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COMPARISON OF SOME RELIABILITY CHARACTERISTICS BETWEEN REDUNDANT SYSTEMS REQUIRING SUPPORTING UNITS FOR THEIR OPERATIONS

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Abstract: In this paper, two different systems both are requiring a supporting unit for their operation are studied. The first system consist of 3-out-of-4 subsystem requiring its support from 2-out-of-4 subsystem for its operation while the other system is two unit cold standby system where each unit is attached to its supporting unit for its operation. Each system is attended by two repairmen, one repairing the main unit and the other repairing the supporting unit. Explicit expressions for mean time to system failure (MTSF) and steady- state availability are developed. We analyzed the system by using linear differential equations. Effect of failure and repair rates on mean time to system failure and steady-state availability have also been discussed graphically. Comparisons are made graphically for specific values of parameters. Furthermore, we compare these reliability characteristics for the two models and found that model I is more reliable and efficient than the remaining models.

Keywords: Redundant system, Reliability characteristics, supporting unit

2000 AMS Subject Classification: 90B25

1. Introduction

Redundancy is a technique used to improve system reliability and availability.

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Reliability is vital for proper utilization and maintenance of any system. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. Studies on redundant system are becoming more and richer day by day due to the fact that numbers of researchers in the field of reliability of redundant system are making huge contributions. Models of redundant systems as well as methods of evaluating system reliability indices such as mean time to system failure (MTSF), system availability, busy period of repairman, profit analysis, etc have been studied in order to improve the system effectiveness (see for example [1] - [4]). Example of such systems are 1-out-of-2, 2-out-of-3,2-out-of-4, or 3-out-of-4 redundant systems. These systems have wide application in the real world. The communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system.

In this paper, we construct two different redundant systems both requiring supporting units/subsystem for their operation. The first system/configuration consists of 3-out-of-4 subsystem requiring its support from 2-out-of-3 subsystem for its operation as can be seen in Fig.1. In the second system/configuration the system is a two unit cold standby where each unit is attached to its supporting unit as can be seen from Fig.3.Each system is attended by two repairmen. One repairing the main unit and the other repairing the supporting unit. Example of such systems can be seen in satellite amplifier redundant system, computer systems, etc. In this paper, we construct two distinct redundant systems and derived their corresponding mathematical models. Furthermore, we study reliability characteristics of each model using linear differential equation method. The focus of our analysis is primarily to capture the effect of both failure and repair rates on the measures of system effectiveness like MTSF, availability and profit. We also looked at the effect of the system design.

The organization of the paper is as follows. In Section 2, we give the notations and assumptions of the study. In Section 3, we give detailed description of the state of the systems.Some reliability characteristics of model I and II are derived in Sections 4. The results of our numerical simulations are presented in Section 5 and discussed in

Section 6. Finally, we make a concluding remark in Section 7.

2. Preliminaries

2.1 Notations and assumptions

Notations

S_i : Transition states, $i = 0,1,2,3,4,5,6,7$ for system I and $i = 0,1,2,3,4,5,6$ for system II

α_1 : repair rate of the main unit for both system I and II

α_2 : repair rate of the supporting unit for both system I and II

β_1 : Failure rate of the main unit for both system I and II

β_2 : Failure rate of the supporting unit for both system I and II

$P(t)$: Probability row vector

$P_i(t)$: Probability that the system is in state S_i

A_i : Main operational units, $i = 1,2,3$ in system I and $i = 1,2$ in system II

P_i : Supporting units $i = 1,2,3$ in system I and $i = 1,2$ in system II

$MTSF_i$: Mean time to system failure, $i = 1,2$

$A_i(\infty)$: System availability, $i = 1,2$

Assumptions

1. System I consist is 3-out-of-4 subsystem and a 2-out-of-3 supporting subsystem
2. System II consist of two cold standby with the supporting attached to each uniy
3. Failure and repair rates are constant
4. Initially two units are in operable condition of full capacity
5. System I failed when the number of working unit goes down beyond one or when two or three of the supporting units failed
6. System II failed when all the units failed
7. Failure and repair time follow exponential distribution

8. Repair is as good as new (Perfect repair).

2.2 Model descriptions

2.2.1 First System / Configuration

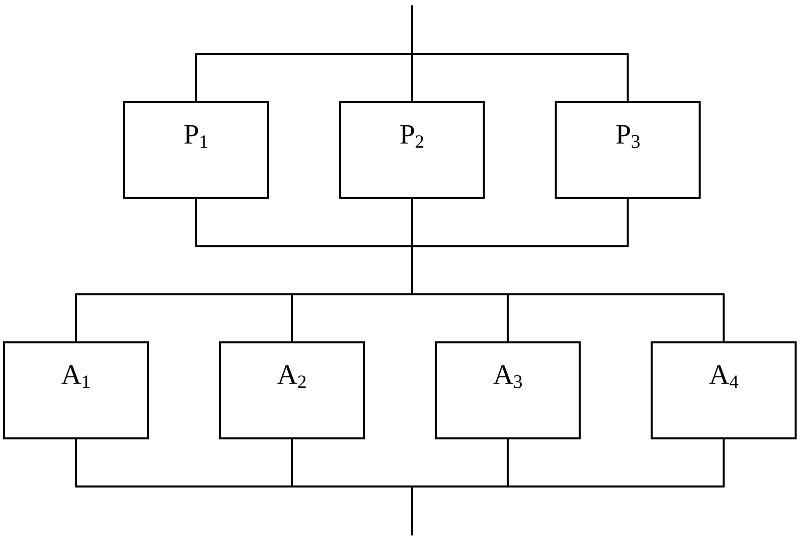

Fig. 1 Reliability block diagram of the System

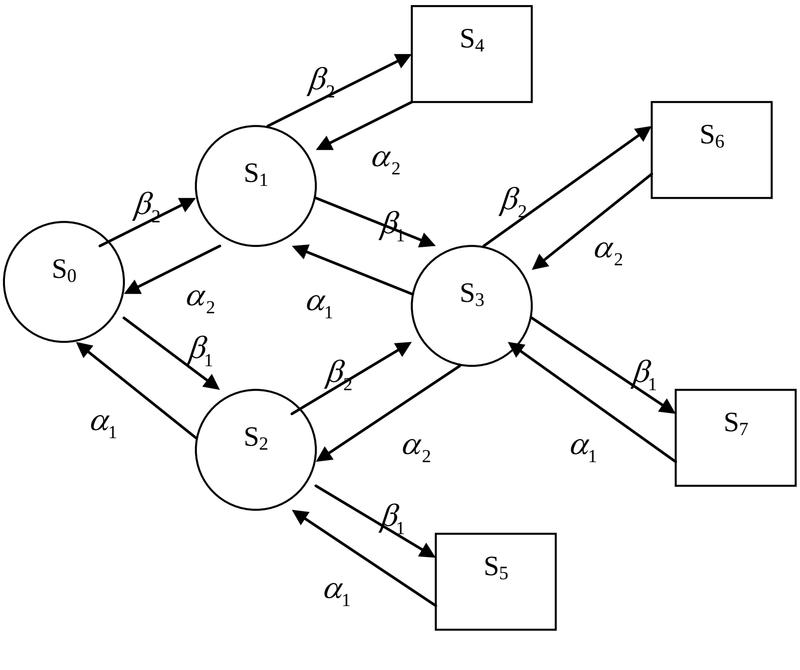

Fig. 2 transition diagram of the first System

State of the System:

State S_0: In subsystem I, two units are operational, one unit is in standby. In subsystem II, three units are operational, one unit is in standby. The system is operational

State S_1: In subsystem I, two units are operational, one is under repair. In subsystem II, three units are operational, one unit is in standby. The system is operational

State S_2: In subsystem I, two units are operational, one unit is in standby. In subsystem II, three units are operational, one unit is under repair. The system is operational

State S_3: In subsystem I, two units are operational, one is under repair. In subsystem II, three units are operational, one unit is under repair. The system is operational

State S_4: In subsystem I, one unit is under repair, one unit is waiting for repair, one unit is good. In subsystem II, all the units are good. The system failed

State S_5: In subsystem I, two units are operational, one unit is in standby. In subsystem II, one unit is under repair, one is waiting for repair and two units are good. The system failed

State S_6: In subsystem I, one unit is under repair, one unit is waiting for repair and one unit is good. In subsystem II, one unit is under repair, three units are good. The system failed.

State S_7: In subsystem I, one unit is under repair, two units are good. In subsystem II, one unit is under repair, one unit is waiting for repair and two units are good. The system failed.

2.2.2 Second System / Configuration

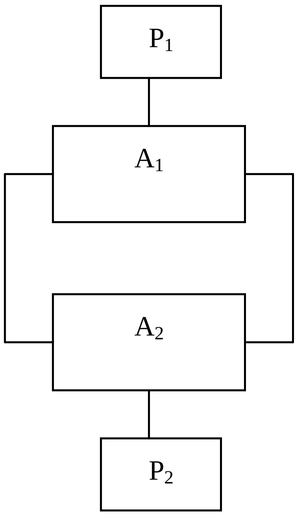

Fig. 3 reliability block diagram of the second system

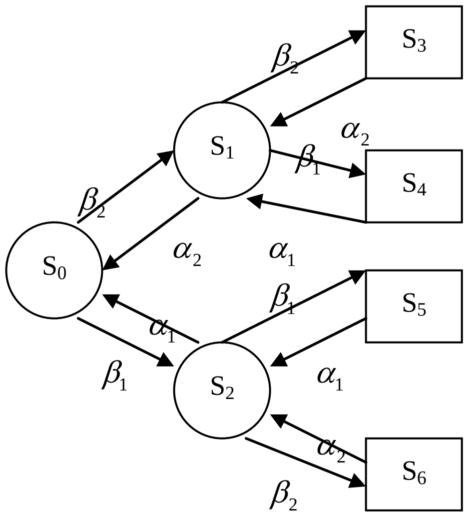

Fig.4 Transition diagram of the second system

States of the System:

S_0: Unit I and supporting unit I are operational, unit II and supporting II are in standby. The system is operational

S_1:Unit I is good, supporting unit I is under repair, unit II and supporting unit II are operational. The system is operational

S_2: Unit I is under repair, supporting I is good, unit II and supporting unit II are operational. The system is operational

S_3: Unit I is good, supporting unit I is under repair, unit II is good, supporting unit II is waiting for repair. The system failed

S_4: Unit I is good, supporting unit I is under repair, unit II is under repair, supporting unit II is good. The system failed

S_5: Unit is under repair, supporting I is good, unit II is waiting for repair, supporting unit II is good. The system failed

S_6: Unit is under repair, supporting I is good, unit II is good, supporting unit II is under repair. The system failed

3. Main results

3.1 Reliability Characteristics of the first System

3.1.1 Mean time to system failure analysis $MTSF_1$

From Fig. 1 above, define $P_i(t)$ to be the probability that the system at time $t, t \geq 0$ is in state S_i. Let $P(t)$ be the probability row vector at time t, the initial condition for this paper are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = \left[1,0,0,0,0,0,0,0\right]$$

we obtain the following differential equations:

$$\frac{dP_0(t)}{dt} = -(\beta_2 + \beta_1)P_0(t) + \alpha_2 P_1(t) + \alpha_1 P_2(t)$$

$$\frac{dP_1(t)}{dt} = -(\beta_2 + \beta_1 + \alpha_1)P_1(t) + \beta_2 P_0(t) + \alpha_1 P_3(t) + \alpha_2 P_4(t)$$

$$\frac{dP_2(t)}{dt} = -(\beta_2 + \beta_1 + \alpha_1)P_2(t) + \beta_1 P_0(t) + \alpha_2 P_3(t) + \alpha_1 P_5(t)$$

$$\frac{dP_3(t)}{dt} = -(\beta_2 + \beta_1 + \alpha_2 + \alpha_1)P_3(t) + \beta_1 P_1(t) + \beta_2 P_2(t) + \alpha_2 P_6(t) + \alpha_1 P_7(t)$$

$$\frac{dP_4(t)}{dt} = -\alpha_2 P_4(t) + \beta_2 P_1(t)$$

$$\frac{dP_5(t)}{dt} = -\alpha_1 P_5(t) + \beta_1 P_2(t)$$

(1)

$$\frac{dP_6(t)}{dt} = -\alpha_2 P_4(t) + \beta_2 P_3(t)$$

$$\frac{dP_7(t)}{dt} = -\alpha_1 P_4(t) + \beta_1 P_3(t)$$

Which is in matrix form as :

$$\dot{P} = A_1 P \qquad (2)$$

$$A_1 = \begin{bmatrix} -(\beta_2+\beta_1) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\beta_2+\beta_1+\alpha_2) & 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\beta_2+\beta_1+\alpha_1) & \alpha_2 & 0 & \alpha_1 & 0 & 0 \\ 0 & \beta_1 & \beta_2 & -(\alpha_1+\beta_1+\alpha_2+\beta_2) & 0 & 0 & \alpha_1 & \alpha_1 \\ 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 \end{bmatrix}$$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q_1 (see El said and El hamid [1, 2], El said [3], Haggag [4], Wang et al [5]) .

The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\int_0^\infty e^{Q_1 t}\,dt \qquad (3)$$

and

$$\int_0^\infty e^{Q_1 t}\,dt = Q_1^{-1}\text{, since } Q_1^{-1} < 0 \qquad (4)$$

For system 1, explicit expression for the $MTSF_1$ is given by

$$E\left[T_{P(0)\to P(absorbing)}\right]=MTSF_1=P(0)(-Q_1^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\frac{N_1}{D_1} \qquad (5)$$

where

$$Q_1=\begin{bmatrix}-(\beta_2+\beta_1) & \beta_2 & \beta_1 & 0\\ \alpha_2 & -(\beta_2+\beta_1+\alpha_2) & 0 & \beta_1\\ \alpha_1 & 0 & -(\beta_2+\beta_1+\alpha_1) & \beta_2\\ 0 & \alpha_1 & \alpha_2 & -(\alpha_1+\beta_1+\alpha_2+\beta_2)\end{bmatrix}$$

$$N_1=2\alpha_1\beta_2^2+3\beta_1\beta_2^2+\beta_2^3+3\alpha_1\beta_1\beta_2+3\beta_1^2\beta_2+3\alpha_2\beta_1\beta_2+\alpha_1^2\beta_2+3\alpha_1\alpha_2\beta_2+\alpha_1\beta_1^2+\beta_1^3+2\alpha_2\beta_1^2+3\alpha_1\alpha_2\beta_1+$$

$$\alpha_2\beta_2^2+\alpha_2^2\beta_1+\alpha_1^2\alpha_2+\alpha_1\alpha_2^2+\beta_2(2\alpha_1\beta_2+2\beta_1\beta_2+\beta_2^2+3\alpha_1\beta_1+\beta_1^2+\alpha_2\beta_1+\alpha_1^2+\alpha_1\alpha_2)+\beta_1(3\alpha_2\beta_2+\alpha_1\beta_2+$$

$$2\beta_1\beta_2+\beta_2^2+\beta_1^2+2\alpha_2\beta_1+\alpha_1\alpha_2+\alpha_2^2)+\beta_1\beta_2(2\beta_2+2\beta_1+\alpha_1+\alpha_2)$$

$$D_1=4\beta_1^3\beta_2+6\beta_1^2\beta_2^2+2\alpha_2\beta_1^3+\alpha_2^2\beta_1^2+2\alpha_1\beta_1^2\beta_2+4\alpha_2\beta_1^2\beta_2+4\alpha_1\beta_1\beta_2^2+4\beta_1\beta_2^3+2\alpha_2\beta_1\beta_2^2+\beta_1^4+\alpha_1^2\beta_2^2+$$

$$\beta_2^4+\alpha_1\alpha_2\beta_1^2+\alpha_1\alpha_2\beta_2^2+2\alpha_1\beta_2^3$$

3.1.2 Steady-State Availability analysis $A_1(\infty)$

For the analysis of availability case of Fig. 1 using the same initial conditions for this problem as

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0)]=[1,0,0,0,0,0,0,0]$$

The differential equations in (1) above can be expressed as

$$\begin{bmatrix}\dot P_0\\ \dot P_1\\ \dot P_2\\ \dot P_3\\ \dot P_4\\ \dot P_5\\ \dot P_6\\ \dot P_7\end{bmatrix}=\begin{bmatrix}-(\beta_2+\beta_1) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0\\ \beta_2 & -(\beta_2+\beta_1+\alpha_2) & 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0\\ \beta_1 & 0 & -(\beta_2+\beta_1+\alpha_1) & \alpha_2 & 0 & \alpha_1 & 0 & 0\\ 0 & \beta_1 & \beta_2 & -(\alpha_1+\beta_1+\alpha_2+\beta_2) & 0 & 0 & \alpha_1 & \alpha_1\\ 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0\\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0\\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_1 & 0\\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix}$$

The steady-state availability is given by

$$A_1(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) \tag{6}$$

In the steady state, the derivatives of the state probabilities become zero so that

$$A_1 P = 0 \tag{7}$$

which in matrix form

$$\begin{bmatrix} -(\beta_2+\beta_1) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\beta_2+\beta_1+\alpha_2) & 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\beta_2+\beta_1+\alpha_1) & \alpha_2 & 0 & \alpha_1 & 0 & 0 \\ 0 & \beta_1 & \beta_2 & -(\alpha_1+\beta_1+\alpha_2+\beta_2) & 0 & 0 & \alpha_1 & \alpha_1 \\ 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1 \tag{8}$$

To obtain $P_0(\infty), P_1(\infty), P_2(\infty), P_3(\infty)$, we substitute (8) in one of the redundant rows of (7) to give the following system of linear equations in matrix form which solved using MATLAB to give $A_1(\infty)$

$$\begin{bmatrix} -(\beta_2+\beta_1) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\beta_2+\beta_1+\alpha_2) & 0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\beta_2+\beta_1+\alpha_1) & \alpha_2 & 0 & \alpha_1 & 0 & 0 \\ 0 & \beta_1 & \beta_2 & -(\alpha_1+\beta_1+\alpha_2+\beta_2) & 0 & 0 & \alpha_1 & \alpha_1 \\ 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Explicit expression for steady state availability $A_1(\infty)$ is:

$$A_1(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) \tag{9}$$

$$= \frac{\alpha_1^2\alpha_2^2 + \alpha_1^2\alpha_2\beta_2 + \alpha_1\alpha_2^2\beta_1 + \alpha_1\alpha_2\beta_1\beta_2}{\alpha_1^2\beta_2^2 + \alpha_1^2\alpha_2\beta_2 + \alpha_1^2\alpha_2^2 + \alpha_1\alpha_2^2\beta_1 + \alpha_1\alpha_2\beta_1\beta_2 + \alpha_2^2\beta_1^2 + \alpha_2\beta_1\beta_2^2 + \alpha_2\beta_1^2\beta_2}$$

3.2 Reliability Characteristics of the second system

3.2.1 Mean time to System failure analysis $MTSF_2$

From Fig. 1 above, define $P_i(t)$ to be the probability that the system at time $t, t \geq 0$ is in state S_i. Let $P(t)$ be the probability row vector at time t, the initial condition for this paper are:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)\right] = \left[1,0,0,0,0,0,0\right]$$

we obtain the following differential equations:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2)P_0(t) + \alpha_2 P_1(t) + \alpha_1 P_2(t)$$

$$\frac{dP_1(t)}{dt} = -(\alpha_2 + \beta_1 + \beta_2)P_1(t) + \beta_2 P_0(t) + \alpha_2 P_3(t) + \alpha_1 P_4(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha_1 + \beta_1 + \beta_2)P_2(t) + \beta_1 P_0(t) + \alpha_1 P_5(t) + \alpha_2 P_6(t)$$

$$\frac{dP_3(t)}{dt} = -\alpha_2 P_3(t) + \beta_2 P_1(t)$$

$$\frac{dP_4(t)}{dt} = -\alpha_1 P_4(t) + \beta_1 P_1(t)$$

$$\frac{dP_5(t)}{dt} = -\alpha_1 P_5(t) + \beta_1 P_2(t)$$

$$\frac{dP_6(t)}{dt} = -\alpha_2 P_6(t) + \beta_2 P_2(t) \tag{10}$$

Which is in matrix form as :

$$\dot{P} = A_2 P \tag{11}$$

$$A_2 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\alpha_2+\beta_1+\beta_2) & 0 & \alpha_2 & \alpha_1 & 0 & 0 \\ \beta_1 & 0 & -(\alpha_1+\beta_1+\beta_2) & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix}$$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q_2 (see El said and El hamid [1, 2], El said [3], Haggag [4], Wang et al [5]) .

The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_2 = P(0)(-Q_2^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{N_3}{D_3} \tag{12}$$

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\int_0^{\infty} e^{Q_2 t}\,dt \tag{13}$$

and

$$\int_0^{\infty} e^{Q_2 t}\,dt = Q_2^{-1} \text{, since } Q_2^{-1} < 0 \tag{14}$$

$$Q_2 = \begin{bmatrix} -(\beta_1+\beta_2) & \beta_2 & \beta_1 \\ \alpha_2 & -(\alpha_2+\beta_1+\beta_2) & 0 \\ \alpha_1 & 0 & -(\alpha_1+\beta_1+\beta_2) \end{bmatrix}$$

For system 2, explicit expression for the $MTSF_2$ is given by

$$MTSF_2 = P(0)(-Q_2^{1})\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$=\frac{(\alpha_2+\beta_1+\beta_2)(\alpha_1+\beta_1+\beta_2)+\beta_2(\alpha_1+\beta_1+\beta_2)+\beta_1(\alpha_2+\beta_1+\beta_2)}{\alpha_2\beta_1^2+\alpha_2\beta_1\beta_2+\beta_1^3+3\beta_1^2\beta_2+\alpha_1\beta_1\beta_2+3\beta_1\beta_2^2+\alpha_1\beta_2^2+\beta_2^3}$$

3.2.2 Steady-State Availability Analysis $A_2(\infty)$

For the analysis of availability case of Fig. 1 using the same initial conditions for this problem as

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0)]=[1,0,0,0,0,0,0]$$

The differential equations in (1) above can be expressed as

$$\begin{bmatrix}\dot{P}_0\\\dot{P}_1\\\dot{P}_2\\\dot{P}_3\\\dot{P}_4\\\dot{P}_5\\\dot{P}_6\end{bmatrix}=\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0\\ \beta_2 & -(\alpha_2+\beta_1+\beta_2) & 0 & \alpha_2 & \alpha_1 & 0 & 0\\ \beta_1 & 0 & -(\alpha_1+\beta_1+\beta_2) & 0 & 0 & \alpha_1 & \alpha_2\\ 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0\\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0\\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0\\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2\end{bmatrix}$$

The steady-state availability is given by

$$A_1(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty) \quad (15)$$

In the steady state, the derivatives of the state probabilities become zero so that

$$A_2P=0 \quad (16)$$

which in matrix form

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\alpha_2+\beta_1+\beta_2) & 0 & \alpha_2 & \alpha_1 & 0 & 0 \\ \beta_1 & 0 & -(\alpha_1+\beta_1+\beta_2) & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)=1 \tag{17}$$

to obtain $P_0(\infty), P_1(\infty), P_2(\infty)$, we substitute (17) in one of the redundant rows of (16) to give the following system of linear equations in matrix form which solved using MATLAB to give $A_2(\infty)$

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_2 & \alpha_1 & 0 & 0 & 0 & 0 \\ \beta_2 & -(\alpha_2+\beta_1+\beta_2) & 0 & \alpha_2 & \alpha_1 & 0 & 0 \\ \beta_1 & 0 & -(\alpha_1+\beta_1+\beta_2) & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_2(\infty) = P_0(\infty)+P_1(\infty)+P_2(\infty) =$$

$$= \frac{\alpha_1^2\alpha_2^2+\alpha_1^2\alpha_2\beta_2+\alpha_1\alpha_2^2\beta_1}{\alpha_1^2\alpha_2^2+\alpha_1^2\alpha_2\beta_2+\alpha_1^2\beta_1^2+\alpha_1\alpha_2^2\beta+\alpha_2^2\beta_1^2+2\alpha_1\alpha_2\beta_1\beta_2}$$

3.3. Numerical simulations of Systems behavior

For the study of system behavior, we plot graphs in Fig. 5 to 8 for MTSF and system availability with respect to α_1 and β_1.

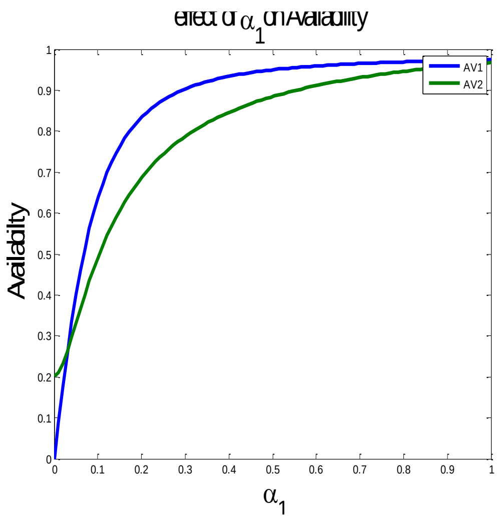

Fig. 5 effect of α_1 on systems availability

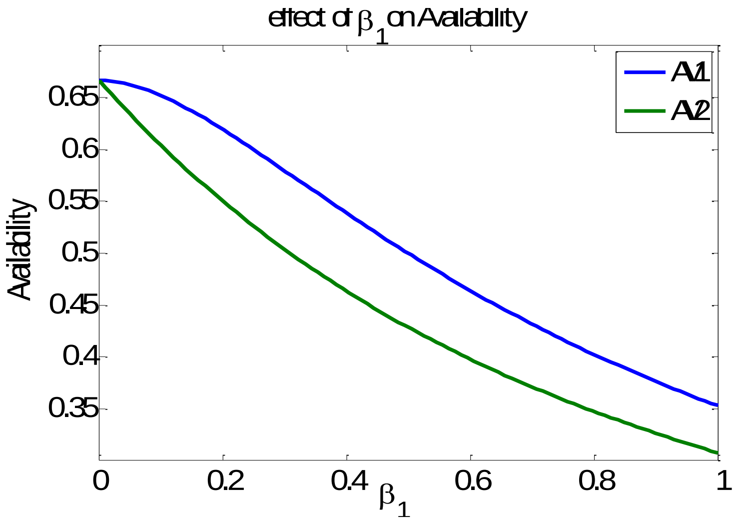

Fig. 6 effect of β_1 on systems availability

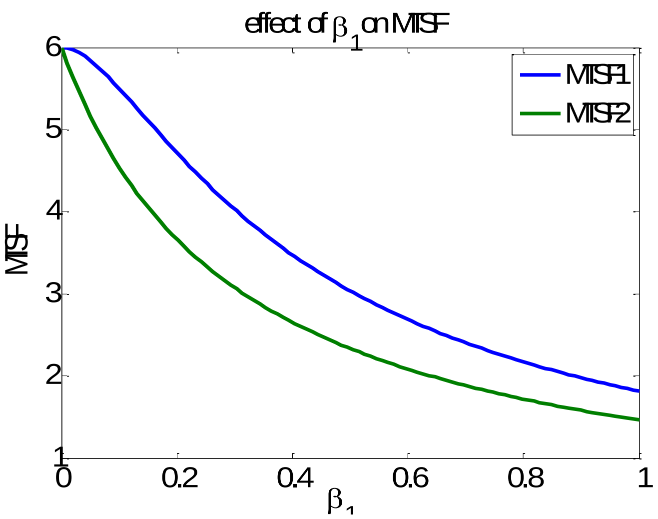

Fig. 7 effect of β_1 on MTSF

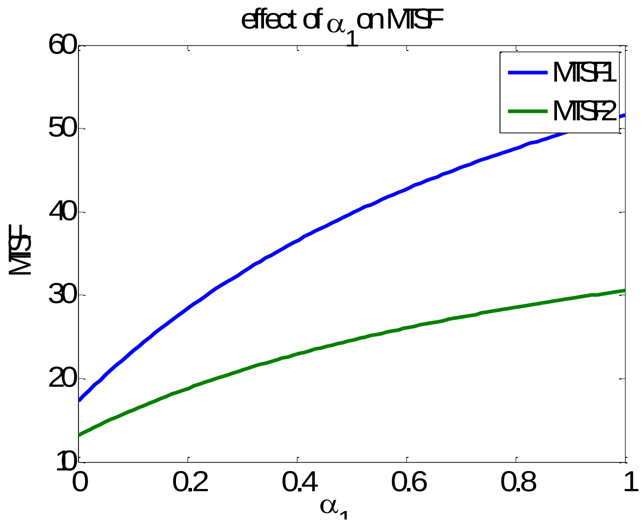

Fig. 8 effect of α_1 on MTSF

3.4 Discussion

For Fig. 5 we fixed α_2=0.69, $\beta_1 = \beta_2 = 0.1$ and vary α_1. From Fig. 5 it is clear that availability increases with increase in the value of α_1 which reflects the effect of repair on availability. This reflects the effect repair rate on main unit (3-out-of-4 subsystem). It is clear that system I tend to increase with increase in α_1 than system II. Thus, $A_1(\infty) > A_2(\infty)$.From Fig. 6, we fixed $\alpha_1 = \alpha_2 = \beta_2 = 0.5$ and vary β_1. It is evident from Fig. 6 that system availability decrease with increase in value of β_1. This depicts the effect failure rate on main unit (3-out-of-4 subsystem). The graph in Fig. 6 below reveals that system II decreases more that system I. From the result in Fig. 6, it is clear that $A_1(\infty) > A_2(\infty)$. Fig. 7 shows the relationship between β_1 and MTSF. In this figure we fixed $\alpha_1 = 0.2, \alpha_2 = 0.5, \beta_2 = 0.5$ and vary β_1 where MTSF decrease as β_1 increase. The result have indicated that $MTSF_1 > MTSF_2$. From Fig. 8, it is evident that MTSF increase as α_1 increases. We fixed α_2=0.69, $\beta_1 = \beta_2 = 0.1$ and vary α_1. From Fig. 8 it is clear that availability increases with increase in the value of α_1 which reflects the effect of repair on availability. It is clear that system I tend to increase with increase in α_1 more than system II. Thus, $MTSF_1 > MTSF_2$

3.5 Conclusion

In this paper, two different repairable systems both with standby unit and requiring a supporting unit for their operations are considered. Explicit expressions for mean time to system failure $MTSF_i$ and steady-state availability $A_i(\infty)$, $i=1,2$ are developed for the two systems and comparative analysis are performed to determine the optimal configuration between the two systems. Graphical studies of the systems behavior have shown that system I is better than system II.

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