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ON PAIRWISE-ω-PERFECT FUNCTIONS

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Abstract. In this article, we will present a new form of functions called pairwise$-$ $\omega-$ perfect functions and pairwise M-$\omega-$perfect functions. We will give some properties of this functions, and we will looking for homeomorphism of different bitopological spaces under the effect these functions. Last but not least, we give the characterizations of product theorems.

Keywords: bitopological spaces; ω-open sets; pairwise perfect functions; pairwise$-\omega-$ perfect functions.

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1. Introduction and Preliminaries

Firstly, Kelly [10] established the bitopological spaces by generalised any characteristics in single topology into bitopological spaces.For examples for these topices,species of Hausdorff space, continuous functions, lindelöf, compactness, countably compact, normal, and others topices that we can,t count it. In this research it will be an abbreviation of pairwise by p-, for example p-perfect functions, it is means pairwise perfect functions.

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If (S,η_1,η_2) is a bitopological space and $N \subseteq S, cl_1(N)$ and $cl_2(N)$ will denote the closure of N with respect to η_1 and η_2 respectively.Let (S,η) be a topological space and let N be a subset of S. A point $s \in (S,\eta_1,\eta_2)$ is called a condensation point of N, if for each K $\in \eta$ with $s \in K$, the set K $\cap$ N is uncountable. Hdeib presented ω-closed sets and ω-open sets as: N is called ω-closed if it contains all its condensation points. The complement of an ω-closed set is called ω-open. also $cl^{\omega}\ N$ will denote the intersection of all ω-closed sets which contains N. The family of all $\omega-$open sets in (S,η) is denoted by$W\ (\eta)$.In [7] Datta defined p-closed functions, $p-$open sets, and in [8] Fletcher presented $p-$continuous functions, in addition of these in [9] Fora and Hdeib gived p- compact and p-lindelöf. Recently, A.Atoom and H.Z.Hdeib constructed the perfect functions in the bitopological spaces by a function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is called $p-$perfect, if Ω is $p-$continuous, $p-$closed, and for each $t \in T$, $\Omega^{-1}(t)$ is $p-$compact. In this work, we will be presenting pairwise- $\omega-$ perfect functions, and characterizations of pairwise- M- $\omega-$ perfect functions.

2. Definitions and Results

Definition 2.1. *A subset N of a bitopological space (S,η_1,η_2) is pairwise- $\omega-$open,(simply $p-\omega-$open) if for each $s \in N$ there exists a pairwise$-$open subset K_s containing s such that $K_s - N$ is a countable set. The complement of a pairwise $\omega-$open is said to be pairwise- $\omega-$closed set(simply $p-\omega-$closed) .The family of all pairwise $\omega-$open (respectively pairwise $\omega-$closed) subsets of a space (S,η_1,η_2) is denoted by p- $\omega - OP(S)$, (respectively $p-\omega-CL(S)$).Also the family of all pairwise- $\omega-$open sets of (S,η_1,η_2) containing s is denoted by $p-\omega-OP(S;s)$.*

Definition 2.2. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is a pairwise- $\omega-$closed function, if it functions pairwise closed sets onto pairwise- $\omega-$closed sets.*

Definition 2.3. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is p-weakly continuous function if for every p-open set $K \subset T$, $\Omega^{-1}(K)$ is p-$\omega-$open.*

Definition 2.4. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is p-strongly$-\omega-$continuous function if for every p-$\omega-$open set $K \subset T$,$\Omega^{-1}(K)$ is p-open.*

Definition 2.5. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is called $p-\omega-$continuous at point $s \in (S,\eta_1,\eta_2)$, if for every p-open set Lcontaining $\Omega(s)$, there is $p-\omega-$open set K contaning s such that $\Omega(K) \subset L$.If $\Omega$is $p-\omega-$continuous at each point of (S,η_1,η_2), then Ω is said to be $p-\omega-$continuous on (S,η_1,η_2).*

Definition 2.6. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is called $p-\omega-$continuous (resp. $p-\omega-$irresolute) if, $\Omega_1 : (S,\eta_1) \to (T,\gamma_1)$ and $\Omega_2 : (S,\eta_2) \to (T,\gamma_2)$ are $\omega-$continuous (resp. $\omega-$irresolute) functions.*

Definition 2.7. *A family $\hat{N}$ of subsets of a bitopological space (S,η_1,η_2) is called $\eta_1\eta_2-\omega-$open if $\hat{N} \subset W(\eta_1) \cup W(\eta_2)$. If, in addition $\hat{N} \cap W(\eta_1) \neq \phi$ and $\hat{A} \cap W(\eta_2) \neq \phi$then $\hat{N}$ is called pairwise $\omega-$open.*

Definition 2.8. *A bitopological space (S,η_1,η_2) is said to be pairwise -$\omega-$compact, (resp. pairwise M-$\omega-$compact) if each p.ω.open (resp. $\eta_1\eta_2$-$\omega-$open) cover of S has a finite subcover.Clearly every p.M-ω.c. space is p.ω.c., and we can easily show that the converse may not be true.*

Definition 2.9. *A space (S,η_1,η_2) is said to be $p-\omega-$lindelöf if every $p-\omega-$open cover of (S,η_1,η_2) has a countable subcover.*

Definition 2.10. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is said to be p-weakly continuos function if for every p-open set $K \subset T$, $\Omega^{-1}(K)$ is p-$\omega-$open.*

Definition 2.11. *A space (S,η_1,η_2) is said to be $p-\omega-I_1$ if for each pair of distinct points s and t of (S,η_1,η_2), there exist $p-\omega-$open sets Kand Lcontaing s and t, respectively such that $t \notin K$, and $s \notin L$.*

Definition 2.12. *A space (S,η_1,η_2) is said to be $p-\omega-I_2$ if for each pair of distinct points s and t of (S,η_1,η_2), there exist $p-\omega-$open sets Kand L in (S,η_1,η_2)such that $s \in K$ and $t \in L$.*

3. Main Results in Pairwise- ω- Perfect Functions

Definition 3.1. *A function $\Omega : (S,\eta_1,\eta_2) \to (T,\gamma_1,\gamma_2)$ is pairwise $-\omega-$perfect, if Ω is pairwise $-\omega-$continuous, pairwise $-\omega-$closed, and for each $t \in T$, $\Omega^{-1}(t)$ is pairwise$-\omega-$compact.*

Definition 3.2. *A function $\Omega : (S, \eta_1, \eta_2) \to (T, \gamma_1, \gamma_2)$ is called pairwise $-\omega-M$- perfect, if Ω is pairwise $-\omega-$continuous, pairwise $-\omega-$closed, and for each $t \in T$,$\Omega^{-1}(t)$ is pairwise $M-\omega-$compact.*

Theorem 3.3. *If $\Omega : (S, \eta_1, \eta_2) \to (T, \gamma_1, \gamma_2)$ is a pairwise $-\omega-$perfect function, then for every pairwise $-\omega-$compact subset $Q \subseteq T$, the inverse image $\Omega^{-1}(Q)$ is a pairwise $-\omega-$compact .*

Proof. Let $\underset{\sim}{K} = \{K_\theta : \theta \in \Psi\}$ be a p-open cover of (S, η_1, η_2) , because Ω is a pairwise $-\omega-$perfect function, then $\forall t \in T$, $\Omega^{-1}(t)$ is pairwise $-\omega-$ compact, there exists a finite subsets Ψ_t, $\overset{*}{\Psi}_t$ of Ψ , s.t $\Omega^{-1}(t) \subseteq \bigcup_{\theta\in\Psi_t} \{L_\theta : \theta \in \Psi_t\} \bigcup \bigcup_{\theta\in\overset{*}{\Psi}_t} \{E_\theta : \theta \in \overset{*}{\Psi}_t\}$, where $\{L_\theta : \theta \in \Psi_t\}$ is η_1 $-\omega-$ open ,$\{E_\theta : \theta \in \overset{*}{\Psi}_t\}$ is $\eta_2 - \omega-$ open. Let $D_t = T - \Omega(S - \bigcup_{\theta\in\Psi_t} L_\theta)$ is a γ_1 $-\omega-$ open set containing t , and $D_t^* = T - \Omega(S - \bigcup_{\theta\in\Psi_t} E_\theta)$ is a $\gamma_2 - \omega-$ open set containing t , where $\Omega^{-1}(D_t) \subseteq \bigcup_{\alpha\in\Psi_y} L_\theta$, Ω^{-1} $(D_t^*) \subseteq \bigcup_{\alpha\in\overset{*}{\Psi}_y} E_\theta$. Let $\underset{\sim}{D} = \{D_t : t \in T\} \bigcup \{D_t^* : t \in T\}$ is a pairwise $-\omega-$ open cover of T. $\underset{\sim}{D}$ is pairwise$-\omega-$open cover of Q .Since Q is pairwise $-\omega-$compact, $Q \subseteq \bigcup_{i=1}^{n}(D_{t_i}) \bigcup \bigcup_{i=1}^{m}(D_{t_j}^*)$. □

Thus, Ω^{-1} $(Q) \subseteq \bigcup_{i=1}^{n}\Omega^{-1}(D_{t_i}) \bigcup \bigcup_{j=1}^{m}\Omega^{-1}(D_{t_j}^*) \subseteq$ union of finite of $\underset{\sim}{K}$, i.e Ω^{-1} (Q) is pairwise $-\omega-$compact.

Corollary 3.4. *A pairwise $-\omega-$compact space is inverse invariant under pairwise $-\omega-$perfect function .*

Theorem 3.5. *If $\Omega : (S, \eta_1, \eta_2) \to (T, \gamma_1, \gamma_2)$ is a pairwise $M-\omega-$ perfect function, then for every pairwise $-\omega-$compact subset $Q \subseteq T$, the inverse image Ω^{-1} (Q) is a pairwise $M-\omega-$compact .*

Proof. We will use the same techniquein theorem [2.8]. □

Corollary 3.6. *A pairwise $M-\omega-$compact space is constant algebraic expression under pairwise $M-\omega-$perfect function .*

Theorem 3.7. *If* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$*,is pairwise* $-\omega-$*perfect function and* $\Theta : (T,\gamma_1,\gamma_2) \rightarrow (Q,\mu_1,\mu_2)$*is pairwise perfect function,*$\Theta\circ\Omega$ *is pairwise* $-\omega-$*perfect function .*

Proof. Suppose N be any $-\omega-$ μ_1- open set in Q, sinceΘ is pairwise$-\omega-$ perfect function ,then $\Theta^{-1}(N)$ is γ_1- open set in (T,γ_1,γ_2). □

Because Ω is pairwise perfect function, then $\Omega^{-1}(\Theta^{-1}(N))$ η_1- open set in S. The same thing, let G be any be any $-\omega-$ μ_2- open set in Q. Hence $\Theta\circ\Omega$ is pairwise$-\omega-$ perfect function .

Corollary 3.8. *If* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$*,is pairwise* $-\omega-$*semi perfect function and* $\Theta : (T,\gamma_1,\gamma_2) \rightarrow (Q,\mu_1,\mu_2)$*is pairwise perfect function* $\Theta\circ\Omega$ *is pairwise* $M-\omega-$*perfect function .*

Proposition 3.9. *If the composition* $\Theta\circ\Omega$ *of the pairwise* $-\omega-$*continuous funcion,* $\Omega : (S,\eta_1,\eta_2) \overset{onto}{\rightarrow} (T,\gamma_1,\gamma_2)$*, and pairwise continuous* $\Theta : (T,\gamma_1,\gamma_2) \overset{onto}{\rightarrow} (Q,\mu_1,\mu_2)$ *is a pairwise* $-\omega-$ *closed , then the function* $\Theta : (T,\gamma_1,\gamma_2) \overset{onto}{\rightarrow} (Q,\mu_1,\mu_2)$ *is pairwise* $-\omega-$ *closed.*

Proof. Let N be a $\gamma_1-\omega-$ closed in T,then Ω^{-1} (N) is $\eta_1-\omega-$ closed in S.Since $\Theta\circ\Omega$ is pairwise $-\omega-$ closed,then $\Theta(\Omega\Omega^{-1}$ $(N))$ is $\rho_1-\omega-$ closed in Q, i.e $\Theta(N)$ is $\mu_1-\omega-$closed in Q.Simillary, we can show that if G be a $\gamma_2-\omega-$closed in T, then $\Theta(G)$ is $\gamma_2-\omega-$closed in Q .Thus Θ is a pairwise $-\omega-$closed function. □

Theorem 3.10. *If the composition* $\Theta\circ\Omega$ *of the pairwise* $-\omega-$*continuous funcion,*

$\Omega : (S,\eta_1,\eta_2) \overset{onto}{\rightarrow} (T,\gamma_1,\gamma_2)$,and pairwise continuous $\Theta : (T,\gamma_1,\gamma_2) \overset{onto}{\rightarrow} (Q,\mu_1,\mu_2)$ is pairwise $-\omega-$ perfect,

then the function $\Theta : (T,\gamma_1,\gamma_2) \overset{onto}{\rightarrow} (Q,\mu_1,\mu_2)$ is pairwise $-\omega-$ perfect.

Proof. For every $q\in Q$, $\Theta^{-1}(q) =$ Ω $((\Theta\circ\Omega)^{-1}(q))$ $=$ pairwise $-\omega-$ compact, because $\Theta\circ\Omega$ is pairwise $-\omega-$ perfect. Since Θ is pairwise $-\omega-$ closed by previous proposition, we get that Θ is pairwise $-\omega-$ perfect . □

Theorem 3.11. *If* $\Omega : (S,\eta_1,\eta_2) \overset{onto}{\rightarrow} (T,\gamma_1,\gamma_2)$ *is pairwise* $-\omega-$*closed function , then for any* $G\subset T$ *the restriction* Ω_B $:\Omega^{-1}(G) \rightarrow G$ *is pairwise* $-\omega-$*closed .*

Proof. Let $G \subset T$ .Consider the function $\Omega : (S, \eta_1) \rightarrow (T, \gamma_1)$,let S be a $\eta_1 - \omega-$closed. Then $\Omega_G\ (S \bigcap \Omega^{-1}(G)) = \Omega(S) \bigcap G$ is $\gamma_1 - \omega-$ closed in G. □

The same thing, we can show that if S a $\gamma_2 - \omega-$closed, $\Omega_G\ (S \bigcap \Omega^{-1}(G)) = \Omega(S) \bigcap G$ is $\sigma_2 - \omega-$ closed in G. Thus $\Omega_B\ : \Omega^{-1}(G) \rightarrow G$ is pairwise $-\omega-$closed.

Theorem 3.12. *If* $\Omega : (S, \eta_1, \eta_2) \overset{onto}{\rightarrow} (T, \gamma_1, \gamma_2)$ *is pairwise* $-\omega-$*perfect function , then for any* $G \subset T$ *the restriction* $f_B\ :\ f^{-1}(B) \rightarrow B$ *is pairwise* $-\omega-$ *perfect .*

Proof. We will use the same technique in the above theorem . □

Theorem 3.13. *A bitopological space* (S, η_1, η_2) *is p.*$-\omega-$*c. if and only if each proper* $\eta_r - \omega-$*closed subset of* (S, η_1, η_2) *is* $\omega-$*compact relative to* (S, η_p),*where* $r, p = 1, 2; r \neq p$.

Proof. The proof comes from last thoerem. □

Theorem 3.14. *If* $\Omega : (S, \eta_1, \eta_2) \overset{onto}{\rightarrow} (T, \gamma_1, \gamma_2)$ *is pairwise* $-\omega-$*perfect , where* (S, η_1, η_2) *is pairwise* $-\omega-$ *compact, and* (T, γ_1, γ_2) *is pairwise* $-\omega-$ *Hausdorff , then* Ω *is pairwise* $-\omega-$ *closed .*

Proof. If N is $\eta_1 - \omega-$ closed subset of (S, η_1, η_2) ,then it is $\eta_2 - \omega-$compact,because (S, η_1, η_2) is pairwise $-\omega-$ compact .Since Ω is pairwise $-\omega-$continuous .$\Omega(N)$ is a $\gamma_2 - \omega-$ compact subset of (T, γ_1, γ_2).Since (T, γ_1, γ_2) is pairwise $-\omega-$ Hausdorff, then $\Omega(N)$ is a $\gamma_1 - \omega-$closed .Simillary if B is a $\eta_2 - \omega-$ closed subset of S, then $\Omega(G)$ is a $\gamma_2 - \omega-$closed subset of (T, γ_1, γ_2). □

Corollary 3.15. *If* $\Omega : (S, \eta_1, \eta_2) \overset{onto}{\rightarrow} (T, \gamma_1, \gamma_2)$ *is pairwise* $M-\omega-$*perfect ,where* (S, η_1, η_2) *is pairwise*$M-\omega-$*compact, and* (T, γ_1, γ_2) *is pairwise* $-\omega-$ *Hausdorff , then* Ω *is pairwise* $-\omega-$*closed .*

Definition 3.16. *A function* $\Omega : (S, \eta_1, \eta_2) \rightarrow (T, \gamma_1, \gamma_2)$ *is called pairwise* $-\omega-$*homeomorphism, if* Ω *is pairwise continous, pairwise* $-\omega-$*closed(pairwise* $-\omega-$*open), and* Ω*is bijection .*

Theorem 3.17. *Let* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ *be a p-continous bijection function. If* (T,γ_1,γ_2) *is pairwise* $-\omega-$ *Hausdorff space, and* (S,η_1,η_2) *is pairwise* $-\omega-$*compact, then* Ω *is pairwise* $-\omega-$ *homeomorphism function.*

Proof. This is enough to prove that Ω is pairwise $-\omega-$ closed.Let H be a η_r-closed proper subset of S, and hence H is proper $\eta_p - \omega-$compact, for $r,p=1,2; r\neq p.$, by using theorem[3.13], and so, $\Omega(H)$ is a $\gamma_p - \omega-$ compact, but (T,γ_1,γ_2) is pairwise $-\omega-$ Hausdorff space, $\Omega(H)$ is $\gamma_r - \omega-$ closed. Hence, Ω is pairwise$-\omega-$ homeomorphism function. □

Definition 3.18. *A function* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ *is called pairwise* $-\omega-$*strongly function(pairwise* $-\omega-$*weakly function), if for every pairwise* $-\omega-$*open cover* $\underset{\sim}{K} = \{K_\theta : \theta \in \Psi\}$ *,there exists pairwise* $-\omega-$*open cover* $\underset{\sim}{L} = \{L_\theta : \theta \in \Psi_t\}$*of* T*, s.t* $\Omega^{-1}(L) \subseteq \bigcup\{K_\theta : \theta \in \Psi_1, \Psi_1 \subset \Psi, finite\}, \forall L_\theta \in \underset{\sim}{L}$.

Theorem 3.19. *Let* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ *be a pairwise* $-\omega-$*strongly onto function, then* (S,η_1,η_2) *is pairwise* $-\omega-$*compact, if* (T,γ_1,γ_2)*is so.*

Proof. Suppose $\underset{\sim}{K} = \{K_\theta : \theta \in \Psi\}$ be a pairwise $-\omega-$ open cover (S,η_1,η_2) .Because Ω is pairwise $-\omega-$ strongly function, there exists pairwise open cover $\underset{\sim}{L} = \{L_\theta : \theta \in \Psi_t\}$ of (T,γ_1,γ_2), such that $\Omega^{-1}(L) \subseteq \bigcup\{K_\theta : \theta \in \Psi_1, \Psi_1 \subset \Psi, \text{finite}\}, \forall L_\theta \in \underset{\sim}{L}$,but (T,γ_1,γ_2) is pairwise $-\omega-$ compact , so there exists $\Psi_1 \subset \Psi$, where $\Psi_1$is finite, such that, $T = \bigcup_{\theta\in\Psi_t} L_\theta$ and so, $S = \bigcup \Omega^{-1}(L_\theta)$. Each $\Omega^{-1}(L_\theta)$ contains of finite members of $\underset{\sim}{K}$, thus S is pairwise $-\omega-$ compact. □

Definition 3.20. *If* $\underset{\sim}{K}$ *and* $\underset{\sim}{F}$ *are pairwise* $-\omega-$*open covers of the bitopological space* (S,η_1,η_2)*, then* $\underset{\sim}{K}$ *is called a parallel refinement of* $\underset{\sim}{F}$*,if each* $K \in \underset{\sim}{K} \cap W(\eta_r)$*is contained in some* $F \in \underset{\sim}{F} \cap W(\eta_r), r=1,2$.

Definition 3.21. *If* $\underset{\sim}{K}$ *and* $\underset{\sim}{F}$ *are pairwise* $\eta_1\eta_2$ $-\omega-$*open covers of the bitopological space* (S,η_1,η_2)*, then* $\underset{\sim}{K}$ *is called a parallel refinement of* $\underset{\sim}{F}$*,if each* $K \in \underset{\sim}{K} \cap W(\eta_r)$*is contained in some* $F \in \underset{\sim}{F} \cap W(\eta_r), r=1,2$.

Definition 3.22. *A family $\underset{\sim}{N}$ of subsets of a space $(S,\eta_1,\eta_2)$is locally finite in $(S,W(\eta))$ if for each $s\in S$ there exists a $\omega-$ open set K such that $s\in K$ and K intersects at most finitely many elements of $\underset{\sim}{N}$.*

Definition 3.23. *A bitopological space (S,η_1,η_2) is called pairwise $M-\omega-$paracompact, if each pairwise $-\omega-$open cover of S has a pairwise$-\omega-$locally finite $\eta_1\eta_2-\omega-$open refinement.*

Definition 3.24. *A bitopological space (S,η_1,η_2) is called pairwise $-\omega-$paracompact, if each pairwise $-\omega-$open cover of S has a pairwise $-\omega-$ocally finite pairwise $-\omega-$open refinement.*

Theorem 3.25. *Let $\Omega:(S,\eta_1,\eta_2)\rightarrow(T,\gamma_1,\gamma_2)$ be a pairwise $-\omega-$perfect function , and (T,γ_1,γ_2) is a pairwise $M-\omega-$paracompact , then (S,η_1,η_2) is so.*

Proof. Suppose $\underset{\sim}{K}=\{K_\theta{:}\theta\in\Psi\}$ be a pairwise $-\omega-$open cover of (S,η_1,η_2) , because Ω is a pairwise$-\omega-$ perfect function,then $\forall t\in T$, $\Omega^{-1}(t)$ is pairwise $-\omega-$compact, Suppose $\underset{\sim}{K}=\{K_\theta{:}\theta\in\Psi\}$ be a p-open cover of (S,η_1,η_2) , since Ω is a pairwise $-\omega-$perfect function, then $\forall t\in T$, $\Omega^{-1}(t)$ is pairwise $-\omega-$ compact, $\exists$ a finite subsets Ψ_t, $\overset{*}{\Psi}_t$ of Ψ , s.t $\Omega^{-1}(t)\subseteq\bigcup\limits_{\theta\in\Psi_t}\{L_\theta:\theta\in\Psi_t\}\bigcup\bigcup\limits_{\theta\in\overset{*}{\Psi}_t}\{E_\theta:\theta\in\overset{*}{\Psi}_t\}$, where $\{L_\theta:\theta\in\Psi_t\}$ is η_1 $-\omega-$ open ,$\{E_\theta:\theta\in\overset{*}{\Psi}_t\}$ is $\eta_2-\omega-$ open. Let $D_t=T-\Omega(S-\bigcup\limits_{\theta\in\Psi_t}L_\theta)$ is a γ_1 $-\omega-$ open set containing t , and $D_t^*=T-\Omega(S-\bigcup\limits_{\theta\in\Psi_t}E_\theta)$ is a $\gamma_2-\omega-$ open set containing t , where $\Omega^{-1}(D_t)\subseteq\bigcup\limits_{\alpha\in\Psi_y}L_\theta$, Ω^{-1} $(D_t^*)\subseteq\bigcup\limits_{\alpha\in\overset{*}{\Psi}_y}E_\theta$. Let $\underset{\sim}{D}=\{D_t:t\in\ T\ \}\bigcup\{D_t^*:t\in\ T\}$ is a pairwise $-\omega-$ open cover of T. Since (T,γ_1,γ_2) is pairwise M$-\omega-$ paracompact, $\underset{\sim}{D}$ has a pairwise locally finite $\eta_1\eta_2-\omega-$open, refinement. say: □

$\underset{\sim}{I}=\{I_Z:Z\in\Xi_1\ \}\bigcup\{I_Z^*:Z\in\Xi_2\ \}$, where $\{I_Z:Z\in\Xi_1\ \}$ is $\eta_1-\omega-$locally finite paracompact of D_t ,and $\{I_Z^*:Z\in\Xi_2\ \}$ is $\eta_2-\omega-$ locally finite paracompact of D_t^*, $\Xi=\Xi_1\bigcup\Xi_2$. Let $J_1=\{\Omega^{-1}(I_Z)\bigcap L_{\theta_r},r=1,2,...,n,Z\in\Xi_1,\theta\in\Psi_t\}$ is $\eta_1-\omega-$ open locally finite refinement of $\{L_\theta:\theta\in\Psi_t\}$,and let $J_2=\{f^{-1}(I_Z^*)\bigcap E_{\theta_r},r=1,2,...,n,Z\in\Xi_2,\theta\in\overset{*}{\Psi}_t\}$ is $\eta_2-\omega-$

open locally finite refinement of $\{E_\theta : \theta \in \overset{*}{\Psi}_t\}$. Let $\underset{\sim}{I} = \{I_1 \bigcup I_2\}$,then $\underset{\sim}{I}$ is pairwise $-\omega-$ locally finite$\eta_1\eta_2 - \omega-$open refinement $\underset{\sim}{U}$ .Hence (S,η_1,η_2) is a pairwise M$-\omega-$ paracompact space .

Corollary 3.26. *Let $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ be a pairwise $-\omega-$perfect function , and (T,γ_1,γ_2) is a pairwise $-\omega-$paracompact , then (S,η_1,η_2) is so.*

Theorem 3.27. *The pairwise $-\omega-$Hausdorff space is constant algebraic expression under pairwise $-\omega-$perfect.*

Proof. Let (S,η_1,η_2) be a pairwise $-\omega-$ Hausdorff space, $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ be a pairwise $-\omega-$ perfect function, and $t_1 \neq t_2$ in (T,γ_1,γ_2), then $\Omega^{-1}(t_1)$,$\Omega^{-1}(t_2)$ are disjoint and pairwise $-\omega-$ compact subset of (S,η_1,η_2) .Since (S,η_1,η_2) be a p-Hausdorff space ,there exists a $\eta_{1_}$neighborhood K of S, and $\eta_{2_}$neighborhood L, s.t $\Omega^{-1}(t_1) \subseteq K$,$\Omega^{-1}(t_2) \subseteq L$,$K \bigcap L = \phi$.Let the sets $T - \Omega(S-K)$ be $\gamma_1 - \omega-$ open set in (T,γ_1,γ_2) and containing $t_1, T - \Omega(S-L)$ be $\gamma_2 - \omega-$open set in(T,γ_1,γ_2) and containing t_2 , s.t $[T - \Omega(S-K) \bigcap T - \Omega(S-L)] = T - [\Omega(S-L) \bigcup \Omega(S-L)] = Y - f(X - U \bigcap V) = T - \Omega(S) = \phi$.Hence (T,γ_1,γ_2) is pairwise $-\omega-$ Hausdorff space. □

Remark 3.28. *The pairwise $-\omega-$Hausdorff space is constant algebraic expression and inverse constant algebraic expression under pairwise M$-\omega-$perfect.*

Lemma 3.29. *In a bitopological space (S,η_1,η_2) , $W(\eta_1)$ is said to be $\omega-$regular with respect to $W(\eta_2)$ if, for each point s in S and each $\eta_1 - \omega-$closed set Csuch that $s \notin C$, there are a $\eta_1 - \omega-$ open set K and a $\eta_2 - \omega-$open set L such that $s \in K, C \subseteq L$ and $K \cap L = \phi$. (S,η_1,η_2) is $p - \omega-$regular if $W(\eta_1)$is $\omega-$regular with respect to $W(\eta_2)$ .Let S be a pairwise $-\omega-$regular space, and N be $\eta_r - \omega-$compact subset of $S, r = 1,2$, then for each $\tau_r - \omega-$ neighbourhood K of N , there exists a $\eta_r - \omega-$open P, such that $N \subset P \subset Cl_{\eta_\varepsilon}(P) \subset U, r, \varepsilon = 1,2, r \notin \varepsilon$.*

Proof. For each $n \in N$,there exist a $\eta_r - \omega -$ neighbourhood $V(n)$ such that $Cl_{\ \eta_\varepsilon} L(n) \subset K$,so $N \subset \bigcup_{\varkappa=1}^{n} L(n_\varkappa) \subset Cl_{\ \eta_\varepsilon} \bigcup_{\varkappa=1}^{n} L(n_\varkappa)$.Let $P = \bigcup_{\varkappa=1}^{n} L(n_\varkappa)$,then P is $\eta_r - \omega -$open, but $Cl_{\ \eta_\varepsilon} P$ $= Cl_{\ \eta_\varepsilon} \bigcup_{\varkappa=1}^{n} L(n_\varkappa) = Cl_{\ \eta_\varepsilon} \cup L(n_\varkappa)$,hence $N \subset P \subset Cl_{\ \eta_\varepsilon}(P) \subset K,\ r,\varepsilon = 1,2, r \notin \varepsilon$. □

Theorem 3.30. *Let* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ *be a pairwise* $-\omega-$*perfect function , and* (S,η_1,η_2) *is a pairwise* $-\omega-$*regular, then* (T,γ_1,γ_2) *is so.*

Proof. Given $\gamma_r - \omega -$ open set $L, t \in L, r,\varepsilon = 1,2, r \notin \varepsilon, \Omega^{-1}(t) \in \Omega^{-1}(L)$ in T,since S is pairwise $-\omega-$ regular,there exists $\eta_r - \omega -$open set K,(by using Lemma 2.52),such that $\Omega^{-1}(t) \in Cl_{\ \eta_\varepsilon} \bigcup_{\varkappa=1}^{n} K \subset \Omega^{-1}(L)$.Since Ω is $\eta_r - \omega$, then there exists $\gamma_r - \omega -$neighbourhood P of t, such that $\Omega^{-1}(t) \in \Omega^{-1}(P) \subset L$,but $P \subset \Omega(Cl_{\ \eta_\varepsilon} K) \subset L$,since $\Omega(Cl_{\ \eta_\varepsilon} K)$ is $\gamma_\varepsilon - \omega -$ closed , $t \in E \subset (Cl_{\ \eta_\varepsilon}(P)) \subset \Omega(Cl_{\ \eta_\varepsilon} K) \subset L$,hence T is pairwise $-\omega-$regular. □

Remark 3.31. *The pairwise* $-\omega-$*regular space is constant algebraic expression and inverse constant algebraic expression under* $M-\omega-$*perfect.*

Definition 3.32. *A bitopological space* (S,η_1,η_2) *is called pairwise* $-\omega-$*normal, if each* $\eta_r -$ $\omega-$*closed set N and* $\eta_\varepsilon - \omega -$ *closed set G, there exists* $\eta_\varepsilon - \omega -$*open set K and* $\eta_r - \omega -$*open set L, such that* $N \subset K, G \subset L, K \cap L = \phi,\ r,\varepsilon = 1,2, r \notin \varepsilon$.

Theorem 3.33. *Let* $\Omega : (S,\eta_1,\eta_2) \rightarrow (T,\gamma_1,\gamma_2)$ *be a pairwise* $-\omega-$*perfect function , and* (S,η_1,η_2) *is a pairwise* $-\omega-$*normal, then* (T,γ_1,γ_2) *is so.*

Proof. It follows by using Lemma [3.32] and theorem [3.33]. □

Theorem 3.34. *Let* (S,η_1,η_2) , (T,γ_1,γ_2), *be any bitopological spaces .If* (S,η_1,η_2) *is pairwise* $M-\omega-$*compact,then the projection function,* Φ : $(S \times T, \eta_1 \times \gamma_1\ , \eta_2 \times \gamma_2) \rightarrow (T,\gamma_1,\gamma_2)$ *is pairwise* $-\omega-$*closed.*

Proof. If (S,η_1,η_2) is pairwise M$-\omega-$ compact, then (S,η_1) is M$-\omega-$compact,(S,η_2) is M$-\omega-$compact, □

thus the projection functions: $\Phi_1 : (S \times T, \eta_1 \times \gamma_1) \rightarrow (T,\gamma_1),\ \Phi_2 : (S \times T, \eta_2 \times \gamma_2) \rightarrow (T,\gamma_2)$,are $\omega-$closed,thus Φ is pairwise $-\omega-$closed.

Corollary 3.35. *Let* (S,η_1,η_2) , (T,γ_1,γ_2) *are pairwise* $M-\omega-$*compact then* $(S\times T,\eta_1\times\gamma_1\ ,\eta_2\times\gamma_2)$ *is pairwise* $M-\omega-$*compact*

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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