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EFFICIENT SIXTH ORDER ITERATIVE METHOD FREE FROM HIGHER DERIVATIVES FOR NONLINEAR EQUATIONS

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Abstract. In this paper, we proposed new iterative sixth order convergence method for solving nonlinear equations. The combination of the Taylor series and composition approach is used to derive the new method. Numerous methods have been developed by many researchers whenever the function's second and higher order derivatives exist in the neighbourhood of the root. Computing the second and higher derivative of a function is a very cumbersome and time consuming task. In terms of low computation cost, the newly proposed method finds the best approximation to the root of non-linear equations by evaluating the function and its first derivative. The proposed method has been theoretically demonstrated to have sixth-order convergence. The proposed method has an efficiency index of 1.56. Several comparisons of the proposed method with the various existing iterative method of the same order have been performed on the number of problems. Finally, the computational results suggest that the newly proposed method is efficient compared to the well-known existing methods.

Keywords: iterative method; non-linear equations; efficiency index.

2010 AMS Subject Classification: 65H05, 65B10.

1. Introduction

The solution of nonlinear equations governing the natural phenomena of real word problems is a very important and pertinent problem in computational sciences. It is almost impossible to

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find exact solutions to many non-linear equations. For such problems, it would be important to develop methods to obtain approximate solutions. Therefore, we investigate an iterative technique for obtaining an approximate solution of the non linear equations of the form:

$$\varphi(x) = 0 \tag{1}$$

where $\varphi : D \subset R \to R$ is substantially differential fucntion in the interval D. The well-known Newton's method to find the approximate solution of the non-linear equation (1) is given by

$$x_{n+1} = x_n - \frac{\varphi(x_n)}{\varphi'(x_n)}, \quad n = 0, 1, 2..... \tag{2}$$

It is one point second-order method and requires evaluation of one function and one derivative at each iteration. The one point iteration method based on the k-function evaluation has maximum order k $[1,2]$. The multipoint iterative approach has surpassed the theoretical limit of the one-point methods in terms of computational efficiency and convergence order. If n be the number of function evaluations per iteration of the method and ρ represents the order of convergence, then the efficiency index [2] of the method can be measured by $\rho^{\frac{1}{n}}$. In the last few decades, the problems of finding an approximation to the root of non-linear equations have been extensively studied. Some surveys and complete literatures in this direction could be found in Argyros et al. [3], Abbasbandy [4], Chun [5, 6], Kogan et al. [7], Kumar et al. [8], Thota [9], Mahesh et al. [10], Sabharwal [11], Sharma et al. [12], Chanu et at. [13] and the references therein. Some of the well-known methods developed recently for solving non-linear equations are detailed below:

In 2007, Noor et al. [14], proposed following modified Housholder iterative method (NM1 for short) for solving nonlinear equations with order six:

$$\begin{aligned} y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\ x_{n+1} &= y_n - \frac{\varphi(y_n)}{\varphi'(y_n)} - \frac{[\varphi(y_n)]^2 \varphi''(y_n)}{2[\varphi'(y_n)]^3} \end{aligned} \tag{3}$$

In 2007, Noor et al. [15], devised the following Predictor-Corrector Halley scheme for nonlinear equations (NM2 for short) with order six:

$$
\begin{aligned}
y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\
x_{n+1} &= y_n - \frac{2\varphi(y_n)\varphi'(y_n)}{2\varphi'(y_n) - \varphi(y_n)\varphi''(y_n)}
\end{aligned} \tag{4}
$$

In 2008, Parhi [16], presented a following new method for nonlinear equations (PM for short) with order six:

$$
\begin{aligned}
y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\
z_n &= x_n - \frac{2\varphi(x_n)}{\varphi'(x_n) + \varphi'(y_n)} \\
x_{n+1} &= z_n - \frac{\varphi(z_n)}{\varphi'(x_n)} \frac{\varphi'(x_n) + \varphi'(y_n)}{3\varphi'(y_n) - \varphi'(x_n)}
\end{aligned} \tag{5}
$$

In 2012, Chun et al. [17], suggested a following new scheme for nonlinear equation (CM for short) with order six:

$$
\begin{aligned}
y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\
z_n &= y_n - \frac{\varphi(y_n)}{\varphi'(x_n)} \frac{1}{[1 - \frac{\varphi(y_n)}{\varphi(x_n)}]^2} \\
x_{n+1} &= z_n - \frac{\varphi(z_n)}{\varphi'(x_n)} \frac{1}{[1 - \frac{\varphi(y_n)}{\varphi(x_n)} - \frac{\varphi(z_n)}{\varphi(x_n)}]^2}
\end{aligned} \tag{6}
$$

In 2014, Singh et al. [18], presented the following new method for nonlinear equations (SM for short) with order six:

$$
\begin{aligned}
z_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\
\bar{x}_{n+1} &= z_n - \frac{\varphi(z_n)}{\varphi'(z_n)} \frac{\varphi(x_n)}{\varphi(x_k) - 2\varphi(z_n)} \\
x_{n+1} &= \bar{x}_{n+1} - \frac{\varphi(\bar{x}_{n+1})(\bar{x}_{n+1} - z_n)}{\varphi(\bar{x}_{n+1}) - \varphi(z_n)}
\end{aligned} \tag{7}
$$

In 2019, Shengfen Li [19], proposed the following fourth order iterative method(LM for short) using Thiele's continued fraction:

$$x_{n+1}=x_n-\frac{\varphi(x_n)(6\varphi'(x_n)^2\varphi''(x_n)-3\varphi(x_n)\varphi''(x_n)^2+2\varphi(x_n)\varphi'(x_n)\varphi'''(x_n))}{2\varphi'(x_n)(3\varphi'(x_n)^2\varphi''(x_n)-3\varphi(x_n)\varphi''(x_n)^2+\varphi(x_n)\varphi'(x_n)\varphi'''(x_n))} \tag{8}$$

Inspired by the recent activities in this direction, we present a new sixth-order approach for solving non-linear equations by altering the Taylor's series expansion and composition techniques. The proposed method uses two evaluations of the function and two evaluations of the first derivative in each iteration. The proposed method is tested through numerical experimentation to support the theory on various non-linear equations. Finally, the newly proposed method is efficient in approximating the roots of non-linear equations. The remaining segment of the present study is organized as follows: The development of the new method is presented in Section 2. The theoretical result about the order of convergence of the proposed method is also established in Section 2. In Section 3, the numerical implementation of the proposed method is presented and the comparison of the results of the new method with other existing techniques of identical orders are summarized in tables. The concluding remarks are presented in Section 4.

2. Construction of the Proposed Method with Analysis of Convergence

Let $\varphi(x)$ be a differentiable real valued function defined on the interval $D\subset R$. Suppose that $\alpha\in D$ is a simple zero for non-linear equation $\varphi(x)=0$ and let x_n be a initial guess sufficiently close to α. By Taylor series quadratic approximation of $\varphi(x)$ about the point x_n, we get

$$\varphi(x)=\varphi(x_n)+(x-x_n)\varphi'(x_n)+\frac{(x-x_n)^2}{2!}\varphi''(x_n) \tag{9}$$

Assuming $\varphi(x_{n+1})=0$ to obtain the next approximation(x_{n+1}) of the root(α) of $\varphi(x)$ in the above equation, we get

$$\varphi(x_n)+(x_{n+1}-x_n)\varphi'(x_n)+\frac{(x_{n+1}-x_n)^2}{2!}\varphi''(x_n)=0 \tag{10}$$

Reordering above equation, we get

$$x_{n+1}=x_n-\frac{\varphi(x_n)}{\varphi'(x_n)}-(x_{n+1}-x_n)^2\frac{\varphi''(x_n)}{2\varphi'(x_n)} \tag{11}$$

Substituting the value of $(x_{n+1}-x_n)$ from equation (8) on the right side of equation (11), we get

$$x_{n+1}=x_n-\frac{\varphi(x_n)}{\varphi'(x_n)}-\left(\frac{\varphi(x_n)}{\varphi'(x_n)}\left(\frac{6J_1-3J_2+2J_3\varphi'''(x_n)}{3J_1-3J_2+J_3\varphi'''(x_n)}\right)\right)^2\frac{\varphi''(x_n)}{2\varphi'(x_n)} \tag{12}$$

where

$$J_1 = \varphi'(x_n)^2\varphi''(x_n),\ J_2 = \varphi(x_n)\varphi''(x_n)^2,\ J_3 = \varphi(x_n)\varphi'(x_n)$$

Using Newton's method as the predictor and equation (12) as the corrector, we get the following method

$$\begin{aligned} y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\ x_{n+1} &= y_n - \frac{\varphi(y_n)}{\varphi'(y_n)} - \left(\frac{\varphi(y_n)}{\varphi'(y_n)}\left(\frac{6K_1 - 3K_2 + 2K_3\varphi'''(y_n)}{3K_1 - 3K_2 + K_3\varphi'''(y_n)}\right)\right)^2 \frac{\varphi''(y_n)}{2\varphi'(y_n)} \end{aligned} \tag{13}$$

where

$$K_1 = \varphi'(y_n)^2\varphi''(y_n),\ K_2 = \varphi(y_n)\varphi''(y_n)^2,\ K_3 = \varphi(y_n)\varphi'(y_n)$$

The third order derivative evaluation is required to implement the method given in equation (13). We introduce an approximation of third derivative to overcome this drawback. Let $y_n = x_n - \varphi(x_n)/\varphi'(x_n)$ and using the Taylor series about the point x_n, we get

$$\varphi(y_n) = \varphi(x_n) + \varphi'(x_n)(y_n - x_n) + \frac{1}{2}\varphi''(x_n)(y_n - x_n)^2 + \frac{1}{6}\varphi'''(x_n)(y_n - x_n)^3 \tag{14}$$

Using $y_n - x_n = -\varphi(x_n)/\varphi'(x_n)$ in above equation, we get

$$\varphi(y_n) = \frac{\varphi(x_n)^2\varphi''(x_n)}{2\varphi'(x_n)^2} - \frac{\varphi(x_n)^3\varphi'''(x_n)}{6\varphi'(x_n)^3} \tag{15}$$

$$\varphi''(y_n) = \varphi''(x_n) - \frac{\varphi(x_n)\varphi'''(x_n)}{\varphi(x_n)} \tag{16}$$

$$\varphi'''(y_n) = \varphi'''(x_n) = \frac{6\varphi(y_n)\varphi'(x_n)^3 - 3\varphi(x_n)^2\varphi'(x_n)\varphi''(y_n)}{2\varphi(x_n)^3} \tag{17}$$

Substituting the value of $\varphi'''(y_n)$ from equation (17) into the equation (13), we get the following method free from third derivative evaluation of the function:

$$\begin{aligned} y_n &= x_n - \frac{\varphi(x_n)}{\varphi'(x_n)} \\ x_{n+1} &= y_n - \frac{\varphi(y_n)}{\varphi'(y_n)} - \left(\frac{\varphi(y_n)}{\varphi'(y_n)}\left(\frac{-2L_1 + L_2 + L_3 * L_7}{2L_1 - 2L_2 - L_3 * L_4}\right)\right)^2 \frac{\varphi''(y_n)}{2\varphi'(y_n)} \end{aligned} \tag{18}$$

where

$$L_1 = \varphi(x_n)^3\varphi'(y_n)^2\varphi''(y_n),\ L_2 = \varphi(x_n)^3\varphi(y_n)\varphi''(y_n)^2, L_3 = \varphi'(x_n)\varphi(y_n)\varphi'(y_n)$$

and $L_4 = \varphi(x_n)^2\varphi''(y_n) - 2\varphi(y_n)\varphi'(x_n)^2$.

The implementation of the proposed method (NPM1) given by equation (18) required the evaluation of second derivative. The evaluation of second derivative of the function is very cumbersome and time-consuming. So, the approximation of second derivative is obtained by applying Hermite's interpolation. We consider that $T(t) = q_1 + q_2(t-y_n) + q_3(t-y_n)^2 + q_4(t-y_n)^3$, where q_1, q_2, q_3 and q_4 are unknowns that can be establish from the following conditions [20]:

$$\varphi(x_n) = T(x_n),\ \varphi(y_n) = T(y_n),\ \varphi'(x_n) = T'(x_n),$$

$$\varphi'(y_n) = T'(y_n),\ \varphi''(y_n) = T''(x_n) \tag{19}$$

The above condition will create a four linear equations with four unknown variable $q_1,\ q_2,\ q_3$ & q_4. The following expression can be obtained by solving those four linear equations [20]:

$$S(x_n, y_n) = \varphi''(y_n) = \frac{2}{(x_n - y_n)}[3\frac{\varphi(x_n) - \varphi(y_n)}{(x_n - y_n)} - 2\varphi'(y_n) - \varphi'(x_n)] \tag{20}$$

Substituting the value of $\varphi''(y_n)$ from equation (20) into equation (18), we get the following iterative scheme free from the evaluation of higher derivatives:

$$y_n = x_n - \frac{\varphi(x_n)}{\varphi'(x_n)}$$

$$x_{n+1} = y_n - \frac{\varphi(y_n)}{\varphi'(y_n)} - \left(\frac{\varphi(y_n)}{\varphi'(y_n)}\left(\frac{-2M_1 + M_2 + M_3 * M_4}{2M_1 - 2M_2 - M_3 * M_4}\right)\right)^2 \frac{S(x_n, y_n)}{2\varphi'(y_n)} \tag{21}$$

where

$$M_1 = \varphi(x_n)^3\varphi'(y_n)^2 S(x_n, y_n),\ M_2 = \varphi(x_n)^3\varphi(y_n)S(x_n, y_n)^2,\ M_3 = \varphi'(x_n)\varphi(y_n)\varphi'(y_n)$$

and $M_4 = \varphi(x_n)^2 S(x_n, y_n) - 2\varphi(y_n)\varphi'(x_n)^2$.

The iterative schemes given by equation (21) has the sixth order of convergence and are denoted as NPM2. The newly proposed method (NPM2) requires two function evaluations and two evaluations of the first derivative per iteration. So, the efficiency index of new proposed method given by equation (21) is $(6)^{\frac{1}{4}} \approx 1.56$. The order of convergence of the preceding method is analyzed in the following Theorem 2.1.

Theorem 2.1. *Let $\alpha \in D$ be a simple root of a substantially differentiable function $\varphi : I \subset R \to R$ in an open interval D. If x_0 be initial guesses substantially nearby α, then the iterative method defined by equation (21) has sixth-order convergence and satisfies the following error equation: $e_{n+1} = -c_2^3 c_3 e_n^6 + O(e_n)^7$, where $e_n = x_n - \alpha$ is the error at n^{th} iteration.*

Proof: Since α be a root of $\varphi(x)$ and $e_n = x_n - \alpha$ is the error at n^{th} iteration. So, we can expand $\varphi(x_n)$ in powers of e_n by Taylor's series expansion as follows:

$$\varphi(x_n) = \varphi(\alpha) + e_n\varphi'(\alpha) + \frac{e_n^2}{2!}\varphi''(\alpha) + \frac{e_n^3}{3!}\varphi'''(\alpha) + ... + \frac{e_n^7}{7!}\varphi^{(7)}(\alpha) + O(e_n^8) \tag{22}$$

Substituting $\varphi(\alpha) = 0$ and simplifying, we have

$$\varphi(x_n) = \varphi'(\alpha)[e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + c_5 e_n^5 + c_6 e_n^6 + c_7 e_n^7 + O(e_n^8)] \tag{23}$$

where, $c_n = \frac{\varphi^n(\alpha)}{(n!)\varphi'(\alpha)}$ for $j = 2, 3, 4.....$
Furthermore, we get

$$\varphi'(x_n) = \varphi'(\alpha)[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + 6c_6 e_n^5 + 7c_7 e_n^6 + O(e_n^7)] \tag{24}$$

Using equation (23) and equation (24), we obtain

$$\begin{aligned}\frac{\varphi(x_n)}{\varphi'(x_n)} &= e_n - (2c_3 + 2c_2^2)e_n^3 + (-3c_4 + 7c_2c_3 - 4c_2^3)e_n^4 \\ &+ (8c_2^4 - 20c_2^2c_3 - 6c_3^2 + 10c_2c_4 - 4c_5)e_n^5 \\ &+ (-16c_2^5 + 52c_2^3c_3 - 33c_2c_3^2 - 28c_2^2c_4 + 17c_3c_4 + 13c_2c_5 - 5c_6)e^6 + O(e_n^7)\end{aligned} \tag{25}$$

By utilizing the equation (25) in the first step of equation (21), we get

$$\begin{aligned}y_n &= c_2e_n^2 + (2c_3 - 2c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 5c_2^3)e_n^4 \\ &+ (12c_2^4 - 24c_2^2c_3 + 6c_3^2 + 10c_2c_4 - 4c_5)e_n^5 \\ &+ (28c_2^5 - 73c_2^3c_3 + 37c_2c_3^2 + 34c_2^2c_4 - 17c_3c_4 - 13c_2c_5 + 5c_6)e_n^6 + O(e_n^7)\end{aligned} \tag{26}$$

We expand $\varphi(y_n)$ and $\varphi'(y_n)$ in powers of e_n by Taylor's series expansion using equation (26), we get

$$\begin{aligned}\varphi(y_n) = \varphi'(\alpha)[&c_2e_n^2 + (2c_3 - 2c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 5c_2^3)e_n^4\\ &+ (12c_2^4 - 24c_2^2c_3 + 6c_3^2 + 10c_2c_4 - 4c_5)e_n^5\\ &+ (28c_2^5 - 73c_2^3c_3 + 37c_2c_3^2 + 34c_2^2c_4 - 17c_3c_4 - 13c_2c_5 + 5c_6)e_n^6 + O(e_n^7)]\end{aligned} \tag{27}$$

and

$$\begin{aligned}\varphi'(y_n) = \varphi'(\alpha)[&1 + 2c_2^2e_n^2 + (4c_2c_3 - 4c_2^3)e_n^3 + (6c_2c_4 - 11c_2^2c_3 + 8c_2^4)e_n^4\\ &+ 4c_2(-4c_2^4 + 7c_2^2c_3 - 5c_2c_4 + 2c_5)e_n^5\\ &+ 2(16c_2^6 - 34c_2^4c_3 + 6c_3^3 + 30c_2^3c_4 - 8c_2c_3c_4 - 13c_2^2c_5 + 5c_2c_6)e_n^6 + O(e_n^7)]\end{aligned} \tag{28}$$

Using equations (23), (24), (26), (27) and (28) in equation (20), we get

$$\begin{aligned}S(x_n,y_n) = \varphi'(\alpha)[&2c_2 + (6c_2c_3 - 2c_4)e_n^2 - 4(3c_2^2 - 3c_3^2 - c_2c_4 + c_5)e_n^3\\ &+ 2(12c_2^3c_3 + c_2^2c_4 + 13c_3c_4 + c_2(-21c_3^2 + c_5) - 3c_6)e_n^4\\ &+ 4(12c_2^4c_3 + 9c_3^3 + 6c_2^3c_4 + 12c_2c_3c_4 - 3c_4^2 - 7c_3c_5 - c_2^2(30c_3^2 + c_5) + 2c_7)e_n^5\\ &+ 2(48c_2^5c_3 + 44c_2^4c_4 - 43c_3^2c_4 + 11c_4c_5 - c_2^3(156c_3^2 + 5c_5) + 15c_3c_6\\ &+ c_2^2(14c_3c_4 + 3c_6) + c_2(99c_3^3 + 10c_4^2 - 30c_3c_5 - c_7) - 5c_8)e_n^6 + O(e_n^7)]\end{aligned} \tag{29}$$

Using equations (24), (25), (28), (29) and (30) in the second step of method given by equation (21), we get

$$x_{n+1} = \alpha - c_2^3c_3e_n^6 + O(e_n)^7 \tag{30}$$

Then we can write equation (30) as

$$e_{n+1} = -c_2^3c_3e_n^6 + O(e_n)^7 \tag{31}$$

Hence, the method given by equation (21) has sixth order convergence.

3. Numerical Experimentation

In this section, we present the efficiency of the new proposed methods by applying the method on some nonlinear test function. The test functions and their initial guesses are listed in Table 1.

TABLE 1. Test functions with initial guesses

Test Function $\varphi(x)$	Initial guesses (x_0)
$\varphi_1(x) = (x)^2 - e^x - 3x + 2$	2
$\varphi_2(x) = (x-1)^3 - 1$	2.5
$\varphi_3(x) = x^3 - 10$	2
$\varphi_4(x) = cos(x) - 10$	1.7
$\varphi_5(x) = sin(x)^2 - x^2 + 1$	1

We compare the novel methods with the existing sixth order method given in (3), (4), (5), (6) and (7) denoted by NM1, NM2, PM, CM and SM respectively. We denote the methods given in equation (18) and (21) by NPM1 and NPM2 respectively. The results of numerical comparison on the test functions with their roots are summarized in Table 2 to Table 6. The absolute residual error ($\mid \varphi(x_n) \mid$) of the corresponding functions, the approximated root (x_n) and the total number of function evaluation (TNFE) after completion of four full iterations of methods is presented from Table 2 to Table 6. From the results available in Table 2 to Table 6, we conclude that the newly proposed method given by equation (21) provides a better estimation of roots than other existing methods.

TABLE 2. CONVERGENCE BEHAVIOUR FOR φ_1

Method	x_n	$\mid \varphi_1(x_n) \mid$	TNFE
NM1	0.257502	3.7207×10^{-565}	20
NM2	0.257502	2.3754×10^{-559}	20
PM	0.257502	1.0256×10^{-384}	20
CM	0.257502	9.5776×10^{-676}	16
SM	0.257502	2.0833×10^{-635}	20
NPM1	0.257502	1.7670×10^{-557}	20
NPM2	0.257502	8.7548×10^{-779}	16

TABLE 3. CONVERGENCE COMPARISON FOR φ_2

Method	x_n	$\mid \varphi_2(x_n) \mid$	TNFE
NM1	2	1.3866×10^{-518}	20
NM2	2	2.8571×10^{-604}	20
PM	2	1.7753×10^{-686}	20
CM	2	1.1237×10^{-474}	16
SM	2	1.1700×10^{-596}	20
NPM1	2	1.6772×10^{-693}	20
NPM2	2	1.6772×10^{-693}	16

TABLE 4. CONVERGENCE COMPARISON FOR φ_3

Method	x_n	$\mid \varphi_3(x_n) \mid$	TNFE
NM1	2.154434	1.1007×10^{-1398}	20
NM2	2.154434	5.8619×10^{-1501}	20
PM	2.154434	2.0492×10^{-1652}	20
CM	2.154434	2.8285×10^{-1272}	16
SM	2.154434	2.0386×10^{-1503}	20
NPM1	2.154434	4.7520×10^{-1579}	20
NPM2	2.154434	4.7520×10^{-1579}	16

TABLE 5. CONVERGENCE COMPARISON FOR φ_4

Method	x_n	$\mid \varphi_4(x_n) \mid$	TNFE
NM1	0.739085	1.1007×10^{-1181}	20
NM2	0.739085	5.8619×10^{-1217}	20
PM	0.739085	2.0492×10^{-888}	20
CM	0.739085	5.1947×10^{-857}	16
SM	0.739085	6.8174×10^{-1099}	20
NPM1	0.739085	1.2111×10^{-1276}	20
NPM2	0.739085	4.7520×10^{-1241}	16

TABLE 6. CONVERGENCE COMPARISON FOR φ_5

Method	x_n	$\mid \varphi_5(x_n) \mid$	TNFE
NM1	1.404491	1.4731×10^{-452}	20
NM2	1.404491	5.4945×10^{-512}	20
PM	1.404491	1.1839×10^{-566}	20
CM	1.404491	1.6063×10^{-285}	16
SM	1.404491	2.8223×10^{-538}	20
NPM1	1.404491	3.4810×10^{-748}	20
NPM2	1.404491	1.1600×10^{-612}	16

4. Concluding Remarks

We have introduced a new sixth-order root-finding scheme for solving non-linear equations with four function evaluations. Taylor's series and composition technique are used to build the proposed scheme. The new iterative approach has an efficiency index of 1.56. Compared to other existing well known sixth-order schemes, numerical experimentation has shown that the newly proposed method is faster, uses fewer total number of function evaluations, and has a very low absolute residual error.

Conflict of Interests

The author(s) declare that there is no conflict of interest.

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