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A DECISION MAKING APPROACH BASED ON WEIGHTED FUZZY SOFT SET

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Abstract. This paper is aimed at developing an approach of a real life decision making problem with respect to an weighted fuzzy soft set with preference. This paper introduces weighted fuzzy soft set and studies some of its properties. This paper also enquires about the relations on weighted fuzzy soft sets. Finally, a real life decision making problem in weighted fuzzy soft set is proposed.

Keywords: weighted fuzzy soft sets; weighted fuzzy soft relations; level set; decision making.

2010 AMS Subject Classification: 94D05, 68U35.

1. Introduction

Many real life problems in society, economics, management and engineering are uncertain and imprecise. To dealing with uncertainty there are many theories, such as probability theory, fuzzy set theory [2], intuitionistic fuzzy set theory [3], interval mathematics, rough set theory [4] etc. But all these theories have their own difficulties due to lack of parametrization of the theories. Molodtsov [1] proposed the notion of soft set theory to deal with the intrinsic flows of the above mention theories. Maji et al. [7, 8] continued the study and gave first practical example of the soft set in decision making problem by constructing comparison table. It is observed that the application of soft set theory is in various areas, such as forecasting [5], decision

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making [7, 11] etc.

Maji et al. [9] introduced the notion of fuzzy soft set and studied some of its properties. Mazumder-Samanta [10] generalized the concept of fuzzy soft set. Feng et al. [5] presented a decision making problem based on fuzzy soft set. Due to its ability to parametrization, the theory and applications of fuzzy soft set obtained lots of interest in past years.

In this paper we introduce weighted fuzzy soft set. It is more realistic as corresponding to each parameter there is a degree of preference. Relations on weighted fuzzy soft set is defined and some of its properties are studied. Finally an adjustable approach to decision making problem is proposed with respect to weighted fuzzy soft set.

2. PRELIMINARIES

Here we have given some basic definitions and results which will be needed later for this paper.

Definition 2.1. [1]. *Let U be the initial universe of objects and P be the set of parameters. Consider a mapping $S : P \to \mathscr{P}(U)$ where $\mathscr{P}(U)$ is the power set of U, then the pair (S,P) is called a soft set over U.*

Definition 2.2. [9]. *Let $[0, 1]^U$ be the set of all fuzzy subsets over U and $A \subset P$ where P is the set of all parameters. Also Let $\bar{S} : A \to [0, 1]^U$ be a mapping then the pair $(\bar{S},A)$ is said to be a fuzzy soft set over U.*

Example 2.3. *Suppose for a farm house protection the owner of the farm house wanted to buy a trained dog. There is five high quality trained dogs are available for consideration $U = \{t_1,t_2,t_3,t_4,t_5\}$ and $A \subset P$ where P is the set of parameters and $A = \{p_1 = \text{Temperment}, p_2 = \text{Life span}, p_3 = \text{Medical issues}\}$. Then the fuzzy soft sets are the collection of followings.*

$\bar{S}(p_1) = \{(t_1,0.6),(t_2,0.3),(t_3,0.69),(t_4,0.5),(t_5,0.8)\}$

$\bar{S}(p_2) = \{(t_1,0.5),(t_2,0.9),(t_3,0.6),(t_4,0.4),(t_5,0.69)\}$

$\bar{S}(p_3) = \{(t_1,0.7),(t_2,0.79),(t_3,0.69),(t_4,0.3),(t_5,0.49)\}$

	p_1	p_2	p_3
t_1	0.6	0.5	0.7
t_2	0.3	0.9	0.8
t_3	0.7	0.6	0.7
t_4	0.5	0.4	0.3
t_5	0.8	0.7	0.5

TABLE 1. Tabular representation of the fuzzy soft set. Now a computation table is formed for the 0.6 level soft set (i.e, at least 60 % capability) with respect to each parameter.

	p_1	p_2	p_3
t_1	1	0	1
t_2	0	1	1
t_3	1	1	1
t_4	0	0	0
t_5	1	1	0

TABLE 2. Tabular representation of 0.6 level soft level. Clearly the owner of the farm house buy the dog t_3 as it satisfies all the paprameters.

Definition 2.4. *([9]). Intrersection of two fuzzy soft sets $(\bar{S},A)$ and $(\bar{T},B)$ over a common universe U is the fuzzy soft set $(\bar{I},C)$ where $C = A \cap B$, and $\forall\, e \in C$, $\bar{I}(e) = \bar{S}(e) \cap \bar{T}(e)$. We write $(\bar{S},A)\bar{\cap}(\bar{T},B) = (\bar{I},C)$.*

Definition 2.5. *([9]). Union of two fuzzy soft sets $(\bar{S},A)$ and $(\bar{T},B)$ over a common universe U is the fuzzy soft set $(\bar{J},C)$ where $C = A \cup B$, and $\forall\, e \in C$,*

$$\bar{J}(e) = \begin{cases} \bar{S}(e), & \text{if } e \in A - B \\ \bar{T}(e), & \text{if } e \in B - A \\ \bar{S}(e) \cup \bar{T}(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(\bar{S},A)\bar{\cup}(\bar{T},B) = (\bar{J},C)$.

Definition 2.6. [9]. *For two fuzzy soft sets* $(\bar{S},A)$ *and* $(\bar{T},B)$ *over a common universe* U*, we say that* $(\bar{S},A)$ *is a fuzzy soft subset of* $(\bar{T},B)$ *if*
(i) $A \subset B$*, and*
(ii) $\forall\, e \in A,\ \bar{S}(e)$ *is a fuzzy subset of* $\bar{T}(e)$.
It is denoted by $(\bar{S},A)\bar{\subset}(\bar{T},B)$.

Definition 2.7. *(*[12]*). A binary operation* $* : [0,1]\times[0,1] \to [0,1]$ *is continuous t-norm if* $*$ *satisfies the following conditions:*
(i) $*$ *is commutative and associative ,*
(ii) $*$ *is continuous ,*
(iii) $a*1 = a,\ \forall\, a \in [0,1]$*,*
(iv) $a*b \leq c*d$ *if* $a \leq c$ *,* $b \leq d$*,* $a, b, c, d \in [0,\, 1]$.

Some examples of continuous t-norm are:
$i)\, a * b = ab$,
$ii)\, a * b = min\{a,b\}$,
$iii)\, a * b = max\{a+b-1,0\}$.

Definition 2.8. *(*[12]*). A binary operation* $\diamond : [0,1]\times[0,1] \to [0,1]$ *is continuous t-conorm if* $\diamond$ *satisfies the following conditions :*
(i) $\diamond$ *is commutative and associative ,*
(ii) $\diamond$ *is continuous ,*
(iii) $a\diamond 0 = a,\ \forall\, a \in [0,1]$*,*
(iv) $a\diamond b \leq c\diamond d$ *whenever* $a \leq c$*,* $b \leq d$ *and* $a, b, c, d \in [0,1]$.

Some examples of continuous t-conorm are:
$i)\, a \diamond b = a+b-ab$,
$ii)\, a \diamond b = max\{a,b\}$,
$iii)\, a \diamond b = min\{a+b,1\}$.

3. Weighted Fuzzy Soft Sets

Throughout this paper, U be the initial universe, and P be the set of parameters related to the elements in U, and $A,B,C \subseteq P$ and α,β,γ are the fuzzy subsets of A,B,C respectively.

Definition 3.1. *Let $\bar{\mathscr{F}} : A \to [0,1]^U$ be a mapping and α be preference fuzzy subset of A. Then consider the mapping $\bar{\mathscr{F}}_\alpha(p) : A \to [0,1]^U \times [0,1]$ where $\bar{\mathscr{F}}_\alpha(p) = (\bar{\mathscr{F}}(p), \alpha(p))$*
$= \Big(\{(x, \mu_{\bar{\mathscr{F}}(p)}(x)) : x \in U\},\ \alpha(p)\Big)$
is called a weighted fuzzy soft set over (U,P)

Example 3.2. *Suppose 5 shortlisted students are under consideration for best student of an engineering college. Let $U = \{b_1,b_2,b_3,b_4,b_5\}$ and $A \subset P$ where P is the set of parameters and $A = \{p_1 =$ problem solving ability$, p_2 =$ innovative idea$, p_3 =$ ability for application to an idea $\}$. If we consider $\alpha : A \to [0,1]$ by $\alpha(p_1) = 0.6,\ \alpha(p_2) = 0.5,\ \alpha(p_3) = 0.7$*
We now define $\bar{\mathscr{F}}_\alpha$ as follows:
$\bar{\mathscr{F}}(p_1) = (\{(b_1,0.6),(b_2,0.3),(b_3,0.7),(b_4,0.5),(b_5,0.8)\},\ 0.6)$
$\bar{\mathscr{F}}(p_2) = (\{(b_1,0.5),(b_2,0.9),(b_3,0.6),(b_4,0.4),(b_5,0.7)\},\ 0.5)$
$\bar{\mathscr{F}}(p_3) = (\{(b_1,0.7),(b_2,0.8),(b_3,0.7),(b_4,0.3),(b_5,0.5)\},\ 0.7)$
Then clearly $\bar{\mathscr{F}}_\alpha$ will be called weighted fuzzy soft set.

Definition 3.3. *Consider two weighted fuzzy soft sets $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ over the soft universe (U,P). Then $\bar{\mathscr{F}}_\alpha$ is called a weighted fuzzy soft subset of $\bar{\mathscr{G}}_\beta$ if the following conditions are satisfied.*
(i) α is a fuzzy subset of β,
(ii) $A \subseteq B$, and
(iii) $\mu_{\bar{\mathscr{F}}(p)}(x) \leq \mu_{\bar{\mathscr{G}}(p)}(x), \forall x \in U$ and $p \in A$ i.e., $\bar{\mathscr{F}}(p)$ is a fuzzy subset of $\bar{\mathscr{G}}(p), \forall p \in A$. It is denoted by $\bar{\mathscr{F}}_\alpha \bar{\subseteq} \bar{\mathscr{G}}_\beta$

Example 3.4. *Suppose $\bar{\mathscr{F}}_\beta$ be a weighted fuzzy soft set over the same universe as in the previous example and $\bar{\mathscr{G}}_\beta$ be defined as the following manner:*
$\bar{\mathscr{G}}_\beta(p_1) = (\{(b_1,0.7),(b_2,0.5),(b_3,0.8),(b_4,0.7),(b_5,0.9)\},\ 0.7)$

$\bar{\mathscr{G}}_\beta(p_2) = (\{(b_1,0.8),(b_2,0.9),(b_3,0.65),(b_4,0.6),(b_5,0.8)\},\ 0.6)$

$\bar{\mathscr{G}}_\beta(p_3) = (\{(b_1,0.75),(b_2,0.85),(b_3,0.7),(b_4,0.5),(b_5,0.6)\},\ 0.75)$

$\bar{\mathscr{G}}_\beta(p_4) = (\{(b_1,0.6),(b_2,0.7),(b_3,0.5),(b_4,0.65),(b_5,0.7)\},\ 0.5)$

where $\beta = \{p_1,p_2,p_3,p_4\}$, p_1,p_2,p_3 are same as in the previous exmple and p_4 stands for 'good academic score'. Now if we take $\bar{\mathscr{F}}_\alpha$ as in the previous example and $\bar{\mathscr{G}}_\beta$ then we see that $\bar{\mathscr{F}}_\alpha \bar{\subseteq} \bar{\mathscr{G}}_\beta$. i.e. $\bar{\mathscr{F}}_\alpha$ is a weighted fuzzy soft subset of $\bar{\mathscr{G}}_\beta$

Definition 3.5. *Let $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ be two weighted fuzzy soft sets over the soft set universe (U,P). Then the intersection of $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ is denoted by $\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta$ and is defined by a weighted fuzzy soft set $\bar{\mathscr{H}}_\gamma$ where $\bar{\mathscr{H}}_\gamma : A \cap B \to F(U) \times [0,1]$ is a function defined by*

$$\bar{\mathscr{H}}_\gamma(p) = \Big(\{(x,\ \mu_{\bar{\mathscr{H}}(p)}(x), : x \in U\},\ \gamma(p)\Big)$$

where $\mu_{\bar{\mathscr{H}}(p)}(x) = \mu_{\bar{\mathscr{F}}(p)}(x) * \mu_{\bar{\mathscr{G}}(p)}(x)$ *and* $\gamma(p) = \alpha(p) * \beta(p)$

Example 3.6. *Consider two weighted fuzzy soft sets $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ in the previous example and suppose the t-norm $*$ is defined by $a * b = ab$. Here $A \cap B = \{p_1,p_2,p_3\}$. Then*

$(\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta)(p_1) = (\{(b_1,0.42),(b_2,0.15),(b_3,0.56),(b_4,0.35),(b_5,0.22)\},\ 0.42)$

$(\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta)(p_2) = (\{(b_1,0.4),(b_2,0.81),(b_3,0.39),(b_4,0.24),(b_5,0.56)\},\ 0.525)$

$(\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta)(p_3) = (\{(b_1,0.525),(b_2,0.68),(b_3,0.49),(b_4,0.195),(b_5,0.35)\},\ 0.3)$

Definition 3.7. *Let $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ be two weighted fuzzy soft sets over the soft set universe (U,P). Then the union of $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ is denoted by $\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta$ and is defined by a weighted fuzzy soft set $\bar{\mathscr{H}}_\gamma$ where $\bar{\mathscr{H}}_\gamma : A \cup B \to F(U) \times [0,1]$ is a function defined by*

$\forall x \in U$,

$$\bar{\mathscr{H}}_\gamma(p) = \begin{cases} \Big(\{(x,\ \mu_{\bar{\mathscr{F}}(p)}(x)\},\ \gamma(p)\Big), & \text{if } p \in A - B \\ \Big(\{(x,\ \mu_{\bar{\mathscr{G}}(p)}(x)\},\ \gamma(p)\Big), & \text{if } p \in B - A \\ \Big(\{(x,\ \mu_{\bar{\mathscr{H}}(p)}(x)\},\ \gamma(p)\Big), & \text{if } p \in A \cap B. \end{cases}$$

where $\mu_{\bar{\mathscr{H}}(p)}(x) = \mu_{\bar{\mathscr{F}}(p)}(x) \diamond \mu_{\bar{\mathscr{G}}(p)}(x)$ *and* $\gamma(p) = \alpha(p) \diamond \beta(p)$

Example 3.8. *Again consider the above example in which the two weighted fuzzy soft sets $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ are defined. Also $A \cup B = \{p_1,p_2,p_3,p_4\}$. Suppose t- conorm is defined as*

$a \diamond b = a + b - ab$. Then

$(\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta)(p_1) = (\{(b_1, 0.88), (b_2, 0.65), (b_3, 0.94), (b_4, 0.85), (b_5, 0.98)\},\ 0.88)$

$(\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta)(p_2) = (\{(b_1, 0.9), (b_2, 0.99), (b_3, 0.86), (b_4, 0.76), (b_5, 0.94)\},\ 0.925)$

$(\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta)(p_3) = (\{(b_1, 0.925), (b_2, 0.97), (b_3, 0.91), (b_4, 0.65), (b_5, 0.8)\},\ 0.8)$

$(\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta)(p_4) = (\{(b_1, 0.6), (b_2, 0.7), (b_3, 0.5), (b_4, 0.65), (b_5, 0.7)\},\ 0.5)$

Proposition 3.9. *Let $\bar{\mathscr{F}}_\alpha$, $\bar{\mathscr{G}}_\beta$ and $\bar{\mathscr{H}}_\gamma$ be any three weighted fuzzy soft sets over (U, P). Then*

(i) $\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta = \bar{\mathscr{G}}_\beta \bar{\cup} \bar{\mathscr{F}}_\alpha$. (Commutative Property)

(ii) $\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta = \bar{\mathscr{G}}_\beta \bar{\cap} \bar{\mathscr{F}}_\alpha$. (Commutative Property)

(iii) $\bar{\mathscr{F}}_\alpha \bar{\cup} (\bar{\mathscr{G}}_\beta \bar{\cup} \bar{\mathscr{H}}_\gamma) = (\bar{\mathscr{F}}_\alpha \bar{\cup} \bar{\mathscr{G}}_\beta) \bar{\cup} \bar{\mathscr{H}}_\gamma$. (Associative Property)

(iv) $\bar{\mathscr{F}}_\alpha \bar{\cap} (\bar{\mathscr{G}}_\beta \bar{\cap} \bar{\mathscr{H}}_\gamma) = (\bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{G}}_\beta) \bar{\cap} \bar{\mathscr{H}}_\gamma$. (Associative Property)

Proof. The proof is obvious as t-norm function and t-conorm functions are commutative and associative.

4. Relations on Weighted Fuzzy Soft Sets

Throughout this section $\bar{\mathscr{F}}_\alpha : A \to F(U)$ and $\bar{\mathscr{G}}_\beta : B \to F(U)$ two weighted fuzzy soft sets over (U, P).

Definition 4.1. *Suppose $\bar{\mathscr{F}}_\alpha$ and $\bar{\mathscr{G}}_\beta$ be two weighted fuzzy soft sets over (U, P). By a weighted fuzzy soft relation $\mathscr{R}$ from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$ we mean a function $\mathscr{R} : A \times B \to F(U) \times [0, 1]$ such that*

$$\mathscr{R}(a, b) \bar{\subseteq} \bar{\mathscr{F}}_\alpha(a) \bar{\cap} \bar{\mathscr{G}}_\beta(b),\ \forall (a, b) \in A \times B.$$

Definition 4.2. *Let $\mathscr{R}$ be a weighted fuzzy soft relation from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$. Then the inverse relation of $\mathscr{R}$ from $\bar{\mathscr{G}}_\beta$ to $\bar{\mathscr{F}}_\alpha$ is denoted by $\mathscr{R}^{-1}$ and is defined by $\mathscr{R}^{-1}(b, a) = \mathscr{R}(a, b)$, $\forall (a, b) \in A \times B$.*

If $\mathscr{R}$ is a weighted fuzzy soft relation from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$ then $\mathscr{R}^{-1}$ is also a weighted fuzzy soft relation from $\bar{\mathscr{G}}_\beta$ to $\bar{\mathscr{F}}_\alpha$.

Theorem 4.3. *If $\mathscr{R}_1$ and $\mathscr{R}_2$ are weighted fuzzy soft relations from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{F}}_\beta$,*
(i) $(\mathscr{R}_1^{-1})^{-1} = \mathscr{R}_1$.
(ii) $\mathscr{R}_1 \subseteq \mathscr{R}_2 \Rightarrow \mathscr{R}_1^{-1} \subseteq \mathscr{R}_2^{-1}$.

Proof. Let $(a,b) \in A \times B$.
Then (i) $\mathscr{R}_1(a,b) = \mathscr{R}_1^{-1}(b,a) = (\mathscr{R}_1^{-1})^{-1}(a,b)$.
Therefore $(\mathscr{R}_1^{-1})^{-1} = \mathscr{R}_1$.
And (ii) $\mathscr{R}_1(a,b) \subseteq \mathscr{R}_2(a,b)$
$\Rightarrow (\mathscr{R}_1^{-1})^{-1}(a,b) \subseteq (\mathscr{R}_2^{-1})^{-1}(a,b)$
$\Rightarrow \mathscr{R}_1^{-1}(b,a) \subseteq \mathscr{R}_2^{-1}(b,a)$.
Therefore, $\mathscr{R}_1^{-1} \subseteq \mathscr{R}_2^{-1}$.

Definition 4.4. *Let $\mathscr{R}_1$ and $\mathscr{R}_2$ be two weighted fuzzy soft relations from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$ and $\bar{\mathscr{G}}_\beta$ to $\bar{\mathscr{H}}_\gamma$ respectively. Then the composition of two weighted fuzzy soft relations from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{H}}_\gamma$ is denoted by $\mathscr{R}_1 \circ \mathscr{R}_2$ and defined as $(\mathscr{R}_1 \circ \mathscr{R}_2)(a,c) = \bar{\bigcup}_{b\in B}(\mathscr{R}_1(a,b) \bar{\cap} \mathscr{R}_2(b,c))$,*
$\forall (a,c) \in A \times C$

Theorem 4.5. *Let $\mathscr{R}_1$ and $\mathscr{R}_2$ be two weighted fuzzy soft relations from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$ and $\bar{\mathscr{G}}_\beta$ to $\bar{\mathscr{H}}_\gamma$ respectively. Then the composition of this two weighted fuzzy soft relations i.e. $(\mathscr{R}_1 \circ \mathscr{R}_2)$ is also a weighted fuzzy soft relation.*

Proof. We know that
$\mathscr{R}_1(a,b) \subseteq \bar{\mathscr{F}}_\alpha(a) \bar{\cap} \bar{\mathscr{G}}_\beta(b)$
$= \{(\{x, \mu_{\bar{\mathscr{F}}(a)}(x) * \mu_{\bar{\mathscr{G}}(b)}(x)\}, \alpha(a) * \beta(b)) : x \in U\}$, $\forall (a,b) \in A \times B$.
$\mathscr{R}_2(b,c) \subseteq \bar{\mathscr{G}}_\beta(b) \bar{\cap} \bar{\mathscr{H}}_\gamma(c)$
$= \{(\{x, \mu_{\bar{\mathscr{G}}(b)}(x) * \mu_{\bar{\mathscr{H}}(c)}(x)\}, \beta(b) * \gamma(c)) : x \in U\}$, $\forall (b,c) \in B \times C$.
Therefore $(\mathscr{R}_1 \circ \mathscr{R}_2)(a,c) = \bar{\bigcup}_{b\in B}(\mathscr{R}_1(a,b) \bar{\cap} \mathscr{R}_2(b,c))$
$= \bar{\bigcup}_{b\in B}\{(\{x, (\mu_{\bar{\mathscr{F}}(a)}(x) * \mu_{\bar{\mathscr{G}}(b)}(x)) * (\mu_{\bar{\mathscr{G}}(b)}(x) * \mu_{\bar{\mathscr{H}}(c)}(x))\}, (\alpha(a) * \beta(b)) * (\beta(b) * \gamma(c))) : x \in U\}$, $\forall (a,c) \in A \times C$.
Now $(\mu_{\bar{\mathscr{F}}(a)}(x) * \mu_{\bar{\mathscr{G}}(b)}(x)) * (\mu_{\bar{\mathscr{G}}(b)}(x) * \mu_{\bar{\mathscr{H}}(c)}(x))$
$\leq \mu_{\bar{\mathscr{F}}(a)}(x) * 1 * 1 * \mu_{\bar{\mathscr{H}}(c)}(x)$
$= \mu_{\bar{\mathscr{F}}(a)}(x) * 1 * \mu_{\bar{\mathscr{H}}(c)}(x)$

$= \mu_{\bar{\mathscr{F}}(a)}(x) * \mu_{\bar{\mathscr{H}}(c)}(x)$

Again, $(\alpha(a) * \beta(b)) * (\beta(b) * \gamma(c))$

$\leq \alpha(a) * 1 * 1 * \gamma(c)$

$= \alpha(a) * 1 * \gamma(c)$

$= \alpha(a) * \gamma(c)$.

Therefore $\bar{\bigcup}_{b \in B} (\mathscr{R}_1(a,b) \bar{\cap} \mathscr{R}_2(b,c)) \subseteq \bar{\mathscr{F}}_\alpha \bar{\cap} \bar{\mathscr{H}}_\gamma$.

Hence $\mathscr{R}_1 \circ \mathscr{R}_2$ is a weighted fuzzy soft relation from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{H}}_\gamma$.

Theorem 4.6. *Let $\mathscr{R}_1$ and $\mathscr{R}_2$ be two weighted fuzzy soft relations from $\bar{\mathscr{F}}_\alpha$ to $\bar{\mathscr{G}}_\beta$ and $\bar{\mathscr{G}}_\beta$ to $\bar{\mathscr{H}}_\gamma$ respectively. Then $(\mathscr{R}_1 \circ \mathscr{R}_2)^{-1} = \mathscr{R}_2^{-1} \circ \mathscr{R}_1^{-1}$*

Proof. $(\mathscr{R}_1 \circ \mathscr{R}_2)^{-1}(c,a) = (\mathscr{R}_1 \circ \mathscr{R}_2)(a,c)$

$= \bar{\bigcup}_{b \in B} (\mathscr{R}_1(a,b) \bar{\cap} \mathscr{R}_2(b,c)) = \bar{\bigcup}_{b \in B} (\mathscr{R}_2(b,c) \bar{\cap} \mathscr{R}_1(a,b))$

$= \bar{\bigcup}_{b \in B} \left(\mathscr{R}_2^{-1}(c,b) \bar{\cap} \mathscr{R}_1^{-1}(b,a)\right) = (\mathscr{R}_2^{-1} \circ \mathscr{R}_1^{-1})(c,a)$.

where $a \in A,\ b \in B,\ c \in C$.

Therefore $(\mathscr{R}_1 \circ \mathscr{R}_2)^{-1} = \mathscr{R}_2^{-1} \circ \mathscr{R}_1^{-1}$.

5. An Application of Weighted Fuzzy Soft Set in a Daily Life Decision Making Problem

In our daily life based problem, we can use application of weighted fuzzy soft set. Suppose a person wants to buy a car depending on some parameters. Here we can help the person to choose the best car for the person according to his preference.

Suppose initially the person shortlisted six cars $U = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and the set of decesion parameters is $P = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}\}$ where each p_i $(i = 1,2,...,11)$ stands respectively for

p_1 = price of the car nearing the budget of the person,

p_2 = chosen color of the car,

p_3 = best resale value of the car,

p_4 = larger interior space,

p_5 = good safety features,

p_6 = nearby service centre,

p_7 = most mileage per litre,

p_8 = good exterior design or looks of the car,

p_9 = minimum maintenence cost per year,

p_{10} = brand or company,

p_{11} = smart technology.

Suppose, that person will choose a car depending on the parameters $p_1,\ p_4,\ p_5,\ p_7,\ p_8,\ p_{10}$, i.e., $A = \{p_1,\ p_4,\ p_5,\ p_7,\ p_8,\ p_{10}\}$.

Consider the fuzzy subset $\alpha : A \to [0,1]$ depending upon the preference of that person is given below

$\alpha(p_1) = 0.7$, $\alpha(p_4) = 0.5$, $\alpha(p_5) = 0.8$, $\alpha(p_7) = 0.6$, $\alpha(p_8) = 0.6$, $\alpha(p_{10}) = 0.4$

Let us consider a weighted fuzzy soft set $\bar{\mathscr{F}}_\alpha$ and it's approximation is given below.

$\bar{\mathscr{F}}_\alpha(p_2) = (\{(v_1,0.7), (v_2,0.6), (v_3,0.8), (v_4,0.7), (v_5,0.5), (v_6,0.9)\},\ 0.7)$

$\bar{\mathscr{F}}_\alpha(p_4) = (\{(v_1,0.8), (v_2,0.4), (v_3,0.44), (v_4,0.7), (v_5,0.6), (v_6,0.9)\},\ 0.5)$

$\bar{\mathscr{F}}_\alpha(p_6) = (\{(v_1,0.9), (v_2,0.8), (v_3,0.7), (v_4,0.7), (v_5,0.64), (v_6,0.7)\},\ 0.8)$

$\bar{\mathscr{F}}_\alpha(p_7) = (\{(v_1,0.6), (v_2,0.7), (v_3,0.8), (v_4,0.7), (v_5,0.9), (v_6,0.8)\},\ 0.6)$

$\bar{\mathscr{F}}_\alpha(p_9) = (\{(v_1,0.5), (v_2,0.6), (v_3,0.7), (v_4,0.54), (v_5,0.7), (v_6,0.8)\},\ 0.6)$

$\bar{\mathscr{F}}_\alpha(p_{10}) = (\{(v_1,0.6), (v_2,0.5), (v_3,0.8), (v_4,0.7), (v_5,0.6), (v_6,0.7)\},\ 0.4)$

Algorithm:

(i) Input a fuzzy soft set in tabular form.

(ii) Give a threshold i.e. give a different level of soft set (or Choose mid-level soft set or input a threshold fuzzy set or input top-bottom-level decision rule or input top-top-level decision rule or input bottom-bottom-level decision rule) to make decision.

(iii) Compute the level soft set with respect to the threshold fuzzy soft set (or the mid-level soft set or the top-bottom-level soft set or the top-top-level soft set or the bottom-bottom-level soft set) in tabular form.

(iv) Compute the weighted score of $v_i,\ \forall i$ in tabular form.

(v) If the maximum score occurs in k-th row then that person will buy the car v_k.

(vi) If k has more than one value then one of v_k may be chosen.

	p_1	p_4	p_5	p_7	p_8	p_{10}
v_1	0.7	0.8	0.9	0.6	0.5	0.6
v_2	0.6	0.4	0.8	0.7	0.6	0.5
v_3	0.8	0.44	0.7	0.8	0.7	0.8
v_4	0.7	0.7	0.7	0.7	0.54	0.7
v_5	0.5	0.6	0.64	0.9	0.7	0.6
v_6	0.9	0.9	0.7	0.8	0.8	0.7

TABLE 3. Tabular representation of the weighted fuzzy soft set

Suppose, there are given threshold of different level of soft sets of (F_α, A) as follows.

Threshold of $p_1 = 0.8$,

threshold of $p_4 = 0.7$,

threshold of $p_5 = 0.85$,

threshold of $p_7 = 0.7$,

threshold of $p_8 = 0.7$,

threshold of $p_{10} = 0.5$

Therefore, the tabular representation of level soft set depending on the above thresholds is:

	p_1	p_4	p_5	p_7	p_8	p_{10}	α_i
v_1	0	1	1	0	0	1	3
v_2	0	0	0	0	0	1	1
v_3	1	0	0	1	1	1	4
v_4	0	1	0	0	0	1	2
v_5	0	0	0	1	1	1	3
v_6	1	1	0	1	1	1	5

TABLE 4. Tabular representation of level soft set with respect to above thresholds

	$(p_1,0.7)$	$(p_4,0.5)$	$(p_5,0.8)$	$(p_7,0.6)$	$(p_8,0.6)$	$(p_{10},0.4)$	Score
v_1	0	0.5	0.8	0	0	0.4	1.7
v_2	0	0	0	0	0	0.4	0.4
v_3	0.7	0	0	0.6	0.6	0.4	2.3
v_4	0	0.5	0	0	0	0.4	0.9
v_5	0	0	0	0.6	0.6	0.4	1.6
v_6	0.7	0.5	0	0.6	0.6	0.4	2.8

TABLE 5. Decision table

Thus here we see that the score of the car v_6 is maximum. So the person will buy the car v_6 otherwise his next better option will be v_3. It is obvious that for different problem we use different type of lavel soft sets, the choice depends on the nature of data under consideration and nature of problem under consideration.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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