Available online at http://scik.org
J. Math. Comput. Sci. 2022, 12:27
https://doi.org/10.28919/jmcs/6922
ISSN: 1927-5307

GENERALIZED SHEHU TRANSFORM

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Abstract. In this paper we have introduced Shehu transform for generalized functions. Further inversion theorem is also given. Lastly uniqueness and characterization theorem is obtained.

Keywords: Shehu transform; generalized function.

2010 AMS Subject Classification: 44A15, 44A20.

1. Introduction and Preliminaries

Various integral transforms have been extended to the space of generalized functions. L. Schwartz [13],[14],[15],[16],[17],[18] was firstly extended Fourier transform to a generalized function. Later on Zemanian A.H. [19],[20] have extended the classical integral transform to generalized functions viz. Using theory of L. Schwartz, Zemanian gave extension of Laplace, Fourier, Millen, Hankel transform etc to the generalized functions.We can see in [1],[6],[8],[9] some generalized integral transforms.

Now from last few years new integral transform [5],[7],[9] have been introduced so because of that researcher have more scope to extend these new integral transforms to the space of generalized functions.

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Received October 22, 2021

The generalized integral transforms are play important role because some integral equations have no solution in classical integral transform theory, but it is solvable by the integral transforms in distributional sense.

In this view we extend here Shehu transform for generalized functions. It's inversion theorem,uniqueness theorem, characterization theorem is also given. for generalized Shehu transform.

Definition: Shehu Maitama [11] define the Shehu transform of exponential order of function f(t) over the set

$$A=\{f(t):\exists N,\eta_1,\eta_2>0,|f(t)|<Nexp(\frac{|t|}{\eta_i}),if\ t\in(-1)^i\times[0,\infty)\}as$$

$$Sh[f(t)]=W(s,x)=\int_0^\infty exp(\frac{-st}{x})f(t)dt=\lim_{\alpha\to\infty}\int_0^\alpha exp(\frac{-st}{x})f(t)dt;s>0,x>0.$$

It converges if the limit of the integral exists otherwise diverges .

The inverse Shehu transform is given by[[11]]

$$Sh^{-1}[W(s,x)]=f(t),t\geq 0$$

Equivalently

$$f(t)=Sh^{-1}[W(s,x)]=\frac{1}{2\pi i}\int_{\alpha-i\beta}^{\alpha+i\beta}\frac{1}{x}exp(\frac{st}{x})W(s,x)ds$$

Here s and x are called the Shehu transform variables, and α is a real constant and the integral of above equation taken along $s=\alpha$ in complex plane $s=x+iy$.

2. Generalized Shehu Transform

In this section we extend Shehu transform for generalized functions.

Testing function space $\mathcal{S}_{a,b}$: Let $\mathcal{S}_{a,b}$ denotes the space of all complex valued smooth functions $\phi(t)$ on $-\infty<t<\infty$ on which the functions $\gamma_k(\phi)$ defined by

$$\gamma_k(\phi)\triangleq\gamma_{a,b,k}(\phi)\triangleq\sup_{0<t<\infty}|K_{a,b}(t)D^k(\phi(t))|<\infty$$

where

$$K_{a,b}(t) = \begin{cases} e^{at}, & 0 \leq t < \infty \\ e^{bt}, & -\infty < t < 0 \end{cases}$$

This $\mathcal{S}_{a,b}$ is linear space under the pointwise addition of function and their multiplication by complex number. Each γ_k is clearly a seminorm on $\mathcal{S}_{a,b}$ and γ_0 is a norm. We assign the topology generated by the sequence of seminorm on $(\gamma_k)_{k=0}^{\infty}$ there by making it a countably multinormed space. Note that for each fixed x , the kernel $e^{\frac{-st}{x}}$ as a function of t is a member of $\mathcal{S}_{a,b}$ iff $a < Re(\frac{s}{x}) < b$. With the usual argument[16]. We can show that $\mathcal{S}_{a,b}$ is complete and hence a frechet space. $\mathcal{S}'_{a,b}$ denotes the dual of $\mathcal{S}_{a,b}$ that is f is member of $\mathcal{S}'_{a,b}$ iff it is continuous linear function on $\mathcal{S}_{a,b}$. Thus $\mathcal{S}'_{a,b}$ is the space of generalized functions. Note that the properties of testing function space $\mathcal{S}_{a,b}$ will follows from [20].

Now we are ready to define the generalized Shehu transform. We denote this by $Sh[f(t)]$ or $W(s,x)$ for a given Shehu transformable generalized function f. We assume Ω_f is the open strip in complex plane which is define as, $\Omega_f \triangleq \{x : w_{1<Re(\frac{s}{x})} < w_2\}, s > 0$ since f or each $x \in \Omega_f, s > 0$ the kernel $e^{\frac{-st}{x}}$ as a function of t is a member of $\mathcal{S}'_{w_1,w_2}$. For $f \in \mathcal{S}'_{w_1,w_2}$. We define the generalized Shehu transform of $f(t)$ as,

$$W(s,x) \triangleq Sh[f(t)] \triangleq < f(t), e^{\frac{-st}{x}} >$$

We call Ω_f the region (or strip) of definition for $Sh[f(t)]$, w_1 *and* w_2 are the abscissas of definition. Note that the properties like linearity and continuity of the generalized Shehu transform will follows from [20].

The boundedness Property for the generalized Shehu transform is given by

$$< f(t), e^{\frac{-st}{x}} > \leq M \max_{0 \leq k \leq r} \sup_t |k_{a,b}(t) D_t^k e^{\frac{-st}{x}}|.$$

3. Inversion Theorem

To obtain the inversion formula for the generalized Shehu transform we need following two lemmas which can easily proved by using Zemanian[20].

3.1. lemma. Let $W(s,x) = Sh[f(t)]$ for $w_1 \leq Re(\frac{s}{x}) \leq w_2$ and let $\phi \in \mathcal{S}$, set $W(s,x) = \int\limits_{-\infty}^{\infty} \phi(t) e^{\frac{-st}{x}} dt$. Then for any fixed real number q with $0 < q < \infty$

$$\int_{-q}^{q} < f(\tau), e^{\frac{-s\tau}{x}} > W(s,x)dw = < f(\tau), \int_{-q}^{q} e^{\frac{-s\tau}{x}} W(s,x)dw > \tag{3.1}$$

Where $s = \sigma + iw$ and σ are fixed with $\sigma_1 < \sigma < \sigma_2$.

Proof If $\phi(t) \equiv 0$ then the proof is very straight forward. Let assume that $\phi(t) \neq 0$. Note that $W(s,x)$ is analytic for $w_1 \leq Re(\frac{s}{x}) \leq w_2$ and $W(s,x)$ is an entire function. Therefore the above integral must exists and

$$\| D_\tau^k \int_{-q}^{q} e^{\frac{-s\tau}{x}} W(s,x)dw \| \leq e^{\frac{-\sigma\tau}{x}} \int_{-q}^{q} \| (\frac{s}{x})^k W(s,x)dw \| \tag{3.2}$$

So that $\int\limits_{-q}^{q} e^{\frac{-s\tau}{x}} W(s,x)dw$ is member of $D(\sigma_1, \sigma_2)$.Now partition the path of integration on the straight line from $s = \sigma - iq$ to $s = \sigma + iq$ into m intervals each of length $\frac{2r}{m}$ and let $s_v = \sigma + iw_a$ be any point in the v^{th}interval.

Consider

$$\Theta_m \triangleq \sum_{v=1}^{m} e^{\frac{-s_v\tau}{x}} W(s_v,x) \frac{2r}{m} \tag{3.3}$$

By applying $f(\tau)$ to above equation term by term, we get

$$< f(\tau), \Theta_m > = \sum_{v=1}^{m} < f(\tau), e^{\frac{-s_v\tau}{x}} > W(s_v,x) \frac{2r}{m} \tag{3.4}$$

$$\rightarrow \int_{-r}^{r} < f(\tau), e^{\frac{-s\tau}{x}} > W(s,x)dwm \rightarrow \infty \tag{3.5}$$

In view of the fact that $< f(\tau), e^{\frac{-s\tau}{x}} > W(s,x)$ is a continuous function of W

Next choose a and b such that $\sigma_1 < a < \delta < b < \sigma_2$ since $f \in \mathfrak{S}_{a,b}$, all that remains to be prove is that Θ_m converges in $L(a,b)$ to $\int\limits_{-q}^{q} < f(\tau), e^{\frac{-s\tau}{x}} > W(s,x)dw$. so we need nearly to prove that,for each fixed k the following quality converges uniformly to zero on $-\infty < \tau < \infty$

$$\mu(\tau,m) \triangleq K_{a,b}(\tau)D_j^k[\Theta_m(\tau) - \int_{-q}^{q} e^{\frac{-s\tau}{x}} W(s,x)dw] \tag{3.6}$$

$$= (-1)^k K_{a,b}(\tau) \sum_{v=1}^{m} (\frac{sv}{x})^k e^{\frac{-s_v\tau}{x}} > W(s_v,x)\frac{2r}{m} - (-1)^k K_{a,b}(\tau) \int_{-q}^{q} (\frac{s}{x})^k e^{\frac{-s\tau}{x}} W(s,x)dw \tag{3.7}$$

Now $|K_{a,b}(\tau)e^{\frac{-s\tau}{x}}| = K_{a,b}(\tau)e^{\frac{-\sigma\tau}{x}} \to 0$ as $|\tau| \to \infty$ because $a < \delta < b$. So any $\varepsilon > 0$, we choose τ so large for all $|\tau| > \tau$.

$$|K_{a,b}(\tau)e^{\frac{-s\tau}{x}}| \leq \frac{\in}{3} |\int_{-q}^{q} (\frac{s}{x})^k W(s,x)dw|^{-1}$$

Since $\phi(t) \neq 0$ the right hand side is finite. Now for all $|\tau| > \tau$, the magnitude of the second term on the right hand side of the equation (3.6) is bounded by $\frac{\in}{3}$. Moreover again for $|\tau| > \tau$ the magnitude of the first term on the right hand side of equation (3.6) is given as follows

$$\frac{\in}{3}[\int_{-q}^{q} (\frac{s}{x})^k W(s,x)dw]^{-1} \sum_{v=1}^{m} |(\frac{s_v}{x})^k W(s,x)| \frac{2r}{m} \tag{3.8}$$

We can choose m_0 so large that for all $m > m_0$ the last expression is less than $\frac{2\in}{3}$. Therefore for all $|\tau| > \tau$ and all $m > m_0$, we have $|\mu(\tau,m)| < \in$.

Finally $|K_{a,b}(\tau)(\frac{s}{x})^k W(s,x)e^{\frac{-s\tau}{x}}|$ is a uniformly continuous function of (τ,w,x) on the domain $-\tau \leq \tau \leq \tau, -r \leq r \leq r$. Therefore in view of equation (3.6) there exists an m_1 such that for all $m > m_1, |\mu(\tau,m)| < \in$ on $-\tau \leq \tau \leq \tau$ as well.

Thus for $m > Max(m_0,m_1)$

$$|\mu(\tau,m)| < \varepsilon, -\infty < \tau < \infty \tag{3.9}$$

3.2. Lemma. Let a,b,σ and r be real numbers with $a < \sigma < b$. Also Let $\phi \in \mathcal{S}$ Then

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \phi(t+\tau) e^{\frac{-\sigma\tau}{x}} \frac{sin(rt)}{t} dt \tag{3.10}$$

converges in $\mathcal{S}_{a,b}$ to $\phi(\tau)$ as $r \to \infty$

Proof Suppose that $r > 0$. It is a fact that $\int_{-\infty}^{\infty} \frac{sin(rt)}{t} dt = \pi$ Thus our objective is to prove that for each k=0,1,2.....

$$B_r(\tau) \triangleq \frac{1}{\pi} K_{a,b}(\tau) D^k_\tau \int_{-\infty}^{\infty} [\phi(t+\tau) e^{\frac{\sigma t}{x}} - \phi(\tau)] \frac{sin(rt)}{t} dt \tag{3.11}$$

Converges uniformly to zero on $-\infty < \tau < \infty$ as $r \to \infty$.

Since ϕ is smooth and bounded support, we may differentiate under the integral sign

$$B_r(\tau) = \frac{K_{a,b}(\tau)}{\pi} \int_{-\infty}^{\infty} [e^{\frac{\sigma\tau}{x}} D^k_j \phi(t+\tau) - D^k_j \phi(\tau)] \frac{sin(rt)}{t} dt$$

$$= \frac{K_{a,b}(\tau)}{\pi} [\int_{-\infty}^{-\delta} [e^{\frac{\sigma\tau}{x}} D^k_j \phi(t+\tau) - D^k_j \phi(\tau)] \frac{sin(rt)}{t} dt + \int_{-\delta}^{\delta} [e^{\frac{\sigma\tau}{x}} D^k_j \phi(t+\tau) - D^k_j \phi(\tau)] \frac{sin(rt)}{t} dt$$

$$+ \int_{\delta}^{\infty} [e^{\frac{\sigma\tau}{x}} D^k_j \phi(t+\tau) - D^k_j \phi(\tau)] \frac{sin(rt)}{t} dt]$$

$B_r(\tau) = I_1 + I_2 + I_3$

Where I_1, I_2, I_3 represents the quantities obtained by integrating over the intervals $-\infty < t < -\delta, -\delta < t < \delta, \delta < t < \infty$ respectively where $\delta > 0$.

Consider I_2 the function

$$H(t,\tau) \triangleq K_{a,b}(\tau) t_{-1} [e^{\frac{\sigma\tau}{x}} D^k_j \phi(t+\tau) - D^k_j \phi(\tau)] \tag{3.12}$$

is continuous function of (t,τ,x) for all τ and $t \neq 0, x \neq 0$. Moreover since ϕ is smooth then above equation tends to

$$K_{a,b}(\tau) [e^{\frac{\sigma t}{x}} D^k_T \phi(t+\tau)]_{t=0} \tag{3.13}$$

as $t \to \infty$ upon assigning the values $(3.13) to\ H(0,\tau)$, we obtain a function $H(t,\tau)$ that is continuous everywhere on the (t,τ) plane. Since ϕ is bounded support, $H(t,\tau)$ is bounded on the domain

$\{(t,\tau) : -\delta < t < \delta, -\infty < \tau < \infty\}$ by a constant M. Thus given an $\in > 0$ we can choose δ so small that,

$$|I_2| = |\frac{1}{\pi} \int_{-\delta}^{\delta} H(t,\tau) sin(rt) dt| \leq \frac{2M\delta}{M} > \in, \ -\infty < \tau < \infty \tag{3.14}$$

fix δ in this way.

Now let I_1

Set $I_1 = Q_1(\tau) - Q_2(\tau)$

$$Where\ Q_1(\tau) = \frac{1}{\pi}\int_{-\infty}^{-\delta} K_{a,b}(\tau)e^{\frac{\sigma t}{x}} D_\tau^k \phi(t+\tau)\frac{sin(rt)}{t}dt$$

and $Q_2(\tau) = \frac{1}{\pi}K_{a,b}(\tau)D_\tau^k\phi(\tau)\int\limits_{-\infty}^{-r\delta}\frac{sinz}{z}dz$.

Since $K_{a,b}(\tau)D_\tau^k\phi(\tau)$ is continuous and bounded support, which is bounded on $-\infty < \tau < \infty$. By convergence of improper integral $\int\limits_{-\infty}^{0}\frac{sinz}{z}dz$, it follows that Q_2 tends uniformly to zero on $-\infty < \tau < \infty$ as $r \to \infty$

Now to show that Q_1 tends to zero, first integrate by parts and use the fact that $\phi(\tau)$ is bounded support to obtain

$$Q_1 = \frac{e^{\frac{-\sigma\delta}{x}}cos(r\delta)}{\pi r\delta}K_{a,b}(\tau)D_\tau^k\phi(\tau-\delta) + \frac{1}{\pi r}\int_{-\infty}^{-\delta} cos(rt)k_{a,b}(\tau)D_\tau[e^{\frac{\sigma t}{x}}D_\tau^k\phi(t+\tau)]dt$$

The first term on right hand side tends uniformly to zero on $-\infty < \tau < \infty$ as $r \to \infty$ because δ and σ are fixed and $K_{a,b}(\tau)\phi(\tau-\delta)$ is bounded function of τ. Moreover

$$K_{a,b}(\tau)D_t[e^{\frac{\sigma t}{x}}D_\tau^k\phi(t+\tau)] = K_{a,b}(\tau)e^{\frac{\sigma t}{x}}(\frac{\sigma}{t} - \frac{1}{t^2})D_\tau^k\phi(t+\tau) + k_{a,b}(\tau)\frac{e^{\frac{\sigma\tau}{x}}}{t}D_\tau^{k+1}\phi(t+\tau)$$

But for every $k, K_{a,b}(\tau)e^{\frac{\sigma\tau}{x}}D_\tau^k\phi(t+\tau)$ is bounded on the $(t+\tau)$ plane. It is because $D_\tau^k\phi(t+\tau)$ is bounded and it's support contained in the strip $\{(t+\tau) : |t+\tau| < A\}$ where A is a sufficiently large number where as $K_{a,b}(\tau)e^{\frac{\sigma t}{x}}$ is bounded on the strip by virture of the inequality $a < \sigma < b$.Thus the last equation is bounded on the domain $\{(t,\tau) : -\infty < t < -\delta, -\infty < \tau < \infty\}$ by a constant N

This results and the assumption that the support of $\phi(\tau)$ is contained in the interval $-A \le \tau \le A$ implies that the second term on the right hand side is bounded by $\frac{2NA}{\pi r}$ Which tends to zero as $r \to \infty$ so truly Q_1 and therefore I_1 tends to zero on $-\infty < \tau < \infty$ as $r \to \infty$. A similar argument shows that I_3 tends zero on $-\infty < \tau < \infty$ as $r \to \infty$. Thus we have $\lim\limits_{r\to\infty}|B_r(\tau)| \le \in$, since $\in > 0$ is arbitrary. Hence the proof is complete.

To find the solution of our original problem in it's original domain we required inversion theorem for every integral transform. On the same line given generalized Shehu transform also has it's inverse generalized Shehu transform, which we prove as follows.

3.3. Theorem(Inverse Shehu Transform). Let $W(s,x) = Sh[f(t)]$ for $r_1 < Re(\frac{s}{x}) < r_2$, let q be any real variable then in the sense of convergence in $\mathcal{S}'$

$$f(t) = \lim_{q\to\infty} \frac{1}{2\pi i} \int_{r-iq}^{r+iq} W(s,x) e^{\frac{st}{x}} ds \tag{3.15}$$

Where r is fixed number such that $r_1 < r < r_2$ $s = r + iw$.

Proof Let $\phi \in \mathcal{S}$, let choose any two real numbers a *and* b such that $r_1 < a < r < b < r_2$. To complete the proof of the theorem it is sufficient to prove that

$$\lim_{q\to\infty} < \frac{1}{2\pi i} \int_{r-iq}^{r+iq} W(s,x) e^{\frac{st}{x}} ds, \phi(t) > = < f(t), \phi(t) > \tag{3.16}$$

Now the integral on s is continuous function of t and therefore the left hand side of above equation can be written as

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(t) \int_{-q}^{q} W(s,x) e^{\frac{st}{x}} dwdt \tag{3.17}$$

where $s = r + iw$. and $q > 0$.

Since $\phi(t)$ is of bounded support and the integrand is a continuous function of (t,w) the order of integration may be interchanged

$$\frac{1}{2\pi} \int_{-q}^{q} W(s,x) \int_{-\infty}^{\infty} \phi(t) e^{\frac{st}{x}} dtdw$$

$$\frac{1}{2\pi} \int_{-q}^{q} < f(\tau), e^{\frac{-s\tau}{x}} > \int_{-\infty}^{\infty} \phi(t) e^{\frac{st}{x}} dtdw$$

By using the above lemma, we get

$$< f(\tau), \frac{1}{2\pi} \int_{-q}^{q} e^{\frac{-st}{x}} \int_{-\infty}^{\infty} \phi(t) e^{\frac{st}{x}} dtdw >$$

Here $\phi(t)$ is have bounded support so the order of integration for the repeated integral may be changed and integrand is a continuous function of (t,w) then we have

$$< f(\tau), \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi(t)\int_{-q}^{q} e^{\frac{s(t-\tau)}{x}} dwdt >$$

$$< f(\tau), \frac{1}{\pi}\int_{-\infty}^{\infty}\phi(t+\tau)\frac{sin(pt)}{t}e^{\frac{rt}{x}} dt >$$

But this expression tends to $< f(t), \phi(t) >$ as $r \to \infty$ due to lemma.

Hence the proof.

4. Uniqueness Theorem

If $Sh[f(t)] = W(s,x)$ for $x \in \Omega_f, s > 0$ and $Sh[h(t)] = H(s,x)$ for $x \in \Omega_h, s > 0$, if $\Omega_f \cap \Omega_h$ is non empty, and if $W(s,x) = H(s,x)$ for $x \in \Omega_f \cap \Omega_h$, then $f \equiv h$ in the sense of equality in $\mathcal{S}^{'}(v,z)$,where $v < w < z(v < w_1\ or\ z > w_2)$ is the restriction of Ω_f, Ω_h with real axis.

Proof Let $\phi \in \mathcal{S}$, to give the proof of this theorem it is sufficient to prove that,

$$< f(t), \phi(t) >=< h(t), \psi(t) >$$

Now from inverse theorem (3.15) we get,

$$< f(t), \phi(t) > = \lim_{q\to\infty} < \frac{1}{2\pi i}\int_{r-iq}^{r+iq} W(s,x)e^{\frac{st}{x}} ds, \phi(t) >$$

$$< f(t), \phi(t) > = \lim_{q\to\infty} < \frac{1}{2\pi i}\int_{r-iq}^{r+iq} H(s,x)e^{\frac{st}{x}} ds, \phi(t) >$$

$$=< h(t), \phi(t) >$$

Here f and h assign the same value to each $\phi \in \mathcal{S}$. Furthermore $\mathcal{S}$ is dense in $\mathcal{S}(v,z)$ and f and h are both member of $\mathcal{S}^{'}(v,z)$ Furthermore, $\mathcal{S}$ is dense in $\mathcal{S}(v,z)$ and f, g

5. CHARACTERIZATION THEOREM

The necessary condition for the function $W(s,x)$ to be the Shehu transform of generalized function $f(t)$ are that $W(s,x)$ is analytic on Ω_f and for each closed strip $\{x : a \leq Re(\frac{s}{x}) \leq b\}\}$. of Ω_f there be polynomial such that $|W(s,x)| \leq P(|\frac{s}{x}|)$ for $a \leq Re(\frac{s}{x}) \leq b$. The polynomial P will depends in general on *a and b*.

Proof The analyticity of $W(s,x)$ has been already proved in the previous theorem. By the definition of Shehu transform, f is a member of $\mathcal{S}'_{a,b}$ where $w_1 < a < b < w_2$ so that there exists a constant M and non-negative integer r such that for

$$\begin{aligned}
|W(s,x)| &= |< f(t), e^{\frac{-st}{x}} >| \\
&\leq M \max_{0\leq k\leq r} \sup_t |K_{a,b}(t) D_t^k e^{\frac{-st}{x}}| \\
&\leq M \max_{0\leq k\leq r} |\frac{s}{x}|^k \sup_t |K_{a,b}(t) D_t^k e^{\frac{-st}{x}}| \\
&\leq P(|\frac{s}{x}|)
\end{aligned}$$

This polynomial $P(|\frac{s}{x}|)$ depends in general on the choices of *a and b*.

6. CONCLUSION

In this article we extend Shehu transform for generalized functions. Also it's inversion formula is proved. lastly uniqueness and characterization theorem for generalized Shehu transform is given.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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