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NEIGHBOURHOOD RESOLVING SETS IN GRAPHS - I

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Abstract. P. J. Slater [21] in 1975 introduced the concepts of locating sets and locating number in graphs. Subsequently with minor changes in terminology, this concept was elaborately studied by Harary and Melter [14], Chartrand et al [5], Robert C. Brigham et al [19], Chartrand et al [10] and Varaporn Saenpholphat and Ping Zhang [28]. Given an k-tuple of vectors, $S = (v_1, v_2, \ldots, v_k)$, the neighbourhood adjacency code of a vertex v with respect to S, denoted by $nc_S(v)$ and defined by $(a_1, a_2, \ldots, a_k)$ where a_i is 1 if v and v_i are adjacent and 0 otherwise. S is called a neighbourhood resolving set or a neighbourhood r-set if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V(G)$. A study of this new concept has been done in [25], [26]. In this paper, the study of minimal neighbourhood resolving sets and neighbourhood irredundant sets in graphs are initiated.

Keywords: locating sets, locating number, neighbourhood resolving sets, neighbourhood irredundant sets.

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1. Introduction

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In the case of finite dimensional vector spaces, every ordered basis induces a scalar coding of the vectors where the scalars are from the base field. While finite dimensional vector spaces have rich structures, graphs have only one structure namely adjacency. If a graph is connected, the adjacency gives rise to a metric. This metric can be used to define a code for the verices. P. J. Slater [21] defined the code of a vertex v with respect to a k-tuple of vertices $S = (v_1, v_2, \dots, v_k)$ as $(d(v, v_1), d(v, v_2), \dots, d(v, v_k))$ where $d(v, v_j)$ denotes the distance of the vertex v from the vertex v_j. Thus, entries in the code of a vertex may vary from 0 to diameter of G. If the codes of the vertices are to be distinct, then the number of vertices in G is less than or equal to $(diam(G)+1)^k$. If it is required to extend this concept to disconnected graphs, it is not possible to use the distance property. One can use adjacenty to define binary codes, the motivation for this having come from finite dimensional vector spaces over Z_2. There is an advantage as well as demerit in this type of codes. The advantage is that the codes of the vertices can be defined even in disconnected graphs. The drawback is that not all graphs will allow resolution using this type of codes.

Given an k-tuple of vectors, $S = (v_1, v_2, \dots, v_k)$, the neighbourhood adjacency code of a vertex v with respect to S is defined as $(a_1, a_2, \dots, a_k)$ where a_i is 1 if v and v_i are adjacent and 0 otherwise. Whereas in a connected graph $G = (V, E)$, V is always a resolving set, the same is not true if we consider neighbourhood resolvability. If u and v are two vertices which are non-adjacent and $N(u) = N(v)$, u and v will have the same binary code with respect to any subset of V, including V. Nevertheless, the neighbourhood resolvability has certain advantages. This concept is introduced and studied in this paper.

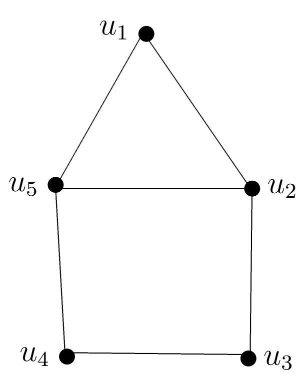
In section 1, definitions, examples of neighbourhood resolving sets are given and minimal neighbourhood resolving sets are studied. In the second section, the neighbourhood resolving number for Complete graphs, paths and cycles are derived. The third section is devoted to the study of neighbourhood irredundant sets. Several interesting results are derived.

2. Neighbourhood Resolving sets in Graphs

Definition 0.1. *Let G be any graph. Let $S \subset V(G)$. Consider the k-tuple $(u_1, u_2, \ldots, u_k)$ where $S = \{u_1, u_2, \ldots, u_k\}$, $k \geq 1$. Let $v \in V(G)$. Define a binary neighbourhood code of v with respect to the k-tuple $(u_1, u_2, \ldots, u_k)$, denoted by $nc_S(v)$ as a k-tuple $(r_1, r_2, \ldots, r_k)$ where* $r_i = \begin{cases} 1, \ if \ v \in N(u_i), 1 \leq i \leq k \\ 0, \ otherwise \end{cases}$. *$S$ is called a neighbourhood resolving set or a neighbourhood r-set if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V(G)$.*
Though neighbourhood is defined with respect to a specific order of the elements of S as a k-tuple (where $|S| = k$), we loosely use the word, "code of u with respect to S" meaning there by code of u with respect to a particular k-tuple from S which is usually written as $(u_1, u_2, \ldots, u_k)$ if $S = \{u_1, u_2, \ldots, u_k\}$.

Example 0.2.

G :



Now $S = V(G) = \{u_1, u_2, u_3, u_4, u_5\}$ is a neighbourhood resolving set of G, since $nc_S(u_1) = (0, 1, 0, 0, 1)$; $nc_S(u_2) = (1, 0, 1, 0, 1)$; $nc_S(u_3) = (0, 1, 0, 1, 0)$; $nc_S(u_4) = (0, 0, 1, 0, 1)$ and $nc_S(u_5) = (1, 1, 0, 1, 0)$.

Observation 0.3. *The above definition holds good even if G is disconnected.*

In the following theorem characterisation of connected graphs which admit neighbourhood resolving sets is given.

Theorem 0.4. [25] *Let G be a connected graph of order $n \geq 3$. Then G does not have any neighbourhood resolving set if and only if there exist two non adjacent vertices u and v in $V(G)$ such that $N(u) = N(v)$.*

Theorem 0.5. *Let $G = (V, E)$ be a simple graph. A neighbourhood resolving subset S of V is minimal if and only if one of the following conditions holds for every $u \in S$.*
(i) There exist $x, y \in V - S$ such that (x and y are either both adjacent to v or both are non-adjacent to v for all $v \in S - \{u\}$) and (x is adjacent to u; y is not adjacent to u or x is not adjacent to u; y is adjacent to u).
(ii) There exists $x \in V - S$ such that x is adjacent to u and x and u are both either adjacent to v or non adjacent to v, for every $v \in S - \{u\}$.
(iii) There exist $x, y \in S - \{u\}$ such that x is not adjacent to y and (x and y are either both adjacent to v or both are non-adjacent to v for all $v \in S - \{u\}$) and (x is adjacent to u; y is not adjacent to u or x is not adjacent to u; y is adjacent to u).
(iv) There exists $x \in S - \{u\}$ and $y \in V - S$ such that x is not adjacent to y and (x and y are either both adjacent to v or both are non-adjacent to v for all $v \in S - \{u\}$) and (x is adjacent to u; y is not adjacent to u or x is not adjacent to u; y is adjacent to u).
(v) If $v \in S - \{u\}$, then u and v are not adjacent and codes of u and v differ at some place corresponding to $w \in S - \{u, v\}$.

Proof : If any one of the conditions hold then $S - \{u\}$ is not a neighbourhood resolving set of G.
Conversely, Suppose S is minimal.
Let $u \in S$. Suppose (i),(ii),(iv) and (v) do not hold.
Suppose (iii) also does not hold. Then for every $x, y \in S - \{u\}$, x is adjacent to y or (if x is adjacent to v then y is not adjacent to v or vice versa for some $v \in S - \{u\}$) or (x is not adjacent to u or y is adjacent to u and x is adjacent to u or y is not adjacent to u).
That is if $x, y \in S - \{u\}$, either x is adjacent to y or if x is adjacent to v then y is not adjacent to v or vice versa for some $v \in S - \{u\}$) or (x and y are both adjacent to u or both not adjacent to u).

If x is adjacent to y then $nc_{S-\{u\}}(x) \neq nc_{S-\{u\}}(y)$.

If x is not adjacent to y and if x is adjacent to v then y is not adjacent to v for some $v \in S - \{u\}$, then x and y are either both adjacent to u or both non adjacent to u. Therefore x and y have the same code value at the place corresponding to x,y and u. Since S is a neighbourhood resolving set of G, there exists a place corresponding to some $v \in S - \{u, x, y\}$ such that x and y have different code values.

Therefore $nc_{S-\{u\}}(x) \neq nc_{S-\{u\}}(y)$. This is true for every $x, y \in S - \{u\}$. Therefore $S - \{u\}$ is a neighbourhood resolving set of G.

Suppose (ii),(iii),(iv) and (v) do not hold.

Suppose (i) also does not hold. Then for every $x, y \in V - S$, (if x is adjacent to v then y is not adjacent to v or vice versa for some $v \in S - \{u\}$) or (x is not adjacent to u or y is adjacent to u and x is adjacent to u or y is not adjacent to u).

If x is adjacent to v then y is not adjacent to v, for some $v \in S - \{u\}$.

Then x, y have different code value in the place corresponding to v. If the other condition holds then x and y are either both adjacent to u or both non-adjacent to u. Therefore x and y have the same code value in the place corresponding to u. Since S is a neighbourhood resolving set of G, there exist $w \in S$, $w \neq u$ such that x and y have different code value in the place of w. Therefore $S - \{u\}$ is a neighbourhood resolving set of G.

Suppose (i),(iii),(iv) and (v) do not hold.

Suppose (ii) also does not hold. Then for every $x \in V - S$, x is not adjacent to u or (there exists $v \in S - \{u\}$ such that x is not adjacent to v and u is not adjacent to v or viceversa). That is x and u have the same code in the place corresponding to u in S. Since S is a neighbourhood resolving set of G, there exists $w \in S$, $w \neq u$ such that x and u have different code value in the place of w. Therefore $nc_{S-\{u\}}(x) \neq nc_{S-\{u\}}(u)$.

If x is adjacent to u, then there exists $v \in S - \{u\}$ such that x is adjacent to v and u is not adjacent to v. Therefore x and u have the different code value in the place corresponding to v. That is $nc_{S-\{u\}}(x) \neq nc_{S-\{u\}}(u)$.

Therefore $S - \{u\}$ is a neighbourhood resolving set of G.

Suppose (i),(ii),(iii) and (v) do not hold.

Suppose (iv) also does not hold. Then for every $x \in S - \{u\}$ and $y \in V - S$, such that x is adjacent to y or (if x is adjacent to v then y is not adjacent to v or vice versa for some $v \in S - \{u\}$) or (x is not adjacent to u or y is adjacent to u and x is adjacent to u or y is not adjacent to u).

If x is adjacent to y, then $nc_{S-\{u\}}(x) \neq nc_{S-\{u\}}(y)$.

If x is not adjacent to y and if x is adjacent to v then y is not adjacent to v for some $v \in S - \{u\}$. Then x and y have different code value in the place corresponding to v. If the other condition holds, then x and y are either both adjacent to u or both non-adjacent to u. Therefore x and y have the same code value in the place corresponding to u.

Since S is a neighbourhood resolving set of G, there exists $w \in S$, $w \neq u$ such that x and y have different code value in the place of w.

Therefore $S - \{u\}$ is a neighbourhood resolving set of G.

Suppose (i),(ii),(iii) and (iv) do not hold.

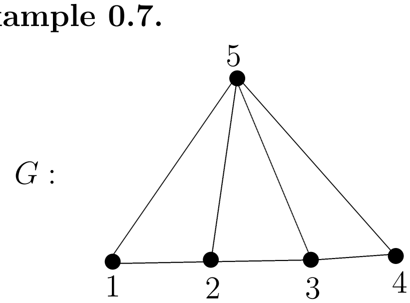
Suppose (v) also does not hold. Then for every $v \in S - \{u\}$, $nc_S(u)$ and $nc_S(v)$ differ only at the place corresponding to u which implies that u and v are adjacent and (u and v are either both adjacent to w or both non adjacent to w for every $w \in S - \{u, v\}$. Therefore $nc_{S-\{u\}}(u) \neq nc_{S-\{u\}}(v)$.

Therefore $S - \{u\}$ is a neighbourhood resolving set of G.

Hence S is minimal. ∎

Definition 0.6. *Let S be a subset of $V(G)$. Let $u \in S$. Then two vertices $x, y \in V$ are said to be privately resolved by u if $nc_S(x)$ and $nc_S(y)$ differ only at the place corresponding to u.*

Example 0.7.



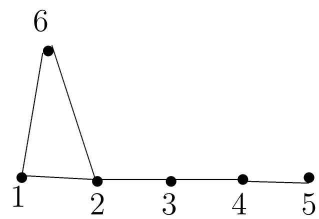
Let $S = \{1, 2, 3\}$. Now the vertices 2 and 4 are privately resolved by 1, since $nc_S(2) = (1, 0, 1)$ and $nc_S(4) = (0, 0, 1)$.
Similarly the vertices 3 and 4 are privately resolved by 2.

The theorem 0.5 can be restated as follows:

Theorem 0.8. *Let S be a neighbourhood resolving set of G. Then S is a minimal neighbourhood resolving set of G if and only if for every $u \in S$, there exist $x, y \in V$ which are privately resolved by u.*

Illustration 0.9.

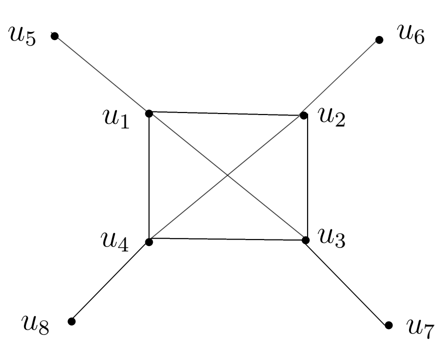
G :



Let $S = \{1, 2, 4\}$. Now $nc_S(1) = (0, 1, 0)$; $nc_S(2) = (1, 0, 0)$; $nc_S(3) = (0, 1, 1)$; $nc_S(4) = (0, 0, 0)$; $nc_S(5) = (0, 0, 1)$; $nc_S(6) = (1, 1, 0)$.
The vertices 1 and 3 are privately resolved by 4; 3 and 5 are privately resolved by 2 ; 1 and 6 are privately resolved by 1.
Therefore S is a minimal neighbourhood resolving set of G.

Observation 0.10. *A minimum neighbourhood resolving set of a graph G is a minimal neighbourhood resolving set, but the converse is not true.*
For: Consider K_4^+.

K_4^+ :



Let S $= \{u_1, u_2, u_5, u_6, u_8\}$.

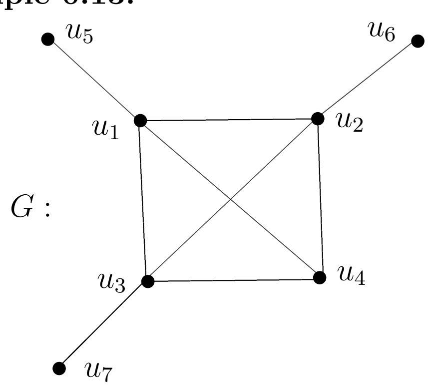
u_1 and u_6 are privately resolved by u_5; u_5 and u_8 are privately resolved by u_1.
u_6 and u_7 are privately resolved by u_2; u_2 and u_5 are privately resolved by u_6.
u_3 and u_4 are privately resolved by u_8.
Therefore S is a minimal neighbourhood resolving set of G, but not minimum.
$\{u_1, u_2, u_3\}$ is a minimum neighbourhood resolving set of G.(Given that $nr(G) = 3$).

Theorem 0.11. *Let S be an nr-set of G. Then for every $x \in S$, there exist at least two vertices in V which are privately resolved by u.*

Proof : Since S is an nr-set of G, S is a minimal nr-set of G. Hence the proof. ∎

Definition 0.12. *The least cardinality of a minimal neighbourhood resloving set of G is called the neighbourhood resolving number of G and is denoted by $nr(G)$. The maximum cardinality of a minimal neighbourhood resolving set of G is called the upper neighbourhood resolving number of G and is denoted by $NR(G)$.*
Clearly $nr(G) \leq NR(G)$. A neighbourhood resolving set S of G is called a minimum neighbourhood resolving set or nr-set if S is a neighbourhood resolving set with cardinality $nr(G)$.

Example 0.13.



Both $\{u_1, u_2, u_3\}$ and $\{u_1, u_2, u_5, u_6, u_7\}$ are minimal neighbourhood resolving sets of G. It can be easily seen that $nr(G) = 3$ and $NR(G) = 5$.

Observation 0.14. *If minimum neighbourhood resolving set exists in a given graph, then it need not be unique. For example, in C_6 with $V(C_6) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, both $\{v_2, v_3, v_4, v_5\}$ and $\{v_1, v_2, v_4, v_5\}$ are minimum neighbourhood resolving sets of C_6.*

3. Neighbourhood resolving number of Standard graphs

Result 0.15. *For a complete graph K_n, $nr(K_n) = n - 1$, $n \geq 2$.*

Proof : Let $V(K_n) = \{u_1, u_2, \dots, u_n\}$.

Let $T = \{u_1, u_2, \dots, u_{n-1}\}$. Now $|T| = n - 1$.

$nc_T(u_i)$, $1 \leq i \leq n$ will receive 0 in the place corresponding to the vertex u_i of T and 1 in all other places corresponding to the vertices of T and $nc_T(u_n)$ will receive the 1-code.

Therefore T is a neighbourhood resolving set of K_n.

Therefore $nr(K_n) \leq |T| = n - 1$.

Suppose T^1 is an nr-set of K_n. Suppose $|T^1| \leq n - 2$.

Let $|T^1| = t \leq n - 2$ and let without loss of generality $T^1 = \{u_1, u_2, \dots, u_t\}$, then u_{n-1} and u_n receive the 1-code with respect to T^1.

Therefore T^1 is not a neighbourhood resolving set of K_n, a contradiction.

Therefore $|T^1| \geq n - 1$. That is $nr(K_n) \geq n - 1$. Hence $nr(K_n) = n - 1$. ■

Lemma 0.16. *Let T be an nr-set of P_n. Then $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$, $n \geq 6$.*

Proof : Let S be a minimal neighbourhood resolving set of P_n.

Claim : Either S has at least four vertices from any six consecutive vertices of P_n or if S has three vertices from a set of six consecutive vertices of P_n, then S has five vertices from another set of six consecutive vertices.

If S has at least four vertices from any six consecutive vertices of P_n, then $|S| \geq \frac{2n}{3}$ and hence $nr(P_n) \geq \frac{2n}{3} \geq \lfloor \frac{2n}{3} \rfloor$.

Suppose $|S \cap \{u_{6i-5}, u_{6i-4}, u_{6i-3}, u_{6i-2}, u_{6i-1}, u_{6i}\}| = 3$, for some i, $1 \leq i \leq m$.

Case(i) : Let $T = \{u_{6i-5}, u_{6i-3}, u_{6i-1}\}$ or $\{u_{6i-3}, u_{6i-2}, u_{6i-1}\}$ or $\{u_{6i-5}, u_{6i-2}, u_{6i-1}\}$ or $\{u_{6i-5}, u_{6i-3}, u_{6i-2}\}$.

Then $nc_S(u_{6i-3}) = nc_S(u_{6i-1})$, a contradiction, since S is a neighbourhood resolving set of P_n.

Case(ii) : Let $T = \{u_{6i-4}, u_{6i-2}, u_{6i}\}$ or $\{u_{6i-4}, u_{6i-3}, u_{6i-2}\}$ or $\{u_{6i-4}, u_{6i-3}, u_{6i}\}$ or $\{u_{6i-3}, u_{6i-2}, u_{6i}\}$.

Then $nc_S(u_{6i-4}) = nc_S(u_{6i-2})$, a contradiction, since S is a neighbourhood resolving set of P_n.

Case(iii) : Let $T = \{u_{6i-5}, u_{6i-4}, u_{6i-3}\}$

Then u_{6i-1} receives 0-code with respect to S.

If $n = u_{6i}$ or $u_{6i+1} \notin S$, then $nc_S(u_{6i-1}) = nc_S(u_{6i})$, a contradiction.

Therefore $u_{6i+1} \in S$.

If $u_{6i+2} \notin S$, then $nc_S(u_{6i-1}) = nc_S(u_{6i+1})$, a contradiction.

Therefore $u_{6i+2} \in S$.

If $u_{6i+3} \notin S$, then $nc_S(u_{6i}) = nc_S(u_{6i+2})$, a contradiction.

Therefore $u_{6i+3} \in S$.

If $u_{6i+4} \notin S$, then $nc_S(u_{6i+1}) = nc_S(u_{6i+3})$, a contradiction.

Therefore $u_{6i+4} \in S$.

If either u_{6i+5} or $u_{6i+6} \in S$, then we are through.

Suppose $u_{6i+5} \notin S$ and $u_{6i+6} \notin S$.

Then if $u_{6i+7} \notin S$, then $nc_S(u_{6i-1}) = nc_S(u_{6i+6})$, a contradiciton.

Therefore $u_{6i+7} \in S$.

Arguing as before, we get $u_{6i+7}, u_{6i+8}, u_{6i+9}, u_{6i+10} \in S$.

If either u_{6i+11} or $u_{6i+12} \in S$, then we are through.

Suppose u_{6i+11} and $u_{6i+12} \notin S$.

Proceeding like this, suppose S contains exactly four elements from every six consecutive vertices from u_{6i+1} to u_{6m}. If $n = 6m$ and both u_{6m-1} and $u_{6m} \notin S$, then $nc_S(u_{6m}) = nc_S(u_{6i-1})$, a contradiction. Therefore either u_{6m-1} or u_{6m} belongs to S. Since $n = 6m$, $u_{6m-1} \in S$. Therefore S contains five vertices from the six consecutive vertices u_{6m-5} to u_{6m}. If $n = 6m+1$, then either $u_{6m}, u_{6m+1} \in S$ or $u_{6m-1}, u_{6m} \in S$. In both the cases S satisfies the claim. if $n = 6m+2$, then either $u_{6m-1}, u_{6m}, u_{6m+1} \in S$ or $u_{6m}, u_{6m+1}, u_{6m+2} \in S$. In both the cases S satisfies the claim. Suppose if $n = 6m+3$, then either $u_{6m-1}, u_{6m}, u_{6m+1}, u_{6m+2} \in S$ or $u_{6m}, u_{6m+1}, u_{6m+2}, u_{6m+3} \in S$. In both the cases S satisfies the claim. If $n = 6m+4$, then $u_{6m}, u_{6m+1}, u_{6m+3}, u_{6m+4} \in S$. Therefore S satisfies the claim. If $n = 6m+5$, then $u_{6m+1}, u_{6m+2}, u_{6m+3}, u_{6m+4} \in S$. Then S contains exactly foue elements from every six consecutive elements starting from u_{6i+1} to u_{6m} and four elements from u_{6m+1} to u_{6m+5}. In all cases, $|S| \geq \frac{2n}{3}$ and hence $nr(P_n) \geq \frac{2n}{3} \geq \lfloor \frac{2n}{3} \rfloor$.

Case (iv) : Let $T = \{u_{6i-2}, u_{6i-1}, u_{6i}\}$ or $\{u_{6i-5}, u_{6i-4}, u_{6i-2}\}$ or $\{u_{6i-3}, u_{6i-1}, u_{6i}\}$. This case is similar to Case (iii).

Case (v) : Let $T = \{u_{6i-5}, u_{6i-4}, u_{6i-1}\}$. If $n = u_{6i}$ or $u_{6i+1} \notin S$, then $nc_S(u_{6i-2}) = nc_S(u_{6i})$, a contradiction. Therefore $u_{6i+1} \in S$.

If $u_{6i+2} \notin S$, then then $nc_S(u_{6i-1}) = nc_S(u_{6i+1})$, a contradiction. Therefore $u_{6i+2} \in S$.

Suppose $u_{6i+3} \in S$. If $u_{6i+4} \notin S$, then $nc_S(u_{6i+1}) = nc_S(u_{6i+3})$, a contradiction. Therefore $u_{6i+4} \in S$.

Then proceeding as in case of Case(iii), we get that S satisfies the claim. Suppose $u_{6i+3} \notin S$. Then $u_{6i+4} \in S$ (otherwise $nc_S(u_{6i+1}) = nc_S(u_{6i+3})$, a contradiction). Similarly $u_{6i+5} \in S$. A similar argument shows that S satisfies the claim.

Case (vi) : Let $T = \{u_{6i-4}, u_{6i-1}, u_{6i}\}$ or $\{u_{6i-4}, u_{6i-2}, u_{6i-1}\}$ or $\{u_{6i-4}, u_{6i-3}, u_{6i-1}\}$. This case is similar to Case(v).

Case (vii) : Let $T = \{u_{6i-5}, u_{6i-4}, u_{6i}\}$. Then u_{6i-2} receives 0-code with respect to S. By similar argument, it is can be shown that $u_{6i+1}, u_{6i+2}, u_{6i+3} \in S$. If u_{6i+4} and u_{6i+5} and $u_{6i+6} \notin S$, then $nc_s(u_{6i+5}) = nc_S(u_{6i-2})$, a contradiction. Therefore either u_{6i+4} or u_{6i+5} or $u_{6i+6} \in S$. Then arguing as in the case of Case (iii) and Case(v), we get S satisfies the claim.

Case (viii) : Let $T = \{u_{6i-5}, u_{6i-1}, u_{6i}\}$ or $\{u_{6i-5}, u_{6i-3}, u_{6i}\}$ or $\{u_{6i-5}, u_{6i-2}, u_{6i}\}$. This case is similar to Case(vii).

Therefore in all case, if S has three vertices from a set of six consecutive vertices of P_n, then S has five vertices from another set of six consecutive vertices and hence $|S| \geq \frac{2n}{3}$. Therefore $nr(P_n) \geq \frac{2n}{3} \geq \lfloor \frac{2n}{3} \rfloor$. ∎

Result 0.17. *For a path P_n, $n \geq 6$, $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.*

Proof : Let $V(P_n) = \{u_1, u_2, \ldots, u_{6m-1}, u_{6m}\}$.

Case (i) : $n = 6m$.

Let $T = \{u_2, u_3, u_4, u_5, u_8, u_9, u_{10}, u_{11}, \ldots, u_{6m-4}, u_{6m-3},$

$u_{6m-2}, u_{6m-1}\}$.

Now $|T| = 4m = 4(\frac{n}{6}) = \frac{2n}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-5} has 1 in the $(4i-3)^{th}$ place and 0 elsewhere; u_{6i-4} has 1 in the $(4i-2)^{th}$ place and 0 elsewhere; u_{6i-3} has 1 in the $(4i-1)^{th}$ and $(4i-3)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i-2)^{th}$ and $(4i)^{th}$ places and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ place and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ place and 0 elsewhere.

Thus T resolves u_{6i-5} to u_{6i}, for every i, $1 \leq i \leq m$.

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n}{3}$.

Since $n = 6m$, $\frac{2n}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.

Case (ii) : $n = 6m + 1$.

Let $T = \{u_2, u_3, u_4, u_5, u_8, u_9, u_{10}, u_{11}, \ldots, u_{6m-4}, u_{6m-3},$
$u_{6m-2}, u_{6m-1}\}$.

Now $|T| = 4m = 4(\frac{n-1}{6}) = \frac{2n-2}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-5} has 1 in the $(4i-3)^{th}$ place and 0 elsewhere; u_{6i-4} has 1 in the $(4i-2)^{th}$ place and 0 elsewhere; u_{6i-3} has 1 in the $(4i-1)^{th}$ and $(4i-3)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i-2)^{th}$ and $(4i)^{th}$ places and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ place and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ place and 0 elsewhere and u_{6m+1} receives 0-code.

Thus T resolves u_{6i-5} to u_{6i}, for every i, $1 \leq i \leq m$ and T resolves u_i and u_{6m+1}, for every i, $1 \leq i \leq 6m$.

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n-2}{3}$.

Since $n = 6m + 1$, $\frac{2n-2}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.

Case (iii) : $n = 6m + 2$.

Let $T = \{u_2, u_3, u_4, u_5, u_8, u_9, u_{10}, u_{11}, \ldots, u_{6m-4}, u_{6m-3}, u_{6m-2},$

$u_{6m-1}, u_{6m+2}\}$.

Now $|T| = 4m + 1 = 4(\frac{n-2}{6}) + 1 = \frac{2n-1}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-5} has 1 in the $(4i-3)^{th}$ place and 0 elsewhere; u_{6i-4} has 1 in the $(4i-2)^{th}$ place and 0 elsewhere; u_{6i-3} has 1 in the $(4i-1)^{th}$ and $(4i-3)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i-2)^{th}$ and $(4i)^{th}$ places and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ place and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ place and 0 elsewhere; u_{6m+1} has 1 in the $(4m+1)^{th}$ place and 0 elsewhere and u_{6m+2} receives 0-code.

Thus T resolves u_{6i-5} to u_{6i}, for every i, $1 \leq i \leq m$; u_{6m+1} and u_{6m+2}; u_{6m+1} and u_i where $1 \leq i \leq 6m$ and u_{6m+2} and u_i where $1 \leq i \leq 6m$.

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n-1}{3}$.

Since $n = 6m + 2$, $\frac{2n-1}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.

Case (iv) : $n = 6m + 3$.

Let $T = \{u_1, u_2, u_4, u_5, u_6, u_7, u_{10}, u_{11}, u_{12}, u_{13}, \ldots, u_{6m-2},$

$u_{6m-1}, u_{6m}, u_{6m+1}\}$.

Now $|T| = 4m = 4(\frac{n-3}{6}) + 2 = \frac{2n}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-3} has 1 in the $(4i-2)^{th}$ and $(4i-1)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i)^{th}$ place and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ and $(4i+1)^{th}$ places and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ and $(4i+1)^{th}$ places and 0 elsewhere; u_{6i+1} has 1 in the $(4i+1)^{th}$ place and 0 elsewhere; u_{6i+2} has 1 in the $(4i+2)^{th}$ place and 0 elsewhere. u_1 has 1 in the second place and 0 elsewhere; u_2 receives 1 in the first place and 0 elsewhere and u_{6m+3} receives 0-code .

Thus T resolves u and v where $u, v \in V(P_n)$ and $u \neq v$

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n}{3}$.

Since $n = 6m + 3$, $\frac{2n}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.

Case (v) : $n = 6m + 4$.

Let $T = \{u_1, u_2, u_4, u_5, u_6, u_7, u_{10}, u_{11}, u_{12}, u_{13}, \ldots, u_{6m-2},$
$u_{6m-1}, u_{6m}, u_{6m+1}, u_{6m+4}\}$.

Now $|T| = 4m = 4(\frac{n-4}{6}) + 3 = \frac{2n+1}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-3} has 1 in the $(4i-2)^{th}$ and $(4i-1)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i)^{th}$ place and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ and $(4i+1)^{th}$ places and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ and $(4i+1)^{th}$ places and 0 elsewhere; u_{6i+1} has 1 in the $(4i+1)^{th}$ place and 0 elsewhere; u_{6i+2} has 1 in the $(4i+2)^{th}$ place and 0 elsewhere. u_1 has 1 in the second place and 0 elsewhere; u_2 receives 1 in the first place and 0 elsewhere; u_{6m+3} has 1 in the last place and 0 elsewhere and u_{6m+4} receives 0-code .

Thus T resolves u and v where $u, v \in V(P_n)$ and $u \neq v$

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n+1}{3}$.

Since $n = 6m + 4$, $\frac{2n+1}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$.

Case (vi) : $n = 6m + 5$.

Let $T = \{u_2, u_3, u_4, u_5, u_8, u_9, u_{10}, u_{11}, \ldots, u_{6m-4}, u_{6m-3}, u_{6m-2},$
$u_{6m-1}, u_{6m+1}, u_{6m+2}, u_{6m+4}\}$.

Now $|T| = 4m = 4(\frac{n-5}{6}) + 3 = \frac{2n-1}{3}$.

With respect to T, for every i, $1 \leq i \leq m$, u_{6i-5} has 1 in the $(4i-3)^{th}$ place and 0 elsewhere; u_{6i-4} has 1 in the $(4i-2)^{th}$ place and 0 elsewhere; u_{6i-3} has 1 in the $(4i-1)^{th}$ and $(4i)^{th}$ places and 0 elsewhere; u_{6i-2} has 1 in the $(4i-2)^{th}$ and $(4i-1)^{th}$ places and 0 elsewhere; u_{6i-1} has 1 in the $(4i-1)^{th}$ place and 0 elsewhere; u_{6i} has 1 in the $(4i)^{th}$ place and 0 elsewhere; u_{6m+1} has 1 in the $(4m+2)^{th}$ place and 0 elsewhere; u_{6m+2} has 1 in the $(4m+1)^{th}$ place and 0 elsewhere; u_{6m+3} has 1 in the $(4m+2)^{th}$ and $(4m+3)^{th}$ places and 0 elsewhere; u_{6m+4} receives 0-code and u_{6m+5} has 1 in the $(4m+3)^{th}$ place and 0 elsewhere. Thus T resolves u and v where $u, v \in V(P_n)$ and $u \neq v$

Therefore T is a neighbourhood resolving set of P_n.

Therefore $nr(P_n) \leq |T| = \frac{2n-1}{3}$.

Since $n = 6m$, $\frac{2n-1}{3} = \lfloor \frac{2n}{3} \rfloor$. Therefore $nr(P_n) \leq \lfloor \frac{2n}{3} \rfloor$ and by previous lemma $nr(P_n) \geq \lfloor \frac{2n}{3} \rfloor$. Hence $nr(P_n) = \lfloor \frac{2n}{3} \rfloor$. ■

Remark 0.18. *For a cycle C_n, $n \geq 6$, $nr(C_n) = \lfloor \frac{2n}{3} \rfloor$.*

4. nr-irrendundant sets in Graphs

Definition 0.19. *A subset S of $V(G)$ is called an nr-irredundant set of G if for every $u \in S$, there exist $x, y \in V$ which are privately resolved by u.*

Observation 0.20. *Any minimal neighbourhood resolving set of G is an nr-irredundant set of G. Converse is not true.*

Example 0.21. *Consider P_6 with $V(P_6) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.*

Let $S = \{u_1, u_2, u_3\}$.

Now S is a neighbourhood irredundant set of P_6.

For : $nc_S(u_1) = (0,1,0)$; $nc_S(u_2) = (1,0,1)$; $nc_S(u_3) = (0,1,0)$;

$nc_S(u_4) = (0,0,1)$ and $nc_S(u_5) = (0,0,0)$.

u_2 and u_4 are privately resolved by u_1; u_3 and u_5 privately resolved by u_2 and u_4 and u_5 privately resolved by u_3.

Let $S_1 = S \cup \{u_4\}$. S_1 does not privately resolve any pair of vertices of $V(P_6)$ with respect to S_1 and let $S_2 = S \cup \{u_5\}$. S_2 does not privately resolve any pair of vertices of $V(P_6)$ with respect to S_2. Hence S is a maximal neighbourhood irredundant set of P_6.

Since $nc_S(u_1) = nc_S(u_3)$, S is not a neighbourhood resolving set of G.

Theorem 0.22. *If S is a neighbourhood resolving irredundant set of G, then any subset of S is also a neighbourhood resolving irredundant set of G.*

Proof : Let S be a neighbourhood resolving irredundant set of G.

Let T be any subset of S. Since S is a neighbourhood resolving irredundant set of G, for every $u \in S$, there exist $x, y \in S$ which are privately resolved by u. Since $y \notin S$, $y \notin T$. Therefore for every $u \in T$ there exist $x, y \in T$ which are privately resolved by u. Therefore T is a neighbourhood resolving irredundant set of G. ■

Observation 0.23. *Neighbourhood resolving irredundance is a hereditary property.*

Observation 0.24. *Let S be an nr-irredundant set of G. Then S is maximal if and only if S is 1- maximal.*

Theorem 0.25. *Every minimal neighbourhood resolving set of G is a maximal neighbourhood resolving irredundant set of G.*

Proof : Let S be a minimal neighbourhood resolving set of G. Then S is a neighbourhood resolving irredundant set of G.
Suppose S is not maximal. Then there exists $u \in V - S$ such that $S \cup \{u\}$ is a neighbourhood resolving irredundant set of G.
Therefore there exist $x, y \in V$ such that x and y are privately resolved by u. Therefore either x and y or y and u have the same code with respect to S, a contradiction, since S is a neighbourhood resolving set of G.
Therefore S is a maximal neighbourhood resolving irredundant set of G. ■

Definition 0.26. *The minimum cardinality of a maximal neighbourhood resolving irredundant set of G is called the neighbourhood resolving irredundance number of G and is denoted by $ir_{nr}(G)$. The maximum cardinality is called the upper neighbourhood resolving irrundance number of G and is denoted by $IR_{nr}(G)$.*

Observation 0.27. *For any graph G, $ir_{nr}(G) \leq nr(G) \leq NR(G) \leq IR_{nr}(G)$.*

Example 0.28. *Consider P_6 with $V(P_6) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.*
Let $S = \{u_1, u_2, u_5\}$.
Now S is a neighbourhood resolving irredundant set of P_6,

since u_1 *and* u_5 *are privately resolved by* u_2*;* u_2 *and* u_5 *are privately resolved by* u_1 *and* u_5 *and* u_6 *are privately resolved by* u_5*.*
Since $S \cup \{u\}$ *where* $u = u_3$ *or* u_4 *or* u_6*, is not a neighbourhood resolving irredundant set of* G*, we have* $ir_{nr}(P_6) = 3$*.*
Clearly $nr(P_6) = 4$*.*

Conclusion 0.29. *Studies of perfect graphs with respect to any two of the above parameters in the inequality chain is under consideration.*

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