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WEAK CONTRACTION CONDITION FOR FAINTLY COMPATIBLE MAPPINGS INVOLVING CUBIC TERMS OF METRIC FUNCTIONS

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Abstract: In this paper, we obtain a generalized common fixed point theorem for four mappings using the conditions of non-compatibility and faint compatibility satisfying a generalized $\emptyset -$weak contraction condition that involves cubic terms of $d(x, y)$. Also, we provide an example in support of our result.

Keywords: $\emptyset -$weak contraction; compatibility; non-compatibility; faint compatibility.

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1. INTRODUCTION

Banach fixed point theorem is the basic tool to study fixed point theory which ensure the existence and uniqueness of a fixed point under appropriate conditions. This theorem provides a technique for solving a variety of applied problems in mathematical sciences and engineering.

In 1969, Boyd and Wong [1] replaced the constant k in Banach contraction principle by a control function ψ as follows:

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Let (X, d) be a complete metric space and $\psi : [0, \infty) \to [0, \infty)$ be an upper semi continuous from the right such that $0 \le \psi(t) < t$ for all $t > 0$. If $T : X \to X$ satisfies $d(T(x), T(y)) \le \psi(d(x, y))$ for all $x, y \in X$, then it has a unique fixed point.

In 1997, Alber and Gueree-Delabriere [2] introduced the concept of weak contraction and Rhoades [3] had shown that the results of Alber et al. equally hold good in complete metric spaces.

A map $T : X \to X$ is said to be weak contraction if for each x, $y \in X$, there exists a function $\emptyset : [0, \infty) \to [0, \infty)$, $\emptyset(t) > 0$ and $\emptyset(0) = 0$ such that

$$d(Tx, Ty) \le d(x, y) - \emptyset(d(x, y)).$$

In 2013, Murthy and Prasad [4] introduced a new type of inequality having cubic terms of $d(x, y)$ that extended and generalized the results of Alber and Gueree-Delabriere [2] and others cited in the literature of fixed point theory.

In 2018, Jain et al. [5] introduced a new type of inequality having cubic terms of $d(x, y)$ that extended and generalized the results of Murthy et al. [4] and others cited in the literature of fixed point theory for two pairs of compatible mappings.

In this paper, we extend and generalize the result of Jain et al. [5] for four mappings using the conditions of non-compatibility and faint compatibility satisfying a generalized $\emptyset -$weak contraction condition that involves cubic terms of $d(x, y)$.

2. PRELIMINARIES

In this section, we give some basic definitions and results that are useful for proving our main results.

The notion of commutativity of mappings in fixed point theory was first used by Jungck [6] to obtain a generalization of Banach's fixed point theorem for a pair of mappings. This result was further generalized, extended and unified by using various types of minimal commutative mappings.

Definition 2.1[6] The pair (f, g) of a metric space (X, d) are said to be commuting if $fgx = gfx$ for all x in X.

The first ever attempt to relax the commutativity of mappings to weak commutative was initiated by Sessa [7] as follows:

Definition 2.2 [7] The pair (f, g) of a metric space (X, d) are said to be weakly commuting if $d(fgx, gfx) \leq d(gx, fx)$ for all x in X.

Remark 2.3 Commutative mappings are weak commutative mappings, but the converse may not be true.

In 1986, Jungck [8] introduced the notion of compatible mappings as follows:

Definition 2.4 The pair (f, g) of a metric space (X, d) are said to be compatible if $lim_n d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_n fx_n = lim_n gx_n = t$ for some t in X.

Definition 2.5 [8] The pair (f, g) of a metric space (X, d) are said to be non-compatible if there exists a sequence $\{ x_n \}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$ but $\lim_{n\to\infty} d(fgx_n, gfx_n)$ is either non-zero or does not exist.

In 1996, Jungck and Rhoades [9] introduced the notion of weakly compatible mappings which is more general than that of compatibility.

Definition 2.6 [9] The pair (f, g) of a metric space (X, d) are said to be weakly compatible if the mappings commute at all of their coincidence points, i.e., $fx = gx$ for some $x \in X$ implies $fgx = gfx$.

In 2008, Al-Thagafi and Shahzad [10] weakened the concept of weakly compatible mappings by giving the new concept of occasionally weakly compatible mappings.

Definition 2.7 [10] The pair (f, g) of a metric space (X, d) are said to be occasionally weakly compatible if there exists a point $x \in X$ which is a coincidence point of f and g at which f and g commute, that is , there exists a point $x \in X$ such that $fx = gx$ and $fgx = gfx$.

In 2010, Pant et al. [11] redefined the concept of occasionally weakly compatible mappings by introducing conditional commutativity.

Definition 2.8 [11] The pair (f, g) of a metric space (X, d) are said to be conditionally commuting if the pair commutes on a non-empty subset of the set of coincidence points whenever the set of coincidences is non empty.

Again, Pant et al. [12] gave the concept of conditional compatibility which is independent of compatibility condition and proved that in case of existence of unique common fixed point/ coincidence point; conditional compatibility cannot be reduced to the compatibility condition.

Further, they also proved that conditional compatibility need not imply Commutativity at the coincidence points.

Definition 2.9 [12] The pair (f, g) of a metric space (X, d) are said to be conditionally compatible iff whenever the set of sequences $\{x_n\}$ satisfying $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n$ is non-empty, there exist a sequence $\{y_n\}$ such that

$$\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = t \text{ and } \lim_{n\to\infty} d(fgy_n, gfy_n) = 0$$

Over the last two decades, there are a number of common fixed/coincidence point theorems for the pair of mappings under different contractive conditions with compatibility and its weaker versions imposed on the mappings(for more details, see [13,14,15,16,17,18] and reference therein).

In 2013, Bisht and Shahzad [18] gave a new notion of conditionally compatible mappings in slightly different settings and named it as faintly compatible mappings.

Definition 2.10 The pair (f, g) of a metric space (X, d) are said to be faintly compatible iff (f, g) is conditionally compatible and (f, g)commutes on a non empty subset of coincidence points whenever the set of coincidences is non empty.

Bisht et al. [18] proved some interesting common fixed point theorems using the concept of faintly compatible mappings on non complete metric spaces under different contractive conditions.

Remark 2.11 Compatibility, weakly compatible, occasionally weakly compatible implies faint compatibility, but converse is not true in general.

Remark 2.12 Faint compatibility and non-compatibility are independent concepts.

In one of the interesting paper, Jungck [19] established a common fixed point theorem for four mappings in a complete metric space. Now, we prove our main result for the existence of common fixed point for four mappings in a non-complete metric space using the concept of faintly compatible mappings which is analogous to the result of Jungck [19].

3. MAIN RESULTS

In this section, we extend and generalize the result of Jain et al. [5] for four mappings using the conditions of non-compatibility and faint compatibility satisfying a generalized $\emptyset -$weak contraction condition that involves cubic terms of $d(x, y)$.

Theorem 3.1 Let A, B, S and T be four continuous self mappings of a complete metric space (X, d) satisfying the following conditions:

(i) pairs (A, S) and (B, T) are non-compatible and faintly compatible;

(ii) $B(X) \subset S(X), A(X) \subset T(X)$;

(iii)

$$[1 + pd(Ax, By)]d^2(Sx, Ty) \leq$$

$$pmax\begin{Bmatrix} \frac{1}{2}[d^2(Ax, Sx)d(By, Ty) + d(Ax, Sx)d^2(By, Ty)], \\ d(Ax, Sx)d(Ax, Ty)d(By, Sx), \\ d(Ax, Ty)d(By, Sx)d(By, Ty) \end{Bmatrix} +$$

$$m(Ax, By) - \emptyset\{m(Ax, By)\}$$

for all $x, y \in X$, where

$$m(Ax, By) = \max\begin{Bmatrix} d^2(Ax, By), d(Ax, Sx)d(By, Ty), d(Ax, Ty)d(By, Sx), \\ \frac{1}{2}[d(Ax, Sx)d(Ax, Ty) + d(By, Sx)d(By, Ty)] \end{Bmatrix}$$

$p \geq 0$ is a real number and $\emptyset$: $[0, \infty) \to [0, \infty)$ is a continuous function with $\emptyset\ (t) = 0 \Leftrightarrow t = 0$ and $\emptyset(t) > 0$ for each $t > 0$.

Then there is a unique point $z \in X$ such that $Az = Bz = Sz = Tz = z$.

Proof. As the pair (A, S) is non compatible, then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$ but $\lim_{n\to\infty} d(ASx_n, SAx_n)$ is either non-zero or non-existent. Since A and S are faintly compatible and $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, there exists a sequence $\{z_n\}$ in X satisfying $\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Sz_n = u$(say)

such that

$$\lim_{n\to\infty} d(ASz_n, SAz_n) = 0. \tag{1}$$

Further, since A is continuous, $\lim_{n\to\infty} AAz_n = Au$ and $\lim_{n\to\infty} ASz_n = Au$. These last three limits together imply $\lim_{n\to\infty} SAz_n = Au$. The inclusion $A(X) \subset T(X)$ implies that $Au = Tv$

for some $v \in X$ and $\lim_{n\to\infty} AAz_n = Tv$, $\lim_{n\to\infty} SAz_n = Tv$.

Similarly, non compatibility of the pair B, T implies that there exists a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t'$ for some $t' \in X$ but $\lim_{n\to\infty} d(BTy_n, TBy_n)$ is either non-zero or non-existent. Now faintly compatibility of B and T will imply that there exists a sequence $\{w_n\}$ in X satisfying $\lim_{n\to\infty} Bw_n = \lim_{n\to\infty} Tw_n = u'$(say) such that

$$\lim_{n\to\infty} d(BTw_n, TBw_n) = 0. \qquad (2)$$

Further, since B is continuous, so $\lim_{n\to\infty} BBw_n = Bu'$ and $\lim_{n\to\infty} BTw_n = Bu'$. These last three limits together imply $\lim_{n\to\infty} TBw_n = Bu'$. The inclusion $B(X) \subset S(X)$ implies that $Bu' = Sv'$ for some $v' \in X$ and $\lim_{n\to\infty} BBw_n = Sv'$, $\lim_{n\to\infty} TBw_n = Sv'$.

Using the condition (iii), we get

$$[1 + pd(Az_n, Bw_n)]d^2(Sz_n, Tw_n) \le$$

$$p\max\left\{\begin{array}{c} \frac{1}{2}\begin{bmatrix} d^2(Az_n, Sz_n)d(Bw_n, Tw_n) \\ +d(Az_n, Sz_n)d^2(Bw_n, Tw_n)\end{bmatrix}, \\ d(Az_n, Sz_n)d(Az_n, Tw_n)d(Bw_n, Sz_n), \\ d(Az_n, Tw_n)d(Bw_n, Sz_n)d(Bw_n, Tw_n)\end{array}\right\} +$$

$$m(Az_n, Bw_n) - \emptyset\{m(Az_n, Bw_n)\}$$

for all $x, y \in X$, where

$$m(Az_n, Bw_n) = \max\left\{\begin{array}{c} d^2(Az_n, Bw_n), d(Az_n, Sz_n)d(Bw_n, Tw_n), d(Az_n, Tw_n)d(Bw_n, Sz_n), \\ \frac{1}{2}[d(Az_n, Sz_n)d(Az_n, Tw_n) + d(Bw_n, Sz_n)d(Bw_n, Tw_n)]\end{array}\right\}$$

Further on solving

$$[1 + pd(u, u')]d^2(u', u') \le$$

$$p\max\left\{\begin{array}{c} \frac{1}{2}[d^2(u, u')d(u', u') + d(u, u')d^2(u', u')], \\ d(u, u')d(u, u')d(u', u'), \\ d(u, u')d(u', u')d(u', u')\end{array}\right\} +$$

$$m(u, u') - \emptyset\{m(u, u')\}$$

for all $x, y \in X$, where

$$m(u, u') = max\left\{\begin{array}{c} d^2(u, u'), d(u, u')d(u', u'), d(u, u')d(u', u'), \\ \frac{1}{2}[d(u, u')d(u, u') + d(u', u')d(u', u')]\end{array}\right\}$$

$\Longrightarrow\ d(u, u') = 0 \Longrightarrow u = u'$.

So $Au = Tv$ and $Bu = Sv'$.

Now, $\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Sz_n = \lim_{n\to\infty} Bw_n = \lim_{n\to\infty} Tw_n =$ u.

Continuity of S and T together with conditions (1) and (2) imply

$\lim_{n\to\infty} SSz_n = \lim_{n\to\infty} SAz_n = Su \Rightarrow \lim_{n\to\infty} SSz_n = \lim_{n\to\infty} ASz_n = Su$,

and $\lim_{n\to\infty} TBw_n = \lim_{n\to\infty} TTw_n = Tu \Rightarrow \lim_{n\to\infty} TTw_n = \lim_{n\to\infty} BTw_n = Tu$.

Now,

$$[1 + pd(ASz_n, BTw_n)]d^2(SSz_n, TTw_n) \leq$$

$$pmax\left\{\begin{matrix} \frac{1}{2}\begin{bmatrix} d^2(ASz_n, SSz_n)d(BTw_n, TTw_n) \\ +d(ASz_n, SSz_n)d^2(BTw_n, TTw_n)\end{bmatrix}, \\ d(ASz_n, SSz_n)d(ASz_n, TTw_n)d(BTw_n, SSz_n), \\ d(ASz_n, TTw_n)d(BTw_n, SSz_n)d(BTw_n, TTw_n)\end{matrix}\right\} +$$

$$m(ASz_n, BTw_n) - \emptyset\{m(ASz_n, BTw_n)\}$$

for all $x, y \in X$, where

$$m(ASz_n, BTw_n) =$$

$$max\left\{\begin{matrix} d^2(ASz_n, BTw_n), d(ASz_n, SSz_n)d(BTw_n, TTw_n), \\ d(ASz_n, TTw_n)d(BTw_n, SSz_n), \\ \frac{1}{2}[d(ASz_n, SSz_n)d(ASz_n, TTw_n) + d(BTw_n, SSz_n)d(BTw_n, TTw_n)]\end{matrix}\right\}$$

Taking $n \to \infty$, we get

$$[1 + pd(Su, Tu)]d^2(Su, Tu) \leq$$

$$pmax\left\{\begin{matrix} \frac{1}{2}\begin{bmatrix} d^2(Su, Su)d(Tu, Tu) \\ +d(Su, Su)d^2(Tu, Tu)\end{bmatrix}, \\ d(Su, Su)d(Su, Tu)d(Tu, Su), \\ d(Su, Tu)d(Tu, Su)d(Tu, Tu)\end{matrix}\right\} + m(Su, Tu) -$$

$$\emptyset\{m(Su, Tu)\}$$

for all $x, y \in X$, where

$$m(Su, Tu) = \ max\left\{\begin{matrix} d^2(Su, Tu), d(Su, Su)d(Tu, Tu), d(Su, Tu)d(Tu, Su), \\ \frac{1}{2}[d(Su, Su)d(Su, Tu) + d(Tu, Su)d(Tu, Tu)]\end{matrix}\right\}$$

On solving we get

$$d(Su, Tu) = 0 \Rightarrow Su = Tu. \tag{3}$$

Also,

$$[1 + pd(Au, BTw_n)]d^2(Su, TTw_n) \leq$$

$$pmax\left\{\begin{matrix} \frac{1}{2}\begin{bmatrix} d^2(Au, Su)d(BTw_n, TTw_n) \\ +d(Au, Su)d^2(BTw_n, TTw_n)\end{bmatrix}, \\ d(Au, Su)d(Au, TTw_n)d(BTw_n, Su), \\ d(Au, TTw_n)d(BTw_n, Su)d(BTw_n, TTw_n)\end{matrix}\right\} +$$

$$m(Au, BTw_n) - \emptyset\{m(Au, BTw_n)\}$$

for all $x, y \in X$, where

$m(Au, BTw_n) =$

$$max\begin{Bmatrix} d^2(Au, BTw_n), d(Au, Su)d(BTw_n, TTw_n), d(Au, TTw_n)d(BTw_n, Su), \\ \frac{1}{2}[d(Au, Su)d(Au, TTw_n) + d(BTw_n, Su)d(BTw_n, TTw_n)] \end{Bmatrix}$$

Taking $n \to \infty$, and on simplification, we get

$$d(Au, Tu) = 0 \Rightarrow Au = Tu. \qquad (4)$$

Using (iii) with $x = y = u$, we get

$[1 + pd(Au, Bu)]d^2(Su, Tu) \leq$

$$pmax\begin{Bmatrix} \frac{1}{2}[d^2(Au, Su)d(Bu, Tu) + d(Au, Su)d^2(Bu, Tu)], \\ d(Au, Su)d(Au, Tu)d(Bu, Su), \\ d(Au, Tu)d(Bu, Su)d(Bu, Tu) \end{Bmatrix} +$$

$$m(Au, Bu) - \emptyset\{m(Au, Bu)\}$$

for all $x, y \in X$, where

$$m(Au, Bu) = \max\begin{Bmatrix} d^2(Au, Bu), d(Au, Su)d(Bu, Tu), d(Au, Tu)d(Bu, Su), \\ \frac{1}{2}[d(Au, Su)d(Au, Tu) + d(Bu, Su)d(Bu, Tu)] \end{Bmatrix}$$

Using (3), (4) and on simplification, we get

$$d(Au, Bu) = 0 \Rightarrow Au = Bu. \qquad (5)$$

From (4), (5) and (6), we have $Au = Bu = Su = Tu$. In fact, u is a common fixed point of A, B, S and T. To see this,

Put $x = z_n$ and $y = u$ in (iii), we get

$[1 + pd(Az_n, Bu)]d^2(Sz_n, Tu) \leq$

$$pmax\begin{Bmatrix} \frac{1}{2}\begin{bmatrix} d^2(Az_n, Sz_n)d(Bu, Tu) \\ +d(Az_n, Sz_n)d^2(Bu, Tu) \end{bmatrix}, \\ d(Az_n, Sz_n)d(Az_n, Tu)d(Bu, Sz_n), \\ d(Az_n, Tu)d(Bu, Sz_n)d(Bu, Tu) \end{Bmatrix} + m(Az_n, Bu) -$$

$$\emptyset\{m(Az_n, Bu)\}$$

for all $x, y \in X$, where

$$m(Az_n, Bu) = \max\begin{Bmatrix} d^2(Az_n, Bu), d(Az_n, Sz_n)d(Bu, Tu), d(Az_n, Tu)d(Bu, Sz_n), \\ \frac{1}{2}[d(Az_n, Sz_n)d(Az_n, Tu) + d(Bu, Sz_n)d(Bu, Tu)] \end{Bmatrix}$$

After simplification, we get

$d(u, Bu) = 0 \Rightarrow Bu = u.$

Hence, $Au = Bu = Su = Tu = u.$

For the uniqueness of the common fixed point, let w be another common fixed point of A, B, S and T, i.e., $Aw = Bw = Sw = Tw = w$.

On putting $x = u \; and \; y = w$ in (iii) and on simplification, we get

$$d(u, w) = 0 \Rightarrow u = w.$$

Now, we give an example in support of our result.

Example 3.2 Let $X = [2,20]$ and d be a usual metric. Define the self mappings A, B, S and T on X by

$$Ax = \begin{cases} 12 & if \quad 2 < x \leq 5 \\ x - 3 & if \quad x > 5 \\ 2 & if \quad x = 2. \end{cases}, \qquad Bx = \begin{cases} 2 & if \quad x = 2 \\ 6 & if \quad x > 2 \end{cases},$$

$$Sx = \begin{cases} 6 & if \quad 2 < x \leq 5 \\ x & if \quad x = 2 \\ 2 & if \quad x > 5. \end{cases} \quad \text{and} \quad Tx = \begin{cases} x & if \quad x = 2 \\ 3 & if \quad x > 2 \end{cases}.$$

Let us consider a sequence $\{x_n\}$ with $x_n = 2$. It is easy to verify that all the conditions of Theorem 3.1 are satisfied. In fact, 2 is the unique common fixed point of S, T, A and B.

Taking $A = B$ and $S = T$ in Theorem 3.1, we obtain the following corollary.

Corollary 3.3 Let A, S be two continuous self mappings of a metric space(X, d). Suppose

(i)$A(X) \subset S(X)$,

(ii) Pairs (A, S) is non-compatible faintly compatible,

(iii)

$$[1 + pd(Sx, Sy)]d^2(Tx, Ty) \leq$$

$$p \, max \begin{Bmatrix} \frac{1}{2}[d^2(Sx, Tx)d(Sy, Ty) + d(Sx, Tx)d^2(Sy, Ty)], \\ d(Sx, Tx)d(Sx, Ty)d(Sy, Tx), \\ d(Sx, Ty)d(Sy, Tx)d(Sy, Ty) \end{Bmatrix}$$

$$+m(Sx, Sy) - \emptyset(m(Sx, Sy)),$$

where

$$m(Sx, Sy) = max \begin{Bmatrix} d^2(Sx, Sy), d(Sx, Tx)d(Sy, Ty), d(Sx, Ty)d(Sy, Tx), \\ \frac{1}{2}[d(Sx, Tx)d(Sx, Ty) + d(Sy, Tx)d(Sy, Ty)] \end{Bmatrix},$$

$p \geq 0$ is a real number and $\emptyset$: $[0, \infty) \rightarrow [0, \infty)$ is a continuous function with $\emptyset(t) = 0$ iff $t = 0$ and $\emptyset(t) > t$ for each $t > 0$.

Then S and A have a unique common fixed point in X.

Taking $T = S = I$ (Identity map) in Theorem 3.1, we obtain the following result as corollary:

Corollary 3.4 Let A, B be two continuous self mappings of a metric space(X, d). Suppose

(i)$A(X) \subset B(X)$,

(ii) Pairs (A, B) is non-compatible faintly compatible,

(iii)

$$[1 + pd(Ax, By)]d^2(x, y) \leq$$

$$pmax\begin{Bmatrix} \frac{1}{2}[d^2(Ax, x)d(By, y) + d(Ax, x)d^2(By, y)], \\ d(Ax, x)d(Ax, y)d(By, x), \\ d(Ax, y)d(By, x)d(By, y) \end{Bmatrix} +$$

$$m(Ax, By) - \emptyset\{m(Ax, By)\}$$

for all $x, y \in X$, where

$$m(Ax, By) = \max\begin{Bmatrix} d^2(Ax, By), d(Ax, x)d(By, y), d(Ax, y)d(By, x), \\ \frac{1}{2}[d(Ax, x)d(Ax, y) + d(By, x)d(By, y)] \end{Bmatrix},$$

$p \geq 0$ is a real number and $\emptyset$: $[0, \infty) \to [0, \infty)$ is a continuous function with $\emptyset(t) = 0$ iff $t = 0$ and $\emptyset(t) > t$ for each $t > 0$.

Then A and B have a unique common fixed point in X.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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