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BERNOULLI'S VACATION OF $M^x/G/1$ QUEUE WITH TWO-TIER SERVICE BASED VOLATILE SERVER

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Abstract: This article describes a Bernoulli's vacation of $M^x/G/1$ queue with two-tier service based volatile server. When the server is at running stage, the server may fail/breakdown and the service line will no longer be available due to downtime and latency. If the client does not come when the server is available, then the server will remain inactive on the system until the queue size increases to the threshold value T. If the client does not come when the server is unavailable, the server will remain inactive, but the threshold of T will decrease. When the queue size is greater than the threshold value of T, then the server immediately starts to do the service of its pending works of its clients. When the queue size is lesser than the threshold value of T, then the server immediately starts to do the re-service of its clients. In general, the distribution of queue sizes by random and departure periods, as well as various performance indicators of the system. After that the server can go on vacation or stay in the system to service the next

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device if necessary. Stationary analysis of the system is extended including the existence of stationary regime, queue size distribution of idle period process, embedded Markov chain steady state distribution along with some system characteristics.

Key Words: Bernoulli's vacation; Markov chain; threshold value; re-service; two tier service; volatile server.

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1. INTRODUCTION

Research in queuing theory patterns with breakdown was growing in the 1950s and some of the earliest articles in this area were written by many researchers. Later few researchers have investigated several discontinuous backup systems with the main assumptions that the service line will be repaired as soon as it fails [7]. In addition, Vedala et al [17] recently investigated several management policies for un-trusted servers, that is, machines with possible errors. All these research states that the server will be repaired immediately after the breakdown [18]. However, in many real-world situations, repairs may not start immediately. Accordingly, the server will have numerous chances to go on vacation if the queue is empty on return. Several authors have discussed various aspects of the Bernoulli vacation model.

In recent years, much attention has been paid to the study of queuing systems $M^x/G/1$, which performs two-stage task in accordance with the Bernoulli's and other vacation schedules. Motivational cause of this type of models lies in the field of Computer networks and communication systems, where messages are processed by the servers in two tier and three tier approaches [8]. As modern communications systems become more complex, and the processing becomes more and simultaneously the complications becomes more [3]. Most of the previous research has assumed that the server is constantly running, and the service has not been denied until it's become breakdown. However, this assumption is unrealistic in implementing in more than one server in practice [2].

Single Server Retrial Queue with Server Vacation is analyzed by Shan Gao et al [15]. Madan K C et al [13] considered a robustness analysis of the model in Bernoulli's vacation schedule, assuming that the server is under repair. In this article, we considered $M^x/G/1$ types of queues and its recommendations, which implements the concept of delay time while getting the service. An initial check of the batch's arrival queue using these recommendations was carried out by Srinivas R et al [16]. They presented a method for achieving an optimal inpatient surgery policy with an appropriate linear cost structure and other outcomes. Similar characteristic of these models was further enhanced by Krishnakumar B, Pavai Madheswari S, et al [11, 12, 14].

In addition, Chowdhury et al [5, 10] investigated this type of two-phase queue arrival model using a Bernoulli's model. Also, Kalita et al [9] discussed Bernoulli's N-Policy vacation's schedule which explored several queuing systems. Optimal control and managing vacation pattern have also received considerable attention in the literature studies of Choudhury and Tadj [4]. In this paper we recommended $M^x/G/1$ queues as a two-stage service and showed Bernoulli's vacation follows the threshold guidelines for unstable servers, downtime, and latency [1]. Further it is embedded with the system of multiple queues connected for batch delivery on the inclusion of additional variables [6].

This paper is organized as follows: Section II focused on a proactive renewal of service hours policy, invoked to the existing customers in the queue. Further the input process, server downtime, server lifetime, server recovery time and server time, server latency and random service time are discussed. In section III, Mathematical modeling for probability generating functions of the system and orbit size is described with suitable conditional equations. Finally, section IV concludes the paper.

2. Proactive Renewal of Service

While applying threshold strategy for unstable servers, downtime and latency, the arrival leads by a complex Poisson process with a degree of arrival λ. The sizes of the queues are based on arrival process and service process of the customers and servers which in turn. When the queue size exceeds its threshold value, the server activates, and each device receives two consecutive heterogeneous phases of service (service for the entire batch in the first phase and individual service will be given in the second phase).

During the maintenance of server, as soon as an error occurs, the server is sent for repair and before the server fails, the served client waits for the recovery to begin, and this can be called delay. If the server is running at any stage of maintenance, it can fail at any time, and the service line can fail for a short period of time (downtime). Collapse, i.e., the server lifetime is generated by an external Poisson process. Once the server is repaired, the remaining customer service can begin at any service level. In other words, a proactive renewal of service hours policy, will be invoked to the existing customers in the queue. After each server's shutdown, the server can go on vacation for an arbitrary period M (vacation time) with probability p. The server shuts down when the system is empty and restarts the server when the queue size increases to T. It is also assumed that the input process, server downtime, server lifetime, server recovery time and server time, server latency and random service time variables are independent.

3. Mathematical Modeling for Probability Generating Functions of the System and Orbit Size

In this section, we generate a system equation of state for a fixed distribution of tail size that includes elapsed vacation time, elapsed service time, elapsed repair time. The server assumes that the system is patched for both phases of maintenance. Now consider the following equations which are used to calculate after each server's shutdown, the server can go on vacation for an arbitrary period M (vacation time) with probability p

$$M(0,c) = p(1-u)\int_0^\infty \pi_1(a,c)\mu_1(a)da + p\int_0^\infty \pi_2(a,c)\mu_2(a)da \tag{3.1}$$

Solving the above PDE

$$M(a,c) = P(0,c)[1-I_1(a)]e^{-\lambda a} \tag{3.2}$$

$$\pi_1(a,c) = \pi_1(0,c)[1-I_1(a)]e^{-F_1(c)a} \tag{3.3}$$

$$\pi_2(a,c) = \pi_2(0,c)[1-I_2(a)]e^{-F_2(c)a} \tag{3.4}$$

$$R_1(a,b,c) = R_1(a,0,c)[1-K_1(b)]e^{-\lambda_0(c)b} \tag{3.5}$$

$$R_2(a,b,c) = R_2(a,0,c)[1-K_2(b)]e^{-\lambda_0(c)b} \tag{3.6}$$

$W(a,c) = W(0,c)[1 - W(a)]e^{-\lambda_0(c)a}$ (3.7)

where $F_1(c) = \lambda_0(c) + \beta_1[1 - K_1^*(\lambda_0(c))], F_2(c) = \lambda_0(c) + \beta_2[1 - K_2^*(\lambda_0(c))]$

$and\ \lambda_0(c) = \lambda(1-c)$ (3.8)

Inserting (4) in (9) we get,

$\pi_1(0,c) = \frac{P(0,c)}{c}[c + (1-c)F^*(\lambda)] + \lambda P_0$ (3.9)

Inserting (5) in (10), we get

$\pi_2(0,c) = u\pi_1(0,c)I_1^*(F_1(c))$ (3.10)

Inserting (5) in (11), we get

$R_1(a,0,c) = \beta_1\pi_1(0,c)(1 - I_1(a))e^{-F_1(c)a}$ (3.11)

Inserting (6) in (2)

$R_2(a,0,c) = \beta_2\pi_2(0,c)(1 - I_2(a))e^{-F_2(c)a}$

(3.12)

Inserting (5), (6) and (9) in (8) we obtain

$M(0,c) = p(1-u)\pi_1(0,c)I_1^*(F_1(c)) + p\pi_2(0,c)I_2^*(F_2(c))$ (3.13)

Inserting (3.3), (3.4), (3.6) in (34) we get

$P(0,c) =$ $c\lambda p_\circ[I_1^*F_1(C)\ (p\bar{u}\mu^*\lambda_\circ(C)+(1-p)\bar{u})+uI_1^*F_1(C)I_2^*F_2(C)(p\mu^*\lambda_\circ(C)) + (1-p)-1]$ /

c-[[c+1-c)$F^*(\lambda)$]{$I_1^*(F_1(C))\ (p\bar{u}\mu^*\lambda_\circ(C)+(1-p)\bar{u} + \bar{u}\ (I_1^*\ F_1(C))(I_2^*\ F_2(C))(p\ \mu^*\lambda_\circ(C)) + (1-$

$p)$}] (3.14)

Substituting (3.14) into (3.9) , we get

$\pi_1(0, c) = \frac{\lambda\ p_\circ}{Vu(c)}[c-1]F^*(\lambda)$ (3.15)

Substituting (3.15) into (3.10), we get

$\pi_1(0,c) = \lambda p_\circ[\frac{c-(c-1)F^*(\lambda))}{Vu(c)}]I_1^*F_1(C)$ (3.16)

Using (3.15) in (3.11) we obtain

R_1(a, o, c) $=\beta_1\lambda$ p$_\circ$[$\frac{(c-1)F^*(\lambda))}{Vu(c)}$](1-$I_1$(a))$e^{-F_1(c)a}$ (3.17)

Substituting (3.16) in (3.12) we get

$R_2(a,o,c) = \beta_2\lambda$ p$_\circ$[$\frac{(c-1F^*(\lambda))}{Vu(c)}$]$(1-I_2$(a))$e^{-F_2(C)a}$ (3.18)

Inserting (3.15), (3.16) in (3.13) we obtain

$P(0,c)=\lambda p_\circ[\frac{(c-1)F^*(\lambda))}{Vu(c)}]\{p\,\bar{u}I_1^*F_1(c)+puI_1^*F_1(c)I_2^*F_2(c)\}$ (3.19)

$$P(a,c) = \frac{c\lambda\, p_\circ[I_1^*F_1(C)p\bar{u}\mu^*\lambda_\circ(C)+p\bar{u})+uI_1^*F_1(C)I_2^*F_2(C))(P\mu^*\lambda_\circ(C)+(1-p))-1](1-F(a))e^{-\lambda a}}{(c+(1-c)F^*(\lambda))\{I_1^*F_1(C)(p\bar{u}\mu^*\lambda_\circ(C))+p\bar{u})+uI_1^*F_1(C)I_2^*F_2(C)(p\,\mu^*\lambda_\circ(C))+(1-p))\}} \quad (3.20)$$

$$\pi_1(a,c) = \lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right](1-I_1(a))e^{-F_1(c)a} \quad (3.21)$$

$$\pi_2(a,c) = u\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]I_1{}^*F_1(c)(1-F_2(a))e^{-F_2(c)a} \quad (3.22)$$

$$R_1(a,b,c) = \beta_1\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right](1-I_1(c))e^{-F_1(c)a}[1-K_1(b)]e^{-\lambda_0(c)b} \quad (3.23)$$

where $\lambda_0(c) = \lambda(1-c)$

$$R_2(a,b,c) = \beta_2 u\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right](1-I_2(c))e^{-F_2(c)a}[1-K_2(b)]e^{-\lambda_0(c)b}I_1{}^*(F_1(c)) \quad (3.24)$$

$$\mu(a,c) = \lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left[(p\bar{u}I_1{}^*F_1(c) + pu\,I_1{}^*(F_1(c))\,I_2{}^*(F_2(c))\right](1-\mu(a))e^{-\lambda_0(c)a} \quad (3.25)$$

$$P(c) = \int_0^\infty p(a,c)\,da$$

$$= \int_0^\infty \lambda c P_0 \frac{T_u(c)}{V_u(c)}(1-F(a))e^{-\lambda a}da$$

$$= \lambda c P_0 \frac{T_u(c)}{V_u(c)}\int_0^\infty e^{-\lambda a}(1-F(a))da$$

$$= \lambda c P_0 \frac{T_u(c)}{V_u(c)}\left[\frac{1-F^*(\lambda)}{\lambda}\right]$$

$$= cP_0\frac{T_u(c)}{V_u(c)}[1-F^*(\lambda)]$$

$$= z(1-F^*(\lambda))P_0\left[\frac{I_1{}^*F_1(c)(p\bar{u}\mu^*\lambda_0(c)+(1-p)\bar{u})+uI_1{}^*F_1(c)I_2{}^*F_2(c)(p\mu^*\lambda_0(c)+(1-p)-1)}{c(c+(1-c)F^*(\lambda))\{I_1{}^*F_1(c)(p\bar{u}\mu^*\lambda_0(c)+(1-p)\bar{u})+uI_1{}^*F_1(c)I_2{}^*F_2(c)(p\mu^*\lambda_0(c)+(1-\bar{p}))\}}\right]$$

(3.26)

$$\pi_1(c)=\int_0^\infty \pi_1(a,c)da=\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\big(1-I_1(a)\big)e^{-F_1(c)a}da \tag{3.27}$$

$$\pi_1(c)=\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left(\frac{1-I_1{}^*F_1(c)}{F_1(c)}\right)$$

$$\pi_2(c)=\int_0^\infty \pi_2(a,c)da=u\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]I_1{}^*F_1(c)\big(1-F_2(a)\big)e^{-F_2(c)a}da$$

$$\pi_2(c)=u\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]I_1{}^*F_1(c)\left(\frac{1-I_2{}^*F_2(c)}{F_2(c)}\right) \tag{3.28}$$

$$R_1(a,c)=\beta_1\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left(\frac{1-K_1{}^*\lambda_0(c)}{\lambda_0(c)}\right)\int_0^\infty\big(1-I_1(a)\big)\,e^{-F_1(c)a}da$$

$$R_1(c)=\beta_1\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left(\frac{1-K_1{}^*\lambda_0(c)}{\lambda_0(c)}\right)\left(\frac{1-I_1{}^*F_1(c)}{F_1(c)}\right) \tag{3.29}$$

$$=\beta_1 P_0\left[\frac{\lambda_0(c)F^*(\lambda)}{V_u(c)}\right]\left(\frac{1-K_1{}^*\lambda_0(c)}{\lambda_0(c)}\right)\left(\frac{I_1{}^*F_1(c)-1}{F_1(c)}\right)$$

$$=\beta_1 P_0[F^*(\lambda)]\left(\frac{1-K_1{}^*\lambda_0(c)}{V_u(c)}\right)\left(\frac{I_1{}^*F_1(c)-1}{F_1(c)}\right)$$

$$R_2(c)=\int_0^\infty R_2(a,c)da$$

$$R_2(c)=\beta_2\lambda P_0 u\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left(\frac{1-K_2{}^*\lambda_0(c)}{\lambda_0(c)}\right)\left(\frac{1-I_2{}^*F_2(c)}{F_2(c)}\right)\big(I_1{}^*F_1(c)\big) \tag{3.30}$$

$$=\beta_2 P_0 u[F^*(\lambda)]\left(\frac{1-K_2{}^*\lambda_0(c)}{V_u(c)}\right)\left(\frac{I_2{}^*F_2(c)-1}{F_2(c)}\right)\big(I_1{}^*F_1(c)\big)$$

$$M(c)=\int_0^\infty M\,(a,c)da$$

$$=\int_0^\infty \lambda\,P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left[\big(p\bar{u}I_1{}^*F_1(c)+pu\,I_1{}^*F_1(c)\big)\,I_2{}^*F_2(c)\right]\big(1-\mu(a)\big)e^{-\lambda_0(c)a}da$$

$$=\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left[\big(p\bar{u}I_1{}^*F_1(c)+pu\,I_1{}^*F_1(c)\big)\,I_2{}^*F_2(c)\right]\int_0^\infty\big(1-\mu(a)\big)e^{-\lambda_0(c)a}da$$

$$M(c)=\lambda P_0\left[\frac{(c-1)F^*(\lambda)}{V_u(c)}\right]\left[\big(p\bar{u}I_1{}^*F_1(c)+pu\,I_1{}^*F_1(c)\big)\,I_2{}^*F_2(c)\right]\left(\frac{1-\mu^*\lambda_0(c)}{\lambda_0(c)}\right) \tag{3.31}$$

The probability generating functions of the system and orbit size are found as:

$$P_0 = \frac{F^*(\lambda) - \lambda E(I_1)\left(1+\beta_1 E(K_1)\right) + uE(I_2)\Big(1+\beta_2 E\big((K_2)\big)\Big) + pE(M)}{F^*(\lambda)} \tag{3.32}$$

$$L(c) = \frac{P_0 F^*(\lambda)\left(I_1{}^* F_1(c)\right)(c-1)\left(\bar{u} + uI_2{}^* F_2(c)\right)}{V_u(c)} \tag{3.33}$$

$$V(c) = \frac{P_0 F^*(\lambda)}{V_u(c)}\,[1-c] \tag{3.34}$$

4. CONCLUSION

In this paper an extensive analysis of Bernoulli's vacation of $M^x/G/1$ queue with two-tier service based volatile server is analyzed. Our methodologies recommended $M^x/G/1$ queues as a two-stage service and showed Bernoulli’s vacation follows the threshold guidelines for unstable servers, downtime, and latency. In addition, it is embedded with the system of multiple queues connected for batch delivery on the inclusion of additional variables along with a complex Poisson process with a degree of arrival λ. It is elaborately discussed that how the proactive renewal of service hours policy, invoked to the existing customers in the queue. Further the input process, server downtime, server lifetime, server recovery time and server time, server latency and random service time are discussed through suitable probability generating functions of the system and orbit size.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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