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WEAKLY PRIME IDEALS IN $\Gamma-$NEARRINGS

AZAR SALAMI[1], WAHEED AHMAD KHAN[2], SAJJAD AHMED[2], ABDELGHANI TAOUTI[1,*]

[1]ETS-Maths and NS Engineering Division, HCT, University City P. O. Box 7947, Sharjah, United Arab Emirates

[2]Department of Mathematics, University of Education Lahore Attock Campus, Pakistan

Abstract. In this note, we introduce and discuss the notion of weakly prime ideals in $\Gamma-$nearrings. We also provide few of their characterizations.

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1. INTRODUCTION

In N. Nobusawa [5] define Γ-ring which is more general structure than a ring. In this chapter we present basic definition and relations of Γ-near ring that will be helpful in our work. Gamma near ring was introduced by B. Satyanarayana in [6]. In Γ-near ring Γ is a subset of M where M is a group under addition (need not be abelian and is a non-empty set). Γ-near R is of the from $M \times \Gamma \times M \to M$ (a triple mapping) which satisfying the condition two condition (i) $(a+b)\alpha c = a\alpha c + b\alpha c$, (ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$ where $a,b,c \in M$ and $\alpha, \beta \in \Gamma$. For every three elements $a_1, b_1, c_1 \in M$ and $\alpha_1, \beta_1 \in \Gamma$. B. Satyanarayana also define I in Γ-near R. Ideal of Γ-near ring is also possess the properties of a subgroup (Normal) of Γ-near ring that satisfying some condition that will be define in chapter three and also discussed some example of ideal in Γ-near

*Corresponding author

E-mail address: ganitaouti@yahoo.com.au

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ring. Prime ideals in gamma near ring introduce by Clay and J. R. [2]. Further Eduard Domi introduced Bi-Prime ideal in Γ-near ring. He also discussed Maximal ideal and prime ideal in Γ-near ring. In [3] Eduard Domi defines relation between prime, bi-prime and maximal ideal in gamma near ring. He proved that if V is a Γ-near $(R,+)$ and $\gamma \in V$ such that γ is unit then every maximal ideal is prime ideal in Γ-near ring.

2. Preliminaries

In this section, we present useful definitions which would help us in understanding the forthcoming section.

Definition 2.1. [5] Suppose that M be any group $a,b,c,\ldots$ are any element of M and Γ be any other additive group such that α,β,γ are elements of Γ. Let $a\gamma b$ be defined as element of M and $\gamma a\beta$ is defined to be an element of Γ. If the product satisfy the following condition,

(1) $(l_1+l_2)\gamma k = l_1\gamma k + l_2\gamma k$,
(2) $l(\gamma_1+\gamma_2)k = l\gamma_1 k + l\gamma_2 k$,
(3) $l\gamma(k_1+k_2) = l\gamma k_1 + l\gamma k_2$
(4) $(l\gamma k)\beta m = l\gamma(k\beta m) = l(\gamma k\beta)m$
(5) If $l\gamma k = 0$ for any $l,k \in M$ and $\gamma = 0$, then M is called Γ-ring.

Definition 2.2. [6] A Γ-near R is a triple $(N,+,\Gamma)$, here

(1) $(N,+)$ is a (not necessarily abelian) group
(2) Γ is a non-empty set of binary operations on N So each $\gamma \in \Gamma$, $(N,+,\gamma)$ is a right near $-R\&$
(3) $(l\gamma k)\mu m = l\gamma(k\mu m)$ for all $l,k,m \in N$ and $\gamma,\mu \in \Gamma$.

Definition 2.3. [6] An ideal P of Γ-near-R M is said prime provided that for every two ideals I,J of M such that $I\Gamma J \subseteq P$ leads that $I \subseteq P$ or $J \subseteq P$.

Definition 2.4. [6] An ideal I of Γ-near-R M is called prime if $\{0\}$ and I is the only ideal of I. Or there is no ideal of I other than I or $\{0\}$. Or there is no proper ideal of I exist.

Definition 2.5. [6] A Γ-near R is said to be B-simple if there is no bi-ideal different from $\{0\}$ and M.

Definition 2.6. [6] If B is an I of Γ-near R than B is called minimal if it is different from zero and it doesn't contain any bi-ideal different from $\{0\}$ or from B itself.

For further concepts about Γ−nearrings, we refer [1, 6, 4].

3. Main Results and Discussions

In this Section, we introduces weakly prime ideals in gamma near rings. We begin with the following definition.

Definition 3.1. A proper ideal P of a Γ-nearring R is said to be a weakly prime if $0 \neq a\gamma b \in P$, where $\gamma \in \Gamma$ implies $a \in P$ or $b \in P$.

Equivalently, weakly prime ideal in Γ-near ring can be defined as.

Definition 3.2. A proper ideal P of a Γ-near ring R is a weakly prime provided that $0 \neq A\Gamma B \subseteq P$, where A and B are the ideals of R implies $B \subseteq P$ or $A \subseteq P$.

Proposition 3.3. *Every prime ideal of a Γ-nearring R is weakly prime.*

Theorem 3.4. *Let N be a Γ-nearring R and P be a weakly prime ideal of N such that P is not a prime ideal then* $P\Gamma P = 0$.

Proof. Assume $P\Gamma P \neq 0$. Now we prove that P is the prime. Suppose A and B are two ideals of a nearring N such that $A\Gamma B \subseteq P$ and let $A\Gamma B \neq 0$, then $B \subseteq P$ or $A \subseteq P$. However, if $A\Gamma B = 0$ and $P\Gamma P = 0$, there might be $l \in A$ and $k \in B$, $p'_o \in < po >$and $q'_o \varepsilon < q_o >$ such that $(l + p'_o)\Gamma(k + q'_o) \notin P$. Which implies that $l\gamma(k + q'_o) \notin P$, but $l\gamma(k + q'_o) = l\gamma(k + q'_o) - l\gamma k \in P$. Hence $A\Gamma B = 0$, a contradiction. Thus $(A + p'_o)\Gamma(B + q'_o) \subseteq P$, which shows that either $B \subseteq P$ or $A \subseteq P$. □

Corollary 3.5. *Let P be a weakly prime ideal of a nearring N. Then, if* $P \subset PN$ *then P is not a prime. However, if* $PN \subset P$ *holds then P is prime.*

Corollary 3.6. *Suppose N is a gamma near-R&P be an ideal of the N. let* $P\Gamma P \neq 0$*, this shows that P is prime iff P is a weakly prime.*

Proposition 3.7. *Let N be a Γ-nearring and P be an ideal of N. Then the following conditions are hold true.*

(1) For all $j,l,k \in N$ with $M \neq j(<l> + <k>) \subset P$, $j \in P$ or l and k in P.

(2) For $a \in N \setminus P$, $(P :< a > + < b >) = P \cup [(0 :< a > + < b >)]$, for all $b \in N$.

(3) For any $a \in N \setminus P$, $(P :< a > + < b >) = P$ or $(P :< a > + < b >) = (0 :< a > + < b >)$.

(4) P is weakly prime.

Proof. (1)$\Rightarrow$ (2) Suppose $t \in (P :< a > + < b >)$, for all $a \in P$ and $b \in N$. Then $t\Gamma(< a > + < b >)P$ such that $t\Gamma(< a > + < b >) = 0$. Which implies $t \in (0 :< a > + < b >)$. So, $0 \neq t\Gamma(< a > + < b >)P$, $t \in P$ by assumption.

(2) $\Rightarrow$ (3) It means that the ideal may be the combination of two ideals. It implies equality between any one of them.

(3) $\Rightarrow$ (4) Suppose A and B are two ideals of the N with $A\Gamma B \subseteq P$. Let A is not a subset of P so there are $l \in A$ and $k \in B$ with $l,k \notin P$. Which implies that $A\Gamma B = 0$.

(4) $\Rightarrow$ (1) Suppose $x_1 \in B$, $A\Gamma(< x > + < x_1 >)P$ implies $A\Gamma(P :< x > + < x_1 >)$. Since we supposed $A\Gamma(< x > + < x_1 >) = 0$ which gives $A\Gamma x_1 = 0$. Thus $A\Gamma B = 0\&$. Hence P is a weakly prime ideal of N.

□

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] G.L. Booth, Characterizing the nil radical of a gamma ring, Quaest. Math. 9 (1986), 55–67.

[2] J. R. Clay, Nearrings: geneses and applications. Oxford University Press, 1992.

[3] E. Domi, Prime ideals and bi-ideals in gamma near–rings. In 1st international symposium on computing in informatics and Mathematics (ISCIM 2011), in Collaba- Ration between EPOKA University and Aleksandr Moisiu University of Durrs on June, pages 2–4, 2011.

[4] A. Kaučikas, R. Wisbauer, On strongly prime rings and ideals. Commun. Algebra, 28 (2000), 5461–5473.

[5] N. Nobusawa, On a generalization of the ring theory. Osaka J. Math. 1 (1964), 81–89.

[6] B. Styanarayana, Contribution to Nearring theory. PhD thesis, Nagarjuna University, 1985.