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PROPERTIES OF MULTI ANTI L-FUZZY QUOTIENT GROUP $\bar{A}$ OF A GROUP G

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Abstract: In this Paper, the notion of multi anti L – fuzzy quotient group $\bar{A}$ of a group G determined by A and K is introduced and discussed its properties.

Keywords: fuzzy set; multi-L-fuzzy subgroup; homomorphism of multi L-fuzzy group; anti homomorphism of multi L-fuzzy group; quotient subgroup; multi L-fuzzy quotient subgroup.

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1. INTRODUCTION

L. A. Zadeh [19] introduced the notion of a fuzzy subset A of a set X as a function from X into I = [0, 1]. Rosenfeld [3] and Kuroki [12] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy sub semi groupoids respectively. The concept of anti – fuzzy subgroup was introduced by Biswas [5]. The Concept

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of multi fuzzy subgroups was introduced by Souriar Sebastian and S. Babu Sundar [17]. In all these studies, the closed unit interval [0, 1] is taken as the Membership lattice. We introduce the notion of a multi L-fuzzy sub group G and discussed some of its properties. The characterizations of a Multi L-fuzzy subgroup under homomorphism and anti homomorphism on multi L-fuzzy quotient subgroup on a group is discussed

2. PRELIMINARIES

In this Section, we review some definitions and some results of Multi L-fuzzy subgroups which will be used in the later sections. Throughout this section we mean that $(G,*)$ is a group, e is the identity of G and xy as $x*y$.

2.1 Definition:

Let X be any nonempty set. A fuzzy set A of X is $A: X \rightarrow [0, 1]$.

2.2 Definition:

Let (G, **.**) be a group. A fuzzy subset A of G is said to be a fuzzy subgroup (FSG) of G if the following conditions are satisfied:

i. $A(xy) \geq \min\{A(x), A(y)\}$,

ii. $A(x^{-1}) = A(x)$, for all x and $y \in G$.

2.3 Definition:

Let (G, **.**) be a group. A fuzzy subgroup A of G is said to be a normal fuzzy subgroup of G if $A(xy) = A(yx)$, for all x and $y \in G$.

2.4 Definition:

A fuzzy subset A of G is said to be a anti fuzzy group of G, if for all $x, y \in G$

i. $A(xy) \leq \max\{A(x), A(y)\}$

ii. $A(x^{-1}) = A(x)$.

2.5 Definition:

An anti fuzzy subgroup A of G is called a anti fuzzy normal subgroup (AFNS) of G if for every $x, y \in G$, $A(xyx^{-1}) \leq A(y)$.

2.6 Definition:

Let X be a non – empty set. A multi L-fuzzy set A in X is defined as a set of ordered sequences, $A = \{(x, A_1(x), A_2(x), ..., A_i(x), ...): x\in X\}$, where $A_i: X \to L$ for all i.

2.7 Definition:

A multi L-fuzzy subset A of G is called a multi L-fuzzy subgroup (MLFS) of G if for every x,y∈G,

i. $A(xy) \geq A(x) \wedge A(y)$
ii. $A(x^{-1}) = A(x)$.

2.8 Definition:

A multi L-fuzzy subset A of G is called a multi anti L-fuzzy subgroup (MALFS) of G if for every x,y ∈G,

i. $A(xy) \leq A(x) \vee A(y)$
ii. $A(x^{-1}) = A(x)$.

2.9 Definition

The function f: G→G′ is said to be a homomorphism if f(xy)= f(x)f(y) ∀ x,y∈G.

2.10 Definition

The function f: G→ G′ (G and G′ are not necessarily commutative) is said to be an anti homomorphism if f(xy)= f(y)f(x) ∀ x,y∈G.

2.11 Definition

Let f be any function from a set X to a set Y, and let A be any L-fuzzy subset of X. Then A is called f-invariant if f(x) = f(y) implies A(x) = A(y), where x,y∈X.

2.12 Definition:

Let (G, **.**) be a group. A multi L-fuzzy subgroup A of G is said to be a multi L-fuzzy normal subgroup of G if A(xy) = A (yx), for all x and y∈G.

2.13 Definition:

Let (G, **.**) be a group. A multi anti L-fuzzy subgroup A of G is said to be a multi anti L-fuzzy normal subgroup of G if A(xy) = A (yx), for all x and y∈G.

2.14 Definition:

Let A be a multi L-fuzzy normal subgroup of G with identity e. Let K = {x ∈ G / A(x) = A(e)}. Consider the map $\overline{A} = G/K \to L^k$ defined by $\overline{A}$(xK) = ∨A(xk) for all k ∈K and x ∈ G. Then, the multi L-fuzzy subgroup $\overline{A}$ of G/K is called a multi L-fuzzy quotient group of A by K.

Remarks:

i. $\overline{A}$ is not a multi L-fuzzy normal quotient group of G/K, Since, $\overline{A}$(xKyK) ≠ $\overline{A}$(yKxK).

ii.Consider the map, $\overline{A}$: $G/K \to$ L defined by $\overline{A}$(xK) = A(x) for all k ∈ K and x ∈ G. Then, $\overline{A}$ is a multi L-fuzzy normal quotient group of G/K.

3. PROPERTIES OF MULTI ANTI L-FUZZY QUOTIENT GROUP $\overline{A}$ DETERMINED BY A AND K

In this section, the properties of multi L-fuzzy quotient group $\overline{A}$ determined by A and K are discussed.

3.1 Theorem:

Let A be a multi anti L-fuzzy normal subgroup of G with identity e. Let K = {x ∈ G / A(x) = A(e)}. Consider the map $\overline{A}$: $G/K \to L^k$ defined by $\overline{A}$(xK) = ∧ A(xk) for all k ∈K and x ∈ G. Then, K is a normal subgroup of G.

i.The map $\overline{A}$ is well defined

ii. $\overline{A}$ is a multi L-fuzzy subgroup of G.

Proof:

Given A is a multi anti L-fuzzy normal subgroup of G and

i. K = {x ∈ G / A(x) = A(e)}. Let x∈G and y∈K, then A(y) = A(e).

Now, $A(xyx^{-1})$ = A(y) =A(e), since A is a normal subgroup of G.

Hence, xyx^{-1} ∈ K.

Hence, K = { x $\in$ G / A(x) = A(e) } is a normal subgroup of G.

ii. Consider the map, $\overline{A} : G/K \to L$ defined by

$\overline{A}(xK) = \wedge A(xK)$ for all k $\in$K and x $\in$ G.

Let Kx = Ky for some x, y $\in$ G. Then $xy^{-1} \in K$. That is, $A(xy^{-1}) = A(e)$.

That is, A(xK) = A(yK)

That is, $\overline{A}(xK) = \overline{A}(yK)$.

Hence, the map $\overline{A}$ is well defined.

iii. Now, $\overline{A}(xKyK) = \overline{A}(xyK) = \wedge A(xyK)$, for all k $\in$K and x, y$\in$ G.

$$\leq \wedge (A(xk_1) \vee A(yk_2)) , k_1,k_2 \in K .$$

$$\leq (\wedge A(xk_1)) \vee (\wedge A(yk_2)) , k_1,k_2 \in K .$$

$$\leq \overline{A}(xK) \vee \overline{A}(yK) .$$

$$\overline{A}(xKyK) \leq \overline{A}(xK) \vee \overline{A}(yK).$$

$$\overline{A}((xK)^{-1}) = \overline{A}(x^{-1}K)$$

$= \wedge A(x^{-1}K)$ for all k $\in$K and x$\in$ G.

$= \wedge A(xK)$ for all k $\in$K and x$\in$ G.

$= \overline{A}(xK)$.

$$\overline{A}((xK)^{-1}) = \overline{A}(xK).$$

Hence, $\overline{A}$ is a multi anti L-fuzzy subgroup of G/K .

3.2 Definition

Let A be a multi anti L-fuzzy normal subgroup of G with identity e. Let K = { x $\in$ G / A(x) = A(e) }. Consider the map $\overline{A} = G/K \to L^k$ defined by $\overline{A}(xK) = \wedge A(xk)$ for all k $\in$K and x $\in$ G. Then, the multi anti L-fuzzy subgroup $\overline{A}$ of G/K is called a multi anti L-fuzzy quotient group of A by K.

Remarks:

i. $\overline{A}$ is not a multi anti L-fuzzy normal quotient group of G/K.

Since, $\overline{A}(xKyK) \neq \overline{A}(yKxK)$.

ii. Consider the map, $\overline{A} : G/K \to L$ defined by $\overline{A}(xK) = A(x)$ for all $k \in K$ and $x \in G$. Then, $\overline{A}$ is a multi anti L-fuzzy normal quotient group of G/K.

3.3 Theorem:

Let $\overline{A} = (\overline{A}_1, \overline{A}_2, \overline{A}_3 \ldots \overline{A}_k)$ is a multi anti L-fuzzy quotient group of a group of G/K, iff $\overline{A}_i$, i = 1, 2, … k, is an anti L-fuzzy quotient group of a group G/K.

Proof:

Let $\overline{A} = (\overline{A}_1, \overline{A}_2, \overline{A}_3 \ldots \overline{A}_k)$ is a multi anti L-fuzzy quotient group of a group of G/K. Then,

$\Leftrightarrow$ $\overline{A}(xy) \leq \overline{A}(x) \vee \overline{A}(y)$ and $\overline{A}(x^{-1}) = \overline{A}(x)$.

$\Leftrightarrow$ $\overline{A}_i(xy) \leq \overline{A}_i(x) \vee \overline{A}_i(y)$ and $\overline{A}_i(x^{-1}) = \overline{A}_i(x)$ for all i = 1,2,…k.

$\Leftrightarrow$ $\overline{A}_i$, i = 1,2,…k , is an anti L-fuzzy quotient group of a group G/K.

Remark:

If $\overline{A} = (\overline{A}_1, \overline{A}_2, \overline{A}_3 \ldots \overline{A}_k)$ is not a multi anti L-fuzzy quotient group of a group G/K, then there is at least one $\overline{A}_i$, i = 1,2,…k , is not an anti L-fuzzy quotient group of a group G/K.

3.4 Theorem:

If $\overline{A}$ is a multi anti L-fuzzy quotient group of a group G/K, then $\overline{A}(xK) \geq \overline{A}(eK)$, for $x \in G$, where $e \in G$ is the identity element of G.

Proof:

Let the element $x \in G$, where $e \in G$ is the identity element of G.

Now, $\overline{A}(e) = \overline{A}(xx^{-1}K)$

$\leq \overline{A}(xK) \vee \overline{A}(x^{-1}K)$

$= \overline{A}(xK)$.

Therefore, $\overline{A}(eK) \leq \overline{A}(xK)$.

3.5 Theorem:

$\overline{A}$ is a multi anti L-fuzzy quotient group of a group G/K if and only if

$\overline{A}(xKy^{-1}K) \leq \overline{A}(xK) \vee \overline{A}(yK)$, for all x and y in G.

Proof:

Assume that $\overline{A}$ is a multi anti L-fuzzy quotient group of a group G/K.

We have, $\overline{A}(xKy^{-1}K) \leq \overline{A}(xK) \vee \overline{A}(y^{-1}K)$

$\leq \overline{A}(xK) \vee \overline{A}(yK)$

Therefore, $\overline{A}(xKy^{-1}K) \leq \overline{A}(xK) \vee \overline{A}(yK)$, for all x and y in G.

Conversely, if $\overline{A}(xKy^{-1}K) \leq \overline{A}(xK) \vee \overline{A}(yK)$,then

$\overline{A}(x^{-1}K) = \overline{A}(ex^{-1}K)$

$\leq \overline{A}(eK) \vee \overline{A}(xK)$

$= \overline{A}(xK)$.

Therefore, $\overline{A}(x^{-1}) \leq \overline{A}(x)$, for all x in G.

Hence, $\overline{A}((x^{-1})^{-1}K) \leq \overline{A}(x^{-1}K)$ and $\overline{A}(xK) \geq \overline{A}(x^{-1}K)$.

Therefore, $\overline{A}(x^{-1}K) = \overline{A}(xK)$, for all x in G.

Now, replace y by y^{-1}, then

$$\overline{A}(xyK) = \overline{A}(x(y^{-1})^{-1}K)$$

$$\leq \overline{A}(xK) \vee \overline{A}(y^{-1}K)$$

$$= \overline{A}(xK) \vee \overline{A}(yK), \text{ for all x and y in G.}$$

Hence, $\overline{A}$ is a multi anti L-fuzzy quotient group of a group G/K .

3.6 Theorem:

If $\overline{A}$ and $\overline{B}$ are two multi anti L-fuzzy quotient groups of a group G/K, then $\overline{A} \cap \overline{B}$ is a multi L-fuzzy quotient group of G/K .

Proof:

It is trivial.

Remark:

The intersection of a family of multi anti L-fuzzy quotient groups of a group G/K**,** is a multi anti L-fuzzy quotient group of a group G/K .

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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