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COMPLEMENTARY CONNECTED DOMINATION AND CONNECTIVITY DOMINATION NUMBER OF AN ARITHMETIC GRAPH $G = V_n$

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Abstract. A subset S of V is said to be a complementary connected dominating set if every vertex not in S is adjacent to some vertex in S and the sub graph induced by $V - S$ is connected. The complementary connected domination number of the graph is denoted by $\gamma_{ccd}(G)$ and is defined as the minimum number of vertices which form a ccd-set. A set S of vertices in a graph G is a *connectivity dominating set* if every vertex not in S is adjacent to some vertex in S and the sub graph induced by $V - S$ is not connected. The *connectivity domination number* $\kappa_\gamma(G)$ is the minimum size of such set.

Keywords: arithmetic graph; complementary connected dominating set; connectivity dominating set.

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1. Introduction

For notation and graph theory terminalogy not given here, we follow [1]. In [6] the super connectivity number of an arithmetic graph is studied by L. Mary Jenitha and S. Sujitha. In this paper connectivity domination and complementary connected domination number of an Arithmetic Graph $G = V_n$ is studied. The concept of complementary connected domination number was introduced by V.R.Kulli and B.Janakiram,they called this parameter as non-split domination

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number in [2]. T.Tamizhchelvam and B.Jaya Prasad have studied the complementary connected domination number of a graph in [9]. Kulli.V.R. and Janakiram.B have introduced the concept of the connectivity dominating set and called this parameter as split dominating set in [3]. The arithmetic graph V_n is defined as a graph with its vertex set is the set consists of the divisors of n(excluding1) where n is a positive integer and $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ where $p_i's$ are distinct primes and $a_i's \geq 1$ and two distinct vertices a,b which are not of the same parity are adjacent in this graph if (a,b) = p_i for some i, $1 \leq i \leq r$. The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance $a = p^2$, $b = p^5$. This concepts was studied from [10]. Also various authors studied different parameters of an arithmetic graph. In [4] the connectivity number of an arithmetic graph is studied by L. Mary Jenitha and S. Sujitha Later, the various parameters of connectivity of an arithmetic graph are studied by the same authors in [5][7]. The following theorems are used in sequel.

Theorem 1.1. [5]For an arithmetic graph G=V_n, n =$p_1^{a_1} \times p_2^{a_2}$ where p_1 and p_2 are distinct primes, $a_1, a_2 \geq 1$ then $\varepsilon = 4a_1a_2 - a_1 - a_2$, where ε is the size of the graph G.

Theorem 1.2. [5] For an arithmetic graph G=V_n, n =$p_1^{a_1} \times p_2^{a_2}$ where p_1 and p_2 are distinct primes, $a_1, a_2 \geq 1$ then G is a bipartite graph.

Theorem 1.3. [4]For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2}$ where p_1 and p_2 are distinct primes, then

$$\kappa(V_n) = \kappa^{'}(V_n) = \begin{cases} 1 & for \text{ a}_i = 1 \ \& \ a_j > 1; i,j = 1,2 \\ 2 & for \text{ a}_i > 1; i = 1,2 \end{cases}$$

Theorem 1.4. [4]For an arithmetic graph $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ where $p_i, i = 1,2,\ldots,r\ r > 2$ are distinct primes and $a_i = 1$ for all $i = 1,2,\ldots,r$ then $\kappa(V_n) = \kappa^{'}(V_n) = r$.

Theorem 1.5. [8]If $m = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, where $a_i's > 1\ i = 1,2,\ldots,r$ then $\gamma_s(V_m) \leq r+1$, where r is the number of distinct prime factors of m.

2. Complementary Connected Domination Number of an Arithmetic Graph

In this section, the complementary connected domination number of an arithmetic graph $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, a_i \geq 1$ is found.

Definition 2.1. [2] A subset S of V is said to be a *complementary connected dominating set* if every vertex not in S is adjacent to some vertex in S and the sub graph induced by $V - S$ is connected. The *complementary connected domination number* of the graph is denoted by $\gamma_{cc}(G)$ and is defined as the minimum number of vertices which form a ccd-set.

Theorem 2.2. In an arithmetic graph $G = V_n$ where $n = p_1 \times p_2$ the complementary connected domination number $\gamma_{cc}(G) = 2$.

Proof. Consider the graph $G = V_n$ where $n = p_1 \times p_2$, then by the definition of arithmetic graph, G is a path with three vertices say $p_1, p_2, p_1 \times p_2$. Since $(p_1, p_1 \times p_2) = p_1$ and $(p_2, p_1 \times p_2) = p_2$, it follows that the vertex $p_1 \times p_2$ is the internal vertex of the path. Hence the vertex $p_1 \times p_2$ dominates the vertices p_1 and p_2. But the induced graph $G - \{p_1 \times p_2\}$ is disconnected so it is necessary to choose one more vertex. Then the complementary connected dominating set S is either $\{p_1, p_1 \times p_2\}$ or $\{p_2, p_1 \times p_2\}$. Now the induced graph $G[V - S]$ is an isolated vertex, which is connected. Since S is minimum, the complementary connected domination number $\gamma_{cc}(G) = 2$. □

Theorem 2.3. In an arithmetic graph $G = V_n$ where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, a_i = 1\ \forall i,\ r > 2$ the complementary connected domination number $\gamma_{cc}(G) = r - 1$.

Proof. Consider the graph $G = V_n$ where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, a_i = 1\ \forall i$
If $r > 2$ then the vertex set consists of primes, product of two primes,..., product of r primes. We claim that the set $S = \{p_1, p_2, \ldots, p_{r-2}, p_{r-1} \times p_r\}$ is a complementary connected dominating set. Let v be any vertex in $V - S$ clearly v is of the form $\{p_{r-1}, p_r, p_1 \times p_2, \ldots, p_{r-2} \times p_{r-1}, p_{r-2} \times p_r, \ldots, p_1 \times p_2 \times \cdots \times p_r\}$. Here $p_{r-1}, p_r \in V - S$ are dominated by $p_{r-1} \times p_r \in S$ and by the definition of arithmetic graph $G = V_n$ all other vertices in $V - S$ are adjacent to atleast

one vertex of $S-\{p_{r-1}\times p_r\}$. Hence S is a dominating set. Further, it is also a complementary connected dominating set. This can be discussed in the following two cases

Case (i) If r is even. Let $X_1,X_2,X_3\subset V-S$ such that $X_1\cup X_2\cup X_3=V-S$ where $X_1=\{u=\prod_{i\in B_1}p_i\ :B_1\subset\{1,2,\dots,r\},|B_1|\leq\frac{r}{2}\}$

$X_2=\{u=\prod_{i\in B_2}p_i\ :B_2\subset\{1,2,\dots,r\},|B_2|\geq\frac{r}{2}+1\}$

$X_3=\{p_{r-1},p_r\}$. In X_1, their exist atleast one path between every pair of vertices. In X_2, no two pair of vertices are adjacent to themselves but all the vertices are either adjacent to p_{r-1} or p_r or both or the vertices in X_1. In X_3, both vertices are adjacent to some vertices of X_1. Hence the induced graph $G[V-S]$ is connected.

Case (ii) If r is odd, Let $Y_1,Y_2,Y_3,Y_4\subset V-S$ such that $Y_1\cup Y_2\cup Y_3\cup Y_4\subset V-S$ where

$Y_1=\{u=\prod_{i\in B_1}p_i\ :B_1\subset\{1,2,\dots,r\},|B_1|\geq\lceil\frac{r}{2}\rceil+1\}$

$Y_2=\{u=\prod_{i\in B_2}p_i\ :B_1\subset\{1,2,\dots,r\},|B_2|=\lceil\frac{r}{2}\rceil\}$

$Y_3=\{u=\prod_{i\in B_3}p_i\ :B_3\subset\{1,2,\dots,r\},|B_3|<\lceil\frac{r}{2}\rceil\}$

$Y_4=\{p_{r-1},p_r\}$.

By the definition of an arithmetic graph, we can easily identify that no two vertices in Y_1 are adjacent, but is adjacent to atleast one vertex of the set Y_3 and Y_4. The vertices of Y_3 are adjacent to some vertices of Y_2 and Y_4. Hence, induced graph $G[V-S]$ is connected. Thus, in both the cases S is a complementary connected dominating set. To prove S is minimum. If we remove any one of $v=p_i;i=1,2,\dots,r-2$ then the vertex $p_i\times p_r$ is not dominated by any of the vertex in $S-\{p_i\}$. Also, if we remove the vertex $p_{r-1}\times p_r$ then the vertices p_r,p_{r-1} are not dominated by any of the vertex in $S-\{p_{r-1}\times p_r\}$. Hence S is minimum and we have $\gamma_{cc}(G)=r-1$. □

Theorem 2.4. If $G=V_n$ is an arithmetic graph $n=p_1^{a_1}\times p_2,a_1>1$ then $\gamma_{cc}(G)=a_1+1$.

Proof. Let $G=V_n$ be an arithmetic graph where $n=p_1^{a_1}\times p_2,a_1>1$ then the vertex set consist of vertices $\{p_1^{\alpha_1},p_2,p_1^{\alpha_1}\times p_2\ ;1\leq\alpha_1\leq a_1\}$. By Theorem 1.2, G is a bipartite graph with partition A and B where $A=\{p_1^{\alpha_1},p_2\ ;1\leq\alpha_1\leq a_1\}$ and $B=\{p_1^{\alpha_1}\times p_2\ ;1\leq\alpha_1\leq a_1\}$. Since both p_1 and $p_2\in A$ are adjacent to all the vertices in partition B and $p_1\times p_2\in B$ is adjacent to all the vertices in partition A. Both the sets $S_1=\{p_1,p_1\times p_2\}$ as well as $S_2=\{p_2,p_1\times p_2\}$ are dominating sets. But $d(p_1^{\alpha_1})=1$ for $1<\alpha_1\leq a_1$ and the neighbour of these vertices is $N(p_1^{\alpha_1})=\{p_1\times p_2\}$ for $1<\alpha_1\leq a_1$. It is clear that, $G[V-S_1]$ is a disconnected graph with

a_1 components in which one of the component is a connected graph and the remaining $a_1 - 1$ components are isolated vertices. By the definition of complementary connected domination, the induced graph $G[V - S_1]$ is to be connected. Hence, we include the isolated vertices to the set S_1. Now, the set $S = S_1 \cup \{p_1^2, p_1^3, \dots, p_1^{a_1}\}$ is a complementary connected dominating set. To prove S is minimum. (i) If we remove any one of the vertex say p_1 from S then the vertices of the form $p_1^{\alpha_1} \times p_2$; $1 < \alpha \leq a_1$ in partition B are not dominated by any vertices in $S - \{p_1\}$.
(ii) If we remove $p_1 \times p_2$. Then the vertex p_2 is not adjacent to any one of the vertex in the set $S - \{p_1 \times p_2\}$.
(iii) If we remove any one of the vertex of the form $p_1^{\alpha_1}; \alpha_1 > 1$ then the induced graph $G[V - \{S - p_1^{\alpha_1}\}]$ is not connected. Hence S is minimum. Similarly $S_2 \cup \{p_1^2, p_1^3, \dots, p_1^{a_1}\}$ is also a minimum complementary connected dominating set, which has $\gamma_{cc}(G) = a_1 + 1$. □

Theorem 2.5. If $G = V_n$ is an arithmetic graph $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, a_i > 1,\ r \geq 2$ for atleast one i then $\gamma_{cc}(G) = r$.

Proof. Consider an arithmetic graph $G = V_n$ where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ and assume that atleast one $a_i > 1$. The vertices of $G = V_n$ are primes, their powers, product of primes, products of powers of prime. Now, we claim that $S = \{p_1, p_2, \dots, p_{r-1}, p_1 \times p_2 \times \cdots \times p_r\}$ is a complementary connected dominating set. First to prove S is a dominating set. Since $(p_i^{\alpha_i}, p_1 \times p_2 \times \cdots \times p_r) = p_i$ the vertices of the form $p_i^{\alpha_i} \in V - S,\ i = \{1, 2, \dots, r\},\ \alpha_i > 1$ are dominated by $p_1 \times p_2 \times \cdots \times p_r$. Also, $(p_i, p_i \times p_j) = p_i$; $i = 1, 2, \dots, r-1, j = 2, 3, \dots, r,\ i \neq j$. Therefore, the product of two primes are dominated by $\{p_1, p_2, \dots, p_r\}$. Similarly, for products of three primes, …, products of $r - 1$ primes are also dominated by $\{p_1, p_2 \dots, p_r\}$. Again $(p_i, p_i^{\alpha_i} \times p_j) = p_i$; $i = 1, 2, \dots, r-1, j = 2, 3, \dots, r,\ i \neq j,\ (p_i, p_i^{\alpha_i} \times p_j^{\alpha_j}) = p_i$; $i = 1, 2, \dots, r-1, j = 2, 3, \dots, r,\ i \neq j$. Also, $(p_i, p_1^{\alpha_1} \times p_2^{\alpha_2} \times \cdots \times p_{r-1}^{\alpha_{r-1}} \times p_r^{\alpha_r}) = p_i$; $i = 1, 2 \dots r-1$. Thus, all the vertices in $V - S$ are dominated by the vertices in S. Hence S is a dominating set. Now to prove $G[V - S]$ is connected. Consider the vertices in $V - S$. The vertices of the form $p_1^{\alpha_1}, p_2^{\alpha_2}, \dots, p_r^{\alpha_r} \in V - S$ are adjacent to atleast one vertices of the form $p_i \times p_j$; $i = 1, 2, \dots, r,\ j = 2, 3, \dots, r$; $i \neq j$ and all the vertices of the form $p_1^{\alpha_1} \times p_2^{\alpha_2} \times \cdots \times p_{r-1}^{\alpha_{r-1}} \times p_r^{\alpha_r}$; $\alpha_i > 1$ for atleast one $i,\ i \in \{1, 2, \dots, r\}$ are adjacent to the vertex p_r. Thus for every pair of vertices in $G[V - S]$ there exist atleast one path. Hence, the induced graph $V - S$ is connected. To prove S

is minimum. Suppose we remove any vertex say $p_i \in S$ then the vertex $p_i \times p_r$ is not dominated by any vertices from $S - p_i$ and hence, it violates the definition of dominating set. Thus S is a complementary connected dominating set. □

3. CONNECTIVITY DOMINATION NUMBER OF AN ARITHMETIC GRAPH

By Theorem 1.5 the connectivity domination number for an arithmetic graph $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ where $a_i's > 1$ is found. Hence, in our study we excluded the above case and the connectivity domination number for an arithmetic graph $G = V_n$, for remaining cases are discussed.

Definition 3.1. [3] A set S of vertices in a graph G is a *connectivity dominating set* if every vertex not in S is adjacent to some vertex in S and the sub graph induced by $V - S$ is not connected. The *connectivity domination number* $\kappa_\gamma(G)$ is the minimum size of such set.

Theorem 3.2. In an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2}$ where $a_1 = a_2 = 1$ has the connectivity domination number $\kappa_\gamma(G) = 1$.

Proof. Consider the arithmetic graph $G = V_n$, where n is the product of two distinct primes. The vertex set of V_n contains three vertices namely $p_1, p_2, p_1 \times p_2$. By the definition of an arithmetic graph G is a path with 3 vertices. Obviously, $(p_1, p_1 \times p_2) = p_1$ and $(p_2, p_1 \times p_2) = p_2$. Hence the internal vertex $p_1 \times p_2$ dominates the vertices p_1 and p_2. Therefore, the set $S = \{p_1 \times p_2\}$ is a dominating set. Also, we can easily observe that the induced graph of $V - S$ is disconnected. Hence proved. □

Theorem 3.3. For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2$ where $a_1 > 1$ then $\kappa_\gamma(G) = 2$.

Proof. By Theorem 1.2, the given arithmetic graph $G = V_n$ is a bipartite graph, with partition $A = \{p_1, p_1^2, \ldots, p_1^{a_1}, p_2\}$ and $B = \{p_1 \times p_2, p_1^2 \times p_2, p_1^3 \times p_2, \ldots, p_1^{a_1} \times p_2\}$. By Theorem 1.3, the set $S_1 = \{p_1 \times p_2\}$ is a minimum vertex cut. Since G is a bipartite graph the dominating set must contain atleast two vertices. Also, the vertex $p_1 \times p_2 \in B$ is adjacent to all the vertices of the partition A and the vertex $p_1 \in A$ is adjacent to all the vertices in partition B. Hence, the set $S = \{p_1, p_1 \times p_2\}$ is a dominating set. Similarly $\{p_2, p_1 \times p_2\}$ is also a dominating set.

Now to prove the induced subgraph $G[V-S]$ is not connected. Since the graph G has $a_1 - 1$ pendant vertices say $p_1^2, p_1^3, \ldots, p_1^{a_1}$ and all these pendant vertices has a common neighbour $p_1 \times p_2$. But $p_1 \times p_2 \notin V(G) - S$ so the induced graph of $V(G) - S$ is disconnected. Also, a single vertex does not dominate a bipartite graph. Hence, S is minimum. Thus $\kappa_\gamma(G) = 2$. □

Theorem 3.4. Let $G = V_n$ be an arithmetic graph where $n = p_1^{a_1} \times p_2^{a_2}$, $a_1, a_2 \geq 2$ then $\kappa_\gamma(G) = 3$.

Proof. Consider an arithmetic graph $G = V_n$, $n = p_1^{a_1} \times p_2^{a_2}$, $a_1, a_2 \geq 2$, then the vertex set consists of primes, power of primes, product of primes, product of prime powers. By Theorem 1.2, G is a bipartite graph with partitions A and B. Partition A contains primes and their powers and partition B contains product of primes and its powers. By Theorem 1.3, the set $S_1 = \{p_1, p_2\}$ is a minimum vertex cut. Since the set S_1 dominates only partition B, we include a vertex from partition B which are adjacent to all vertices in partition A. Let it be $p_1 \times p_2$. Now we claim that the set $S = \{p_1, p_2, p_1 \times p_2\}$ is a connectivity dominating set. It is obvious that the set S is a dominating set. Now to show that $[V(G) - S]$ is disconnected, the minimum degree of the graph $\delta(G) = 2$, since by the proof of Theorem 1.1, the vertices $\{p_1^{\alpha_1} \times p_2^{\alpha_2}; 1 < \alpha_1 \leq a_1, 1 < \alpha_2 \leq a_2\}$ are adjacent only to p_1 and p_2. Moreover, these vertices are minimum degree vertices. So it is clear that $[V(G) - S]$ is disconnected. Hence, the set $S = \{p_1, p_2, p_1 \times p_2\}$ is a connectivity dominating set. To prove S is a minimum connectivity dominating set, Suppose not $S' \subset S$ is a minimum connectivity dominating set let us assume that $v \in S$ and $v \notin S'$ if (i) $v = p_1$ or p_2 by our assumption the induced graph of $V(G) - S'$ is disconnected this shows that their exist atleast one pair of vertices in $[V(G) - S']$ having no path, which is a contradiction to the adjacency condition of arithmetic graph. (ii) $v = p_1 \times p_2$ then the vertices p_1 and p_2 dominates all the vertices in $[V(G) - S']$ this means that $p_1^{\alpha_1}$ and p_1 are adjacent which is a contradiction to the definition of an arithmetic graph. Therefore, S is a minimum connectivity dominating set and hence we have $\kappa_\gamma(G) = 3$. □

Theorem 3.5. For an arithmetic graph $G = V_n$, $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, $r > 2$ and $a_i = 1$, for all $i \in \{1, 2, \ldots, r\}$. Then $\kappa_\gamma(G) = r$.

Proof. Consider an arithmetic graph $G = V_n$, where n is a product of primes without powers, then the vertex set consist of primes, product of two primes, product of three primes, $\ldots$, product of r primes. By Theorem 1.4 $S = \{p_1, p_2, \ldots, p_r\}$ is a minimum vertex cut and $\kappa(G) = r$. By the definition of arithmetic graph the vertices in the set $V(G) - S$ is adjacent to atleast one vertex of the set S. Hence the set S is a dominating set. Also, the minimum degree vertex of G is $p_1 \times p_2 \times \cdots \times p_r$ and $N(p_1 \times p_2 \times \cdots \times p_r) = \{p_1, p_2, \ldots, p_r\}$ hence it is clear that the induced graph of $V(G) - S$ is a disconnected graph with two components. Hence the set S is a connectivity dominating set. To prove S is minimum. Suppose $S_1 \subset S$ is a connectivity dominating set. Let us assume that $p_i \in S$ and $p_i \notin S_1$, since S_1 is a connectivity dominating set $< V - S_1 >$ is a disconnected graph this shows that $(p_i, p_1 \times p_2 \times \cdots \times p_r) \neq p_i$ which is a contradiction. Hence we have $\kappa_\gamma(G) = |S| = r$. □

Remark 3.6. For an arithmetic graph $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ and $a_i \geq 1$, for $i \in \{1, 2, \ldots, r\}$, the connectivity domination number is strictly less than complementary connected domination number. Also, it is found that, for arithmetic graphs other than $G = V_n,\ n = p_1^{a_1} \times p_2$; $a_1 > 2$, we have $\kappa_\gamma(G) = \gamma_{cc}(G) + 1$.

Following example shows that the connectivity domination number is equal to the complementary connected domination number plus one.

Example 3.7. Consider an arithmetic graph $G = V_{210}$ where $210 = 2 \times 3 \times 5 \times 7$ given in Figure 1. The set $S_1 = \{2, 3, 5, 7\}$ is a connectivity dominating set. Hence, we have $\kappa_\gamma(G) = 4$. Also the set $S_2 = \{2, 3, 5 \times 7\}$ is a complementary connected dominating set and the set is minimum so we have $\gamma_{cc}(G) = 3$. Thus we can easily observe that $\kappa_\gamma(G) = \gamma_{cc}(G) + 1$.

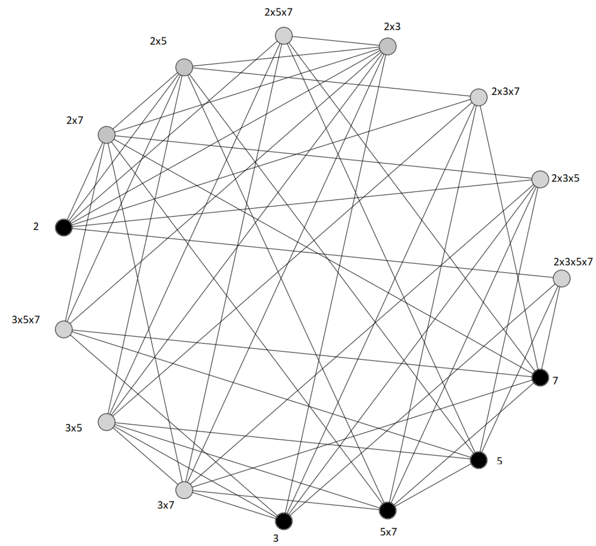

FIGURE 1. Arithmetic Graph $G = V_{210}$

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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