepsilon_t;\theta) = nlog(\alpha) - \frac{n}{2}\log(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\ ) + (\alpha-1)\sum_{t=1}^{n} log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right] + \sum_{t=1}^{n} log\left\{exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\} + \sum_{t=1}^{n}\left[exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right] - \frac{1}{2}nlog(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\ ) \quad (10)$$

Differentiating the above equation (10) partially with respect to $\alpha, \omega, \theta_1, \gamma and \beta_1$ then equating to zero

Let

$$u = exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right), w = \frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}$$

So,

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{t=1}^{n} log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right] = 0 \quad (11)$$

$$\frac{\partial l}{\partial \omega} = -\frac{n}{2}\left(\frac{1}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) + (\alpha-1)\sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1-\exp(-u)}\right)\left(\frac{\varepsilon_t}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] = 0 \quad (12)$$

$$\frac{\partial l}{\partial \theta_1} = -\frac{n}{2}\left(\frac{\varepsilon_{t-1}^2}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) + (\alpha - 1)\sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{\varepsilon_t\varepsilon_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] + \sum_{t=1}^{n}\left[\left(\frac{-\exp(-u)}{\exp(-u)}\right)\left(\frac{\varepsilon_t\varepsilon_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right)\right] + \sum_{t=1}^{n}\left[(-\exp(-u))\left(\frac{\varepsilon_t\varepsilon_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] - \frac{ne_{t-1}^2}{2}\left(\frac{1}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) = 0 \quad (13)$$

$$\frac{\partial l}{\partial \gamma} = -\frac{n}{2}\left(\frac{\varepsilon_{t-1}^2}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) + (\alpha - 1)\sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{\varepsilon_t\varepsilon_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] - \frac{ne_{t-1}^2}{2}\left(\frac{1}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) = 0 \quad (14)$$

$$\frac{\partial l}{\partial \beta_1} = -\frac{n}{2}\left(\frac{\sigma_{t-1}^2}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) + (\alpha - 1)\sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{\varepsilon_t\sigma_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] + \sum_{t=1}^{n}\left[\left(\frac{-\exp(-u)}{\exp(-u)}\right)\left(\frac{\varepsilon_t\sigma_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right)\right] + \sum_{t=1}^{n}\left[(-\exp(-u))\left(\frac{\varepsilon_t\sigma_{t-1}^2}{(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2)^2}\right) exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right] - \frac{ne_{t-1}^2}{2}\left(\frac{1}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right) = 0 \quad (15)$$

3.2 **EGARCH (1,1) model:**

Nelson [3] introduced EGARCH (1,1) model as $\sigma_t^2 = \omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$.

Now, Let σ_t^2 be EGARCH (1,1) model given as $\sigma_t^2 = \omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$ (16)

Then, Substitute equation (16) into the equation (8) general form of the log-likelihood function of SEGEID and differentiate it partially with respect to the parameter below ω, θ_1, γ and β_1

$$L(\varepsilon_t;\theta) = nlog(\alpha) - \frac{n}{2}\log\left(\omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\right)$$
$$+ (\alpha - 1)\sum_{t=1}^{n} log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right]$$
$$+ \sum_{t=1}^{n} log\left\{exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}$$
$$+ \sum_{t=1}^{n}\left[exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right]$$
$$- \frac{1}{2}nlog\left(\omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\right) \quad (17)$$

Differentiating the above equation (17) partially with respect to $\alpha, \omega, \theta_1, \gamma\ and \beta_1$ then equating to zero

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{t=1}^{n} log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{\alpha + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2}\right)\right\}\right] = 0 \quad (18)$$

$$\frac{\partial l}{\partial \omega} = -\frac{n}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)}$$
$$+ (\alpha - 1)\sum_{t=1}^{n}\left[\begin{array}{c}\frac{\exp(-u)}{1 - \exp(-u)}\left(\frac{\varepsilon_t}{\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2}\right) \\ exp\left(-\frac{\varepsilon_t}{\alpha + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2}\right)\end{array}\right]$$
$$= 0 \quad (19)$$

$$\frac{\partial l}{\partial \theta_1} = -\frac{n\left(\varepsilon_{t-1} - \sqrt{2\pi}\right)}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)}$$

$$+ (\alpha - 1)\sum_{t=1}^{n}\left[\begin{array}{c}\frac{\exp(-u)}{1-\exp(-u)}\left(\frac{\left(\varepsilon_{t-1} - \sqrt{2\pi}\right)\varepsilon_t}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right) \\ exp\left(-\frac{\varepsilon_t}{\alpha + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2}\right)\end{array}\right]$$

$$+ \sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1-\exp(-u)}\left(\frac{n\left(\varepsilon_{t-1} - \sqrt{2\pi}\right)}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right)\right)\right]$$

$$+ \sum_{t=1}^{n}\left[\begin{array}{c}-\exp(-u)\left(\frac{\varepsilon_t\left(\varepsilon_{t-1} - \sqrt{2\pi}\right)}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right) \\ exp\left(-\frac{\varepsilon_t}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)}\right)\end{array}\right]$$

$$- \frac{n\left(\varepsilon_{t-1} - \sqrt{2\pi}\right)}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)} = 0 \qquad (20)$$

$$\frac{\partial l}{\partial \gamma} = -\frac{n(\varepsilon_{t-1})}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)}$$

$$+ (\alpha - 1)\sum_{t=1}^{n}\left[\begin{array}{c}\frac{\exp(-u)}{1-\exp(-u)}\left(\frac{(\varepsilon_{t-1})\varepsilon_t}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right) \\ exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2}\right)\end{array}\right]$$

$$+ \sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{1-\exp(-u)}\left(\frac{n(\varepsilon_{t-1})}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right)\right)\right]$$

$$+ \sum_{t=1}^{n}\left[\begin{array}{c}-\exp(-u)\left(\frac{\varepsilon_t(\varepsilon_{t-1})}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)^2}\right) \\ exp\left(-\frac{\varepsilon_t}{\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)}\right)\end{array}\right]$$

$$- \frac{n(\varepsilon_{t-1})}{2\left(\omega + \theta_1\left(\varepsilon_{t-1} - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2\right)} = 0 \qquad (21)$$

Let,

$$Q = \omega + \theta_1\left(\varepsilon_{t-1}^2 - \sqrt{2\pi}\right) + \gamma\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2 \qquad (22)$$

And

$$u = exp\left(-\frac{\varepsilon}{Q}\right) \quad (23)$$

$$\frac{\partial l}{\partial \beta_1} = -\frac{n\varepsilon_{t-1}^2}{2Q} + (\alpha-1)\sum_{t=1}^{n}\left[\frac{\exp(-u)}{1-\exp(-u)}\left(\frac{\varepsilon_t\sigma_{t-1}^2}{Q^2}\right)exp\left(-\frac{\varepsilon_t}{Q}\right)\right] + \sum_{t=1}^{n}\left[\left(\frac{\exp(-u)}{\exp(-u)}\left(\frac{\varepsilon_t\sigma_{t-1}^2}{Q^2}\right)\right)\right] + \sum_{t=1}^{n}\left[-\exp(-u)\left(\frac{\varepsilon_t\sigma_{t-1}^2}{Q^2}\right)exp\left(-\frac{\varepsilon_t}{Q}\right)\right] - \frac{n\sigma_{t-1}^2}{2(Q)} = 0 \quad (24)$$

3.3 **TGARCH (1,1) model**

Zakoian [11] proposed TGARCH (1,1) model as $\sigma_t^2 = \omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$. Therefore, Let σ_t^2 be TGARCH (1,1) model be given as $\sigma_t^2 = \omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$ (25)

Then, Substitute equation (25) into the equation (8) general form of the log-likelihood function of SEGEID and differentiate it partially with respect to the parameter below $\alpha, \omega, \theta_1, \gamma$ and β_1.

$$L(\varepsilon_t;\theta) = nlog(\alpha) - \frac{n}{2}\log(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2) + (\alpha-1)\sum_{t=1}^{n}log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right] + \sum_{t=1}^{n}log\left\{exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\} + \sum_{t=1}^{n}\left[exp\left\{-exp\left(-\frac{\varepsilon_t}{\omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}\right)\right\}\right] - \frac{1}{2}nlog(\omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2) \quad (26)$$

Let,

$$R = \omega + \theta_1\varepsilon_{t-1}^2 + \gamma S_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (27)$$

$$u = exp\left(-\frac{\varepsilon_t}{R}\right) \quad (28)$$

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{t=1}^{n} log\left[1 - exp\left\{-exp\left(-\frac{\varepsilon_t}{R}\right)\right\}\right] = 0 \quad (29)$$

$$\frac{\partial l}{\partial \omega} = -\frac{n}{2R} + (\alpha - 1)\sum_{t-1}^{n}\left[\left(\frac{exp(-u)}{1 - exp(-u)}\right)\left(\frac{\varepsilon_t}{R}\right) exp\left(-\frac{\varepsilon_t}{R}\right)\right] = 0 \quad (30)$$

$$\frac{\partial l}{\partial \theta_1} = -\frac{n\varepsilon_{t-1}^2}{2R} + (\alpha - 1)\sum_{t-1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{\varepsilon_{t-1}^2\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right)\right] + \sum_{t-1}^{n}\left[\left(-\frac{\exp(-u)}{\exp(-u)}\right)\left(\frac{\varepsilon_{t-1}^2\varepsilon_t}{R^2}\right)\right] + \sum_{t=1}^{n} -\exp(-u)\left(\frac{\varepsilon_{t-1}^2\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right) - \frac{n}{2}\varepsilon_{t-1}^2\left(\frac{1}{R}\right) = 0 \quad (31)$$

$$\frac{\partial l}{\partial \gamma} = -\frac{nS_{t-1}\varepsilon_{t-1}^2}{2R} + (\alpha - 1)\sum_{t-1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{(S_{t-1}\varepsilon_{t-1}^2)\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right)\right] + \sum_{t-1}^{n}\left[\left(-\frac{\exp(-u)}{\exp(-u)}\right)\left(\frac{S_{t-1}\varepsilon_{t-1}^2\varepsilon_t}{R^2}\right)\right] + \sum_{t=1}^{n} -\exp(-u)\left(\frac{S_{t-1}\varepsilon_{t-1}^2\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right) - \frac{n}{2}S_{t-1}\varepsilon_{t-1}^2\left(\frac{1}{R}\right) = 0 \quad (32)$$

$$\frac{\partial l}{\partial \beta_1} = -\frac{n\sigma_{t-1}^2}{2R} + (\alpha - 1)\sum_{t-1}^{n}\left[\left(\frac{\exp(-u)}{1 - \exp(-u)}\right)\left(\frac{\sigma_{t-1}^2\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right)\right] + \sum_{t-1}^{n}\left[\left(-\frac{\exp(-u)}{\exp(-u)}\right)\left(\frac{\sigma_{t-1}^2\varepsilon_t}{R^2}\right)\right] + \sum_{t=1}^{n} -\exp(-u)\left(\frac{\sigma_{t-1}^2\varepsilon_t}{R^2}\right) exp\left(-\frac{\varepsilon_t}{R}\right) - \frac{n}{2}\sigma_{t-1}^2\left(\frac{1}{R}\right) = 0 \quad (33)$$

The GRJ-GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) models above are the partial derivatives of the parameters of these selected asymmetric GARCH models but due to the complication involved we were unable to get the precise solution of the parameters. hence, we will apply a method of numerical solution BFGS, that is, algorithm published by Broyden, Fletcher, Goldfarb, and Shannon [12] to obtain the estimated values of the parameters.

4. CONCLUSIONS

This paper uses proposed error innovation called Standardized Exponentiated Gumbel Error Innovation Distribution (SEGEID) to model the theoretical property of some selected asymmetric volatility models.

A method of log likelihood function of the error innovation was applied by using partial derivative to obtain the parameters of the volatility selected.

The newly proposed volatility models will be applied to model some financial investment data in the further research and then compare with the existing Error innovation distribution.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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