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FIXED POINTS FOR WEAKLY COMPATIBLE MAPS WITH EA AND CLR PROPERTIES

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Abstract: Common fixed point theorems using weakly compatible maps with E.A property as well as common limit range property have been established for the mappings satisfying generalized $\Omega -$contraction condition. For the validation of the results an application as well as an example has been given.

Keywords and phrases: weakly compatible mappings; E.A. property; common limit range property.

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1. INTRODUCTION

Fixed point theorems work as a tool for solution of a number of problems in mathematical sciences as well as engineering fields. Banach fixed point theorem has been spontaneously generalized by several authors in different ways. There have been extensive studies for over a century now, regarding fixed point for a pair of mappings or a family of mappings. This led to several motivating as well as elegant results in this direction by various authors.

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Boyd and Wong[4] used a control function instead of constant k in contraction principle. Different control functions have been considered by different authors. Boyd and Wong used the control function which is non-decreasing, continuous function $\Omega(t) < t$, where $\Omega:[0, \infty) \to [0, \infty)$ with $t > 0$. The generalization given by Jungck [9] for Banach's fixed point theorem provided new direction to fixed point theory. The main drawback of contraction principle is that mapping involved in this principle is uniformly continuous, in fact it is continuous. Subsequently there came up many research papers involving contractive definition with no requirement of continuity of T. Further generalizations as well as extensions of this result were given in different manner by several authors. S. Sessa [18] conceived the notion of weak commutativity and proved common fixed point theorem for such maps. In 1996 Jungck [11] enlarged the notion of commuting, weakly commuting mappings to compatible mappings. Clearly commuting, weakly commuting mappings are compatible but neither implication is reversible. Various authors have given a lot of fixed-point theorems for compatible mappings satisfying contractive type conditions and assuming continuity of at least one of mappings.

Researchers are still trying to weaken the commutativity /minimal commutative type mappings and continuity of mappings. Jungck [11] in 1996 enlarged compatible mappings concept to weakly compatible mappings. For metric space (X, d), self-mappings f and g are said to be weakly compatible, whenever $fu = gu$,$u \in X$ gives $fgu = gfu$.

Pant [16] in 1994 brought out the concept of R-weakly commuting mappings.

Let (f, g) be self-mappings of a metric space (X, d), the pair (f, g) is said to be R-weakly commuting if there exists some real number $R > 0$ such that

$d(fgx, gfx) \leq Rd(fx, gx)$, for all $x \in X$.

A pair (f, g) of self-mappings of a metric space (X, d) is said to be pointwise R-weakly commuting on X if, for a given $x \in X$, there exists $R > 0$ such that

$$d(fgx, gfx) \leq Rd(fx, gx).$$

Pathak et al. [14], brought out an improvised version of R-weak commutativity of mappings and named them as R-weak commutativity of type (A_g) and (A_f).

Let (f, g) be a pair of self-maps of metric space (X, d). The pair (f, g) is

(i) R-weakly commuting of type (A_g) if $\exists$ a real number $R > 0$ for which

$d(g\, fx, ffx) \leq R\, d(f\, x, gx)\ \forall x \in X$

(ii) R-weakly commuting of type (A_f) if $\exists$ a real number $R > 0$ for which

$d(fgx, ggx) \leq R\, d(fx, gx)\ \forall x \in X$

Moreover at their coincidence points, these mappings commute.

Aamri and El-Moutawakil [2] brought out concept of E.A. property for self-maps.

Self-maps f and g of a metric space (X, d)satisfy E.A property if there is a sequence $\{x_n\}$ in X so that

$$\lim_n fx_n = \lim_n gx_n = t, \text{ for some t inX.}$$

It is worth mentioning that weak compatibility and E.A. property are independent of one another. The notion of E.A. property was further generalized by Sintunavarat and Kumam [19] who brought out the notion of common limit in the range property (CLR property).

Maps f, g over metric space (X, d)satisfy the common limit in the range of g property if

$$\lim_n fx_n = \lim_n gx_n = gt \text{ for some } t \in X.$$

We shall denote the common limit in the range of g property, hence onwards by CLRg property. The implication of the CLR property and E.A. property lies in the fact that:

(a)In both these properties the hypothesis of continuity of involved maps is relaxed along with relaxation of condition of containment of range subspace of a map into range subspaces of other maps, that's mostly needed for construction of joint iterates sequences in results related to fixed points.

(b) For the E.A. property, closed range subspace of mapping condition replaces need of completeness of space (or range subspaces of involved maps), whereas (CLR) property makes it possible to entirely relax and not replace by any other condition, the need of space completeness (or that of involved maps range subspaces).

The results given here extend, generalize and improve the results of Arora and Kumar [1] and Branciari [5] to weakly compatible maps.

2. WEAKLY COMPATIBLE MAPPINGS

The new generalizedΩ −contraction for pair of mappings which recently appeared in [1] is already known to us now:

Consider a metric space (X, d). Define self-maps A , B, S and T on a metric space (X,d) such that:

$S(X) \subset B(X), T(X) \subset A(X)$;

$$d^3(Sx, Ty) \leq \Omega\, max \begin{Bmatrix} d^2(Ax, Sx)d(By, Ty), d(Ax, Sx)d^2(By, Ty), \\ d(Ax, Sx)\, d(Ax, Ty)\, d(By, Sx), \\ d(Ax, Ty)\, d(By, Ty)\, d(By, Sx) \end{Bmatrix} x, y \in X \text{ , where function}$$

Ω: [0, ∞) → [0, ∞) is continuous non-decreasing such that $\Omega(t) < t$, $\forall t > 0$.

Recently, Arora and Kumar [1] brought out proof of fixed point theorem for weakly compatible mappings by making use of generalized $\Omega-$contraction condition as defined before. The notation X stands for complete metric spaces.

Theorem 2.1. [1] Let A,BS,T be maps from X to X satisfying the conditions which follow:

(M1) If $B(X)$ contains $S(X)$, $A(X)$contains $T(X)$;

(M2) $$d^3(Sx,Ty) \leq \Omega\, max \begin{Bmatrix} [d^2(Ax,Sx)d(By,Ty), d(Ax,Sx)d^2(By,Ty)], \\ d\,(Ax,\ Sx)\,d(\ Ax,\ Ty\ )\,d(\,By,\ S\,x\,), \\ d\,(Ax,Ty\,)\,d(By,Sx)\,d(By,Ty\,) \end{Bmatrix}$$

for all $x, y \in X$, where function Ω: [0, ∞) → [0, ∞) is continuous non-decreasing such that $\Omega(t) < t$, $\forall t > 0$,

(M3) either of AX,X, SX ,TX is complete.

Then there exists only one fixed point of all mappings A,B, S,$T$with condition that pairs (B,T) and (A,S) are weakly compatible.

The following theorem is proved now as an outcome of Theorem2.1:

Theorem 2.2 Let the finite families $\{A_1, A_2, \ldots, A_m\}$, $\{B_1, B_2, \ldots, B_n\}$, $\{S_1, S_2, \ldots, S_p\}$ and$\{T_1, T_2, \ldots, T_q\}$ be self-maps families of metric space$(\,X\,,\ d)$ so that$A\ =\ A_1A_2\ldots A_m$, $B\ =\ B_1B_2\ldots B_n$, $S\ =\ S_1S_2\ldots S_p$ and $T\ =\ T_1T_2\ldots T_q$ satisfy condition (M1), (M2) and the following : If one of subspaces of X , $A(\,X\,)$, $B(\,X\,)$, $S(\,X\,)$, or $T(\,X\,)$ is complete, then

(i) For$A$ and S there is a coincidence point.

(ii)For $B$and T there is a coincidence point.

Also, if$A_iA_j\ =\ A_jA_i$, $B_kB_l\ =\ B_lB_k$, $S_rS_s\ =\ S_sS_r$, $T_tT_u\ = T_uT_t$, $A_iS_r\ =\ S_rA_i$ and $B_kT_t\ =\ T_tB_k$ $\forall\ \ i,j\ \in \{\,1\,,2\,,\ldots,m\} = I_1, say$; $k,l \in \{1\,,\ 2\ \,,\ldots,n\} = I_2, say$; $r,s\ \in \{1\,,2\ \,,\ldots,p\} = I_3$,say ; $t,u\ \in \{1\,,2\,,\ldots,q\} =\ I_4, say$, then $(\forall i\ \in I_1,\ k \in I_2, r \in I_3\ and\ t\ \in I_4)$ A_i, S_r, B_k and T_t have a fixed point in common.

Proof. As all of Theorem 2.1 conditions hold, for A, S, B, T, so assertions in (i) and (ii) are obvious. Appealing to various pairs commutativity, component-wise, it is readily proved that $AS\ =\ SA$; $BT\ =\ TB$, so (A,S) , (B,T) are weakly compatible pairs. As conditions of Theorem2.1 hold, so uniqueness of fixed point in common, say,z is established. It now remains to be shown that all the component maps have z as a fixed point. So take

$$S(S_r z) = ((S_1S_2\ldots S_p)S_r)z = (S_1S_2\ldots S_{p-1})((S_pS_r)z) = (S_1S_2\ldots S_{p-1})(S_rS_pz)$$
$$= (S_1S_2\ldots S_{p-2})(S_{p-1}S_r(S_pz)) = (S_1S_2\ldots S_{p-2})(S_rS_{p-1}(S_pz)) = \ldots$$
$$= S_1S_r(S_2S_3S_4\ldots\ldots S_pz) = S_rS_1(S_2S_3\ldots\ldots S_pz) = S_r(Sz) = S_rz.$$

Similarly, one can show that

$$A(S_rz) = S_r(Az) = S_rz, A(A_iz) = A_i(Az) = A_iz\ ,$$
$$S(A_iz) = A_i(Sz) = A_iz, B(B_kz) = B_k(Bz) = B_kz,$$
$$B(T_tz) = T_t(Bz) = T_tz, T(T_tz) = T_t(Tz) = T_tz$$

and $T(B_kz) = B_k(Tz) = B_kz.$

Thus we conclude: (for all i, r, k and t) A_iz and S_rz are other fixed points of pair (A, S) while for pair (B, T)other fixed points are z and T_tz. Now for the fixed points in common, appealing to their uniqueness, for each of the pair taken in a separate manner, gives

$$z = A_iz = S_rz = B_kz = T_tz,$$

which gives A_i, S_r, B_k,$T_t$have a fixed point z in common, $\forall i, r, k, t$.

On setting $A = A_1 = A_2 = \ldots = A_m$, $B = B_1 = B_2 = \ldots = B_n$, $S = S_1 = S_2 = \ldots = S_p$and $T = T_1 = T_2 = \ldots = T_q$.

Now the following conclusion can be deduced:

Corollary 2.1 For four self-mappings A, B, S ,T be of metric space(X, d) so that A_m, B_n, S_p,T_q satisfy the conditions (M1) and (M2), if one of the subspaces of X, out of $A_m(X), B_n(X), S_p(X)$ or $T_q(X)$ is complete, then there is a fixed point in common which is unique for A, B, S, T, as long as (A, S), (B, T) commute.

Theorem 2.3 If the conditions in Theorem 2.1 of weakly compatible mapping(keeping intact remaining assumptions) are replaced by any one of the conditions

a) R- weakly commuting property,

b) R-Weakly commuting mappings of type (A_S),

c) R-Weakly commuting mappings of type (A_T),

d) R-Weakly commuting mappings of type (P),

e) Weakly commuting.

then Theorem 2.1 still holds.

Proof. As all the hypotheses of Theorem 2.1 are met, so for both pairs coincidence points are assured. Let for the pair (A, S) , w be arbitrary coincidence point. By the use of R-weak commutativity we get

$$d(ASw, SAw) \leq Rd(Aw, Sw)$$

implies $ASw = SAw$. Thus pair (A, S) is weakly compatible. In a similar manner, at all of its coincidence points, the pair (B, T) commutes. It can be concluded that S, T, A, B have a fixed point in common and which is unique, by using Theorem 2.1.

The case when (A, S) is R-weakly commuting map of type (A_S), gives

$$d(ASw, SSw) \leq Rd(Aw, Sw),$$

which amounts to saying that $ASw = SSw$.

Also $d(ASw, SAw) \leq d(ASw, SSw) + d(SSw, SAw) = 0 + 0 = 0,$

giving thereby $ASw = SAw$.

In the same manner, pair(A, S) being R-weakly commuting mappings of type (A_T) or of type (P) or being weakly commuting, gives that at their points of coincidence,(A, S) also commutes. In the same manner, it is shown that pair (B, T) too is coincidentally commuting. By virtue of Theorem 2.1, A, B, S and T have a fixed point in common which is unique which makes the proof complete.

A common fixed point theorem for weakly compatible mappings accompanying E.A property is proved now.

Theorem 2.4 Let (X, d) be a complete metric space. Let A, B, S,T be maps of X into X which satisfy (M1), (M2) and the conditions which follow:

(M4) one out of AX,BX,SX,TX, subspaces of X is closed in X,

(M5)(A, S), (B, T) are weakly compatible pairs,

(M6) (A, S) ,(B, T) satisfy E.A. property pairs,

Then there is a fixed point in common and which is unique for A, S, B, T.

Proof: Let the pairs A, S satisfy E.A. property then $\exists$ a sequence $\{x_n\}$ in X such that $\lim_n Ax_n = \lim_n Sx_n = z$ for some z in X.As $S(X) \subset B(X)$, $\exists$ a sequence $\{y_n\}$ in X such that $By_n = Sx_n$. Hence $\lim_n By_n = z$. Also $T(X) \subset A(X)$ so $\exists$ sequence $\{w_n\}$ in X so that $Tw_n = Ax_n$. So $\lim_n Tw_n = z$.

Suppose now that $BX \subset X$ is closed, then $\exists$ u in X so that $z = Bu$. Eventually, it follows

$$\lim_n Ax_n = \lim_n Sx_n = \lim_n Tw_n = \lim_n By_n = z = Bu.$$

for some u in X.

It is firstly claimed: Tu = z.

Putting $x = x_n$,$y = u$ in (M2)

$$d^3(Sx_n, Tu) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(Ax_n, Sx_n)d(Bu, Tu) + d(Ax_n, Sx_n)d^2(Bu, Tu)], \\ d(Ax_n, Sx_n)d(Ax_n, Tu)d(Bz, Sx_n), \\ d(Ax_n, Tu)d(Bu, Sx_n)d(Bu, Tu) \end{Bmatrix}.$$

Therefore, we get

$$[d^3(z, z)] \leq \quad \psi\, max \begin{Bmatrix} \frac{1}{2}[0 + 0\,], \\ 0\,, \\ 0 \end{Bmatrix}$$

We get $z = Tu$ and so $z = Tu = Bu$. As $T(X) \subset A(X)$, so $\exists\, v \in X$ such that $Tu = z = Av$.

We claim now: $Sv = z$.

Setting $x = v$,$y = u$ in (M2) we have

$$d^3(Sv, Tu) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(Av\,, Sv)\, d(Bu\,, Tu) \;+\; d(\,Av\,, Sv\,)d^2(\,Bu\,, Tu)], \\ d(\,Av, Sv)\, d(\,Av, Tu)\, d(\,Bu\,, Sv), \\ d\,(Av\,, Tu\,)\, d(\,Bu\,, Sv\,)\, d(\,Bu\,, Tu\,) \end{Bmatrix}$$

Therefore, we get

$$d^3(Sv, z) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(z, Sv)\;\; d(z, z) + \;d(z, Sv)\; d^2(z, z)], \\ d(z, Sv)\;\; d\,(z, z)\;\; d\,(z, Sv), \\ d\,(z, z)\;\; d\,(z, Sv)\;\; d\,(z, z) \end{Bmatrix}.$$

This implies that $S\,v = z$ and hence $Sv = Av = z$ so $Av = Sv = Tu = Bu = z$. As pairs A, S;B, T are weakly compatible with v;u as their respective coincidence points, so we get

$$Az = A\,(Sv) = S\,(Av) = Sz, \quad Bz = B(Tu) = T\,(Bu) = Tz.$$

We prove now that A, S, B, Thave a fixed point z in common. We therefore prove $Sv = Tz$.

Setting in (M2),$x = v$ and $y = z$, we have

$$d^3(Sv, Tz) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(Av, Sv)d(Bz, Tz) + d(Av, Sv)d^2(Bz, Tz)], \\ d(Av, Sv)d(Av, Tz)d(Bz, Sv), \\ d(Av, Tz)d(Bz, Sv)d(Bz, Tz) \end{Bmatrix}.$$ Therefore, we get

$$d^3(Sv, Tz) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(z, z)\, d(Bz, Tz) + d(z, z)d^2(Bz, Tz)], \\ d\,(z, z)\, d\,(Sv, Tz)\, d\,(Tz, Sv), \\ d\,(Sv, Tz)\, d\,(Tz, Sv)\, d\,(Bz, Tz) \end{Bmatrix}.$$

This implies that $S\, v = T\, z$ and hence $z = S\, v = Tz$; $z = Tz = Bz$. So $B$and T have a common fixed point z.

It can also be proved that A and S have a common fixed point $Sv = z$.

In the same manner the proof can be completed for cases when the subsets of X; AX or SX or TX is closed. Uniqueness comes readily. This makes the proof complete.

A common fixed point theorem for weakly compatible mappings with common limit range property is now proved.

Theorem 2.5. Let (X, d) be a metric space which is complete. Let A, S, B , T be maps of X into X so that (M1), (M2), (M4), (M5) and the conditions which follow are satisfied:
(M7) Pair(A, S) satisfies CLR_A property or pair(B, T) satisfies $CL\ R_B$ property.
Then there is a fixed point in common and which is unique for$A$,S, B, T.
Proof: If CLR_B property is satisfied by pair B,T; it follows in X, ∃ sequence $\{x_n\}$ so that $\lim_n Bx_n = \lim_n Tx_n = z \in BX$. SinceT(X)is contained in A(X) so for every $\{x_n\}$ in X there exists corresponding sequence $\{y_n\}$ in X so that $Tx_n = Ay_n$. Hence, $\lim_n Ay_n = \lim_n Tx_n = z \in BX$. Thus we have $\lim_n Ay_n = \lim_n Bx_n = \lim_n Tx_n = z$.

Now let $BX \subset X$ be closed, then ∃ a point u in X for which Bu=z.

It is now shown: $\lim_n Sy_n = z$. Putting $x = y_n$ and$y = x_n$. We have

$$d^3(Sy_n,Tx_n) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(Ay_n,Sy_n)d(Bx_n,Tx_n)+d(Ay_n,Sy_n)d^2(Bx_n,Tx_n)], \\ d(Ay_n,Sy_n)d(Ay_n,Tx_n)d(Bx_n,Sy_n), \\ d(Ay_n,Tx_n)d(Bx_n,Sy_n)d(Bx_n,Tx_n) \end{Bmatrix}.$$

$$d^3(Sy_n,z) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(z,Sy_n)d(z,z)+d(z,Sy_n)d^2(z,z)], \\ d(z,Sy_n)\ d(z,z)\ d(z,Sy_n), \\ d(z,z)\ d(z,Sy_n)\ d(z,z) \end{Bmatrix}$$

$$d^3(Sy_n,z) \leq \psi\, max \begin{Bmatrix} \frac{1}{2}[d^2(z,Sy_n)d(z,z)+d(z,Sy_n)\ d^2(z,z)], \\ d(z,Sy_n)\ d(z,z)\ d(z,Sy_n), \\ d(z,z)\ d(z,Sy_n)\ d(z,z) \end{Bmatrix},$$

which implies that $lim_n d(Sy_n,z)=0$. Hence $lim_n Ay_n = lim_n Bx_n = lim_n Tx_n = lim_n Sy_n = z = Bu$ for some u in X. It can be easily proved that A, B, S, T have fixed point in common. Also it is obvious that pair A, S satisfies CLR_A property.

For cases when subset of X; AX or TX or SX is closed, the proof can be completed in same way. Uniqueness follows easily. Thus proof is complete.

Example 2.1 Let $X=[\ 2\ ,\ 20\ ]$ equipped with usual metric d.Constructing self-maps on X , that is, $A,B,S\ ,T$as:

$$Ax = \begin{cases} 12 & if \quad 2 < x \leq 5 \\ x-3 & if \quad x > 5 \\ 2 & if \quad x = 2. \end{cases}, \qquad Bx = \begin{cases} 2 & if \quad x = 2 \\ 6 & if \quad x > 2 \end{cases}$$

$$Sx = \begin{cases} 6 & if \quad 2 < x \leq 5 \\ x & if \quad x = 2 \\ 2 & if \quad x > 5. \end{cases}, \quad Tx = \begin{cases} x & if \quad x = 2 \\ 3 & if \quad x > 2 \end{cases}.$$

Defining continuous, non-decreasing function Ω having domain and range as $[0, \infty)$ so that $\Omega(t) < t$, $\forall t > 0$.Letsequence $< x_n >= 5 + \frac{1}{n}$.Then we get (S,A),(B,T) to be compatible weakly maps pairs. So conditions of Theorem2.1 are ensured, with fixed point 2 which is common as well as unique for maps$A$,S,T, B

In Theorem2.1, taking $S=T$, the result ensues.

3. APPLICATION

A fixed point theorem was presented byBranciari [5] in 2002 for a map for which analogous contraction principle for integral type inequality is satisfied.

Theorem 3.1. Let X be a complete metric space and P: X → X be a map satisfying the following :

For all x, y in X and $0 < c < 1$,

$\int_0^{d(Px,Py)} \xi(t)\, dt \leq c \int_0^{d(x,y)} \xi(t)\, dt$, *where* ξ : R+ → R+ is Lebesgue-integrable function which is non-negative, summable and such that given any positive ε >0 over all subsets of R+ which is compact and $\int_0^{\epsilon} \xi(\mathrm{t})\mathrm{dt} > 0$. Then P possesses one and only one fixed point z ∈ X so that, for every x in X, $\lim_{n\to\infty}(\mathrm{P}^n)$ = z.

As an application, we extend and generalize the Theorem 2.1 for pairs of weakly mappings satisfying a contractive inequality of integral type.

Theorem 3.2. Let self-mapsA, S, B, T on X satisfy (M1), (M2), (M3) with

$$\int_0^{d^3(Sx,\ Ty)} \varphi\,(t)\, dt \leq \int_0^{M(x,\ y)} \varphi\,(t)\, dt$$

$$M(x,y) = \psi\, max \begin{Bmatrix} [d^2(Ax, Sx)\, d(By, Ty) + d(Ax, Sx) d^2(By, Ty)]/2, \\ d(Ax, Ty)\ d(Ax, Sx)\, d(By, Sx), \\ d(By, Sx)\ d(Ax, Ty)\, d(By, Ty) \end{Bmatrix} \text{where,}$$

for all $x, y \in X, \psi$: [0, ∞) → [0, ∞) with $\psi(t) < t, t > 0$,continuous, non-decreasing function; $\emptyset$: [0, ∞) → [0, ∞) being continuous :$\emptyset\,(t)$ = 0 iff t = 0 and $\emptyset(t) > 0, \forall t > 0$. Also when φ : R+ → R+ is non-negative, Lebesgue-integrable over R+ function, summable over all compact subsets of R+ and so that ,

$$\int_0^{\epsilon} \varphi\,(\mathrm{t})\, \mathrm{dt} \text{ is positive, } \forall \text{positive } \varepsilon$$

Then ∃ fixed point in common which is unique, for A, S, B, T provided pairs (A, S), (B, T) are weakly compatible pairs.

Proof. Taking φ (t) = 1 in Theorem 2.1, the proof ensues.

Remark 3.1. In every integral type contractive condition, a corresponding contractive condition not involving integrals is automatically included by setting φ(t) = 1 over R+.

Remark 3.2. Theorem 2.1 can be improved considerably by setting $\varphi(t) = 1$ in Theorem 2.1. We also extend and generalize the theorem of Branciari [5] for a pair of weakly compatible mappings. Many results associated with contractive conditions of similar kind can be generalized in the same way.

CONCLUSION

The common fixed point theorems using weakly compatible maps with E.A property as well as common limit range property for the mappings satisfying generalized $\Omega-$contraction condition have been established. The results given here extend, generalize and improve the results of Arora and Kumar [1] and Branciari [5] to weakly compatible maps.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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