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SOLUTION FOR PROJECTILE MOTION IN TWO DIMENSIONS WITH NONLINEAR AIR RESISTANCE USING LAPLACE DECOMPOSITION METHOD

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Abstract. Laplace decomposition method (LDM) is utilized to obtain an approximate solution of two-dimensional projectile motion with linear air resistance as well as to derive a formalism to obtain the solutions for any order of nonlinearity in the air resistance. The projectile trajectory was obtained using LDM method in three cases: without air resistance, with linear air resistance, and with quadratic air resistance. The solutions were used to illustrate the effect of the order of non-linearity on the basic parameters related to the motion, like Ranges, time of flight, maximum high and some other parameters. The available literature does not provide an exact solution to this motion when higher nonlinearities are involved. Nevertheless, the results show that such method is effective and powerful in getting approximate solutions for problems involving nonlinear behavior.

Keywords: projectile motion; Adomian decomposition method; Laplace decomposition method; high order air resistance.

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1. Introduction

The problem of the projectile motion is very well known for. Physics students and researchers are familiar with the solution of this problem when the air resistance is neglected. In practice, the air resistance is not negligible, and it seems necessary to develop solutions for motion where air resistance is present. In this paper, the projectile motion was solved using LDM method in three cases: without air resistance, with linear air resistance, and with quadratic air resistance. Then the results of some relations were plotted to compare the effects of the nonlinearity order of resistance on the path of the motion [1, 2, 3, 4, 5, 6, 7]. Yabushita [4] considered the two dimensional problem in which the air resistance is proportional to the square of the velocity. It has been considered that the equations of motion in this case are unsolvable for a general projectile angle. Warburton [5] studied three regimes of approximation: low-angle trajectory where the horizontal velocity is assumed to be much larger than the vertical velocity; high-angle trajectory where the vertical velocity is assumed much larger than the horizontal velocity; and split-angle trajectory. Parker [6] derived a simple approximate solution to these equations for both short and long times and a numerical example was used to compare these solutions with accurate results obtained by numerical integration from an exact but implicit approach. The Adomian decomposition method, introduced by Adomian in 1980's [8], has been powerful method to find the approximate solutions for a wide class of ordinary differential equations. The LDM is a successive technique developed with the help of the Adomian decomposition [9, 10]. It is used to solve nonlinear ordinary and partial differential equations. The method is very well suited in physical problems since it can solve nonlinear problems without linearization, perturbation or discretization methods. On the other hand it requires less computation work than traditional approaches.

Many papers used this method to solve various nonlinear partial differential equations. Khuri [11] used this method for the approximate solution of a class of nonlinear ordinary differential equations. Elgazery [12] exploited this method to solve Falkner-Skan equation. Handibag and Karande [13] applied this method for the solution of the linear and nonlinear heat equation.

The LDM was employed in to get approximate analytical solutions of the linear and the nonlinear fractional diffusion-wave equations, nonlinear Schrödinger Equation and Klien Gordon equation [14, 15, 17].

In this paper, the aim is to obtain an approximate solution for the projectile motion equations and to derive a general formula for the solution of any order of nonlinearity for air resistance term using an easy but powerful technique. The solution will be as a form of summation, which yields an exact solution for infinite number of terms.

2. Equations of Motion

A projected object of mass (m) will be considered in this paper. Suppose that the force of gravity affects the object together with the force of air resistance, see Figure 1, which is proportional to the speed of the projected mass and is directed opposite the velocity vector. The resistance force will be in the form (kv^{γ}), where k is the proportionality factor, v is the velocity and γ is the power factor of nonlinearity.

$$m\dot{v} = -mg - kv^{\gamma}, \ \gamma = 0, 1, 2, \ldots \tag{1}$$

According to many researches, the proportionality factor (k) depends on the medium density, the dimension of the projected mass, the cross sectional area of the projectile [16]. For $\gamma = 0$, the projectile motion without air resistance is solved by many researches [7]. In this paper, the effect of nonlinearity on the final trajectory of the object will be handled using LDM formalism. It is clear from Figure 1 that the path of projectile with air resistance is not parabola and it depends on the value of γ to determine the behavior of the trajectory. The LDM will be discussed in next section, we show how it can be used to obtain the solution for Equation 1 with different nonlinear coefficient (γ).

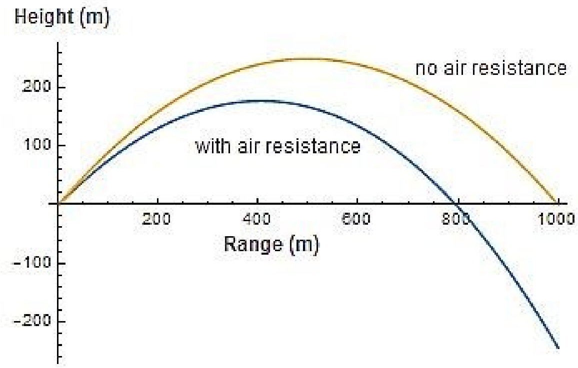

FIGURE 1. The flight of a projectile with and without air resistance, $\gamma = 1,\ v_0 = 100\ m/s, k = 0.1,$ and $m = 10g$

3. LAPLACE DECOMPOSITION METHOD

Suppose that we are interested in solving the following differential equation:

$$F(u(t)) = h(t),\ u(0) = u_0, \tag{2}$$

where F is linear or nonlinear operator, u is the unknown function and u_0 is the initial condition. The main idea of LDM is to decompose the operator F into linear and nonlinear operators in the form:

$$F = L + R + N,$$

where $L+R$ is linear, N is nonlinear and L is invertible with L^{-1} as its inverse. Using this decomposition, Equation 2 becomes

$$Lu(t) + Ru(t) + Nu(t) = h(t), \tag{3}$$

where operator L is defined as $L = \frac{d(.)}{dt}$ and $L^{-1} = \int_0^t (.)ds$. Applying Laplace transform to both sides of Equation 3, we get

$$\mathscr{L}\{Lu(t)\} + \mathscr{L}\{Ru(t)\} + \mathscr{L}\{Nu(t)\} = \mathscr{L}\{h(t)\}, \tag{4}$$

this yields,

$$s\mathscr{L}\{u(t)\} = u_0 + \mathscr{L}\{h(t)\} - \mathscr{L}\{Nu(t)\} - \mathscr{L}\{Ru(t)\}, \tag{5}$$

and thus,

$$\mathscr{L}\{u(t)\} = \frac{u_0}{s} + \frac{1}{s}\mathscr{L}\{h(t)\} - \frac{1}{s}\mathscr{L}\{Nu(t)\} - \mathscr{L}\{Ru(t)\}. \tag{6}$$

We seek a solution of Equation 2 in the form:

$$u(t) = \sum_{n=0}^{\infty} u_n(t). \tag{7}$$

The key of this technique is to decompose the nonlinear term Nu in Equation 3 into a particular series of polynomials:

$$Nu(t) = \sum_{n=0}^{\infty} A_n \tag{8}$$

where $A_n = A_n(u_0, u_1, u_2, \ldots, u_n)$ are called Adomian polynomials and defined as:

$$A_n(u_0, u_1, u_2, \ldots, u_n) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \ \lambda = 0, 1, 2, \ldots \tag{9}$$

The first five Adomian polynomials for the variable $Nu = f(u)$ are given by:

$$\begin{aligned}
&A_0 = f(u_0),\ A_1 = u_1 f'(u_0)\\
&A_2 = u_2 f'(u_0) + \frac{1}{2!}u_1^2 f''(u_0),\ A_3 = u_3 f'(u_0) + u_1 u_2 f''(u_0) + \frac{1}{3!}u_1^3 f^{(3)}(u_0)\\
&A_4 = u_4 f'(u_0) + \left(u_1 u_3 + \frac{1}{2!}u_2^2\right) f''(u_0) + \frac{1}{2!}u_1^2 u_2 f^{(3)}(u_0) + \frac{1}{4!}u_1^4 f^{(4)}(u_0)
\end{aligned}$$

In the next section, we will illustrate how we can use this methodology to solve the projectile motion Equation 1 and obtain a general formula for the velocity of the projected mass at any point in the trajectory for any value of γ.

4. METHOD OF SOLUTION

One of the easiest ways to discuss the projectile motion is to analyze the motion in each direction separately. In other words, we will use one set of equations to describe the vertical motion and another set of equations to describe the horizontal one. In projectile motion, the only acceleration will be in z-direction and therefore, the horizontal velocities components will be constants. If air resistance is present, which directly depends on the speed, forces on the projectile motion, then Newton's equation of motion is:

$$m\frac{d\vec{v}}{dt} = -mg\hat{z} - kv^{\gamma}\hat{v}, \tag{10}$$

with initial conditions $v(0) = u_0$.

Rewrite Equation 10 for the linear case ($\gamma = 1$), we get:

$$\frac{d\vec{v}}{dt} = -g\hat{z} - \frac{k}{m}v\hat{v} \tag{11}$$

Match Equation 3 with Equation 11, we get

$$L = \frac{d}{dt},\ R = 0,\ h = -g,\ Nu = -\frac{k}{m}v(t).$$

Apply Laplace transform to both sides of Equation 11, we have

$$\mathscr{L}\{\dot{v}(t)\} = \mathscr{L}\{-g\} - \frac{k}{m}\mathscr{L}\{v(t)\}, \tag{12}$$

and then,

$$sv(s) - v(0) = -\frac{g}{s} - \frac{k}{m}\mathscr{L}\{v(t)\},$$

so,

$$v(s) = \frac{u_0}{s} - \frac{g}{s^2} - \frac{k}{ms}\mathscr{L}\{v(t)\}. \tag{13}$$

Define

$$v(s) = \sum_{n=0}^{\infty} v_n(s), \tag{14}$$

where

$$\sum_{n=0}^{\infty} v_n(s) = \frac{u_0}{s} - \frac{g}{s^2} - \frac{k}{ms}\mathscr{L}\{v(t)\}.$$

Define

$$v(t) = \sum_{n=0}^{\infty} A_n(u), \tag{15}$$

therefore,

$$\sum_{n=0}^{\infty} v_n(s) = \frac{u_0}{s} - \frac{g}{s^2} - \frac{k}{ms}\mathscr{L}\left\{\sum_{n=0}^{\infty} A_n(u)\right\}. \tag{16}$$

Now, compute Adomian polynomials A_n. $A_0 = v_0,\ A_1 = v_1,\ A_2 = v_2,\ A_3 = v_3, \dots$.

If we match both sides of Equation 16, we get

$$v_0(s) = \frac{u_0}{s} - \frac{g}{s^2}, \tag{17}$$

hence,

$$v_0(t) = \mathscr{L}^{-1}\left\{\frac{u_0}{s} - \frac{g}{s^2}\right\} = u_0 - gt. \tag{18}$$

$$v_1(s) = -\frac{k}{ms}\mathscr{L}\{A_0\} = -\frac{k}{ms}\mathscr{L}\{v_0(t)\}, \tag{19}$$

and so,

$$v_1(t) = -\frac{k}{m}\left(u_0 t - \frac{1}{2}gt^2\right), \tag{20}$$

$$v_2(t) = \frac{k^2}{m^2}\left(\frac{u_0 t^2}{2!} - \frac{gt^3}{3!}\right). \tag{21}$$

Similarly, we can find the rest of the terms using:

$$v_n(t) = \frac{(-1)^n k^n}{m^n}\left(\frac{u_0 t^n}{n!} - \frac{gt^{n+1}}{(n+1)!}\right). \tag{22}$$

To obtain the exact solution we take the summation for all terms, this yields

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} (a_1 u_0 t^n - a_2 g t^{n+1}). \tag{23}$$

Which is the result of applying Laplace Transform, where a_1 and a_2 are constants. The horizontal motion will have same steps except that $h = 0$ in Equation 3, then the solution for the horizontal motion will be

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} (a_1 u_0 t^n) \tag{24}$$

Now we discuss the projectile forced by an air resistance ($\gamma = 2$), which depends on the speed, then the Newton's equation is:

$$\dot{v} = -g - \frac{k}{m} v^2, \; v(0) = u_0. \tag{25}$$

After applying Laplace transform to both sides and then taking the inverse Laplace, we get

$$v(s) = \frac{u_0}{s} - \frac{s}{s^2} - \frac{k}{ms} \mathscr{L}\{v^2\}. \tag{26}$$

Use the definition in Equation 14 and $v^2 = \sum_{n=0}^{\infty} A_n(u)$, we get

$$\sum_{n=0}^{\infty} v_n(s) = \frac{u_0}{s} - \frac{s}{s^2} - \frac{k}{ms} \mathscr{L}\left\{ \sum_{n=0}^{\infty} A_n(u) \right\}. \tag{27}$$

The related Adomian polynomials are :

$$A_0 = v_0^2, A_1 = 2v_0 v_1, A_2 = v_1^2 + 2v_0 v_2, A_3 = 2v_1 v_2 + 2v_0 v_3, \ldots,$$

then,

$$v_0(t) = \mathscr{L}^{-1}\left\{ \frac{u_0}{s} - \frac{g}{s^2} \right\} = u_0 - gt, \tag{28}$$

$$\mathscr{L}\{v_1(t)\} = -\frac{k}{ms} \mathscr{L}\{v_0^2\}, \tag{29}$$

and hence,

$$v_1(t) = -\frac{k}{m}\left(u_0^2 t - g u_0 t^2 + \frac{g^2}{3} t^3\right), \tag{30}$$

similarly,

$$v_2(t) = \frac{k^2}{m^2}\left(u_0^3 t^2 - \frac{8 g u_0^2 t^3}{3!} + \frac{16 u_0 g^2 t^4}{4!} - \frac{16 g^3 t^5}{5!}\right), \tag{31}$$

$$v_3(t) = -\frac{k^3}{m^3}\left(u_0^4 t^3 - \frac{5}{3} u_0^3 g t^4 + \frac{17}{15} u_0^2 g^2 t^5 - \frac{17}{45} u_0 g^3 t^6 + \frac{17}{315} g^4 t^7\right), \tag{32}$$

$$v_4(t) = \frac{k^4}{m^4}\left(u_0^5 t^4 - 2 g u_0^4 t^5 + \frac{77}{45} u_0^3 g^2 t^6 - \frac{248}{315} u_0^2 g^3 t^7 + \frac{62}{315} u_0 g^4 t^8 - \frac{62}{2835} g^5 t^9\right), \tag{33}$$

and

$$v_n(t) = \frac{(-1)^n k^n}{m^n}\left(u_0^{n+1} g^0 t^n - u_0^n g^1 t^{n+1} + u_0^{n-1} g^2 t^{n+2} - \cdots + u_0^0 g^{n+1} t^{2n+1}\right). \tag{34}$$

The solution of Equation 25 for vertical motion is

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n}\left(b_1 u_0^{n+1} g^0 t^n - b_2 u_0^n g^1 t^{n+1} + b_3 u_0^{n-1} g^2 t^{n+2} - \cdots + b_\alpha u_0^0 g^{n+1} t^{2n+1}\right) \tag{35}$$

In addition, the solution for the same equation for horizontal motion is

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n}\left(b_1 u_0^{n+1} t^n\right), \tag{36}$$

where $b_1, \ldots, b_\alpha$ are all constants.

Using successive technique we can obtain solutions for higher order of γ and hence we generalize the solution in a general formula. As expected from the solutions and Figure 1 we can see that the higher order of nonlinearity is the shorter vertical height and horizontal range. To discuss the effect of the order of γ on the time of flight we will obtain more solutions and compare their plots. For cubic air resistance ($\gamma = 3$), the nonlinear equations in terms of Adomian polynomials is

$$\sum_{n=0}^{\infty} v_n(s) = \frac{u_0}{s} - \frac{g}{s^2} - \frac{k}{ms}\mathscr{L}\left\{\sum_{n=0}^{\infty} A_n(u)\right\}, \tag{37}$$

where $A_0 = v_0^3, A_1 = 3v_1v_0^2, A_2 = 3v_2v_0^2 + 3v_0v_1^2, A_3 = 3v_3v_0^2 + 6v_0v_1v_2 + \frac{1}{2!}v_1^3, \ldots,$.

In addition, the terms of the solution for vertical motion can be easily computed as follows:

$$v_0(t) = u_0 - gt, \tag{38}$$

$$v_1(t) = -\frac{k}{m}\left(u_0^3t - \frac{3}{2}gu_0^2t^2 + g^2u_0t^3 - \frac{1}{4}g^3t^4\right), \tag{39}$$

$$v_2(t) = \frac{k^2}{m^2}\left(\frac{3u_0^5t^2}{2} - \frac{gu_0^4t^3}{3} + \frac{15u_0^3g^2t^4}{4} - \frac{9u_0^2g^3t^5}{4} + \frac{3u_0g^4t^6}{4} - \frac{45g^5t^7}{2}\right), \tag{40}$$

$$\begin{aligned} v_3(t) = -\frac{k^3}{m^3}\Bigg(&\frac{4u_0^7t^3}{3} - \frac{7gu_0^6t^4}{2} + \frac{71u_0^5g^2t^5}{10} - \frac{107u_0^4g^3t^6}{12} \\ &+ \frac{171u_0^3g^4t^7}{28} - \frac{171g^5u_0^2t^8}{16} + \frac{247g^6u_0t^9}{16} - \frac{1083g^7t^{10}}{160}\Bigg), \end{aligned} \tag{41}$$

$$v_n(t) = \frac{(-1)^nk^n}{m^n}\left(u_0^{2n+1}g^0t^n - u_0^{2n}g^1t^{n+1} + u_0^{2n-1}g^2t^{n+2} - \cdots + u_0^0g^{2n+1}t^{3n+1}\right), \tag{42}$$

and,

(43)
$$v(t) = \sum_{n=0}^{\infty}\frac{(-1)^nk^n}{m^n}\left(c_1u_0^{2n+1}g^0t^n - c_2u_0^{2n}g^1t^{n+1} + c_3u_0^{2n-1}g^2t^{n+2} - \cdots + c_\alpha u_0^0g^{2n+1}t^{3n+1}\right),$$

and the horizontal motion

$$v(t) = \sum_{n=0}^{\infty}\frac{(-1)^nk^n}{m^n}\left(c_1u_0^{2n+1}t^n\right). \tag{44}$$

Following the same steps above we can obtain the Adomian polynomials for each value of γ related to the motion equation. For $\gamma = 4$, the Adomian polynomials are:

$$A_0 = v_0^4, A_1 = 4v_1v_0^3, A_2 = 4v_2v_0^3 + 6v_0^2v_1^2, A_3 = 4v_3v_0^3 + 12v_0^2v_1v_2 + 4v_0v_1^3, \dots$$

and the he solution can be obtained using the following formula:

(45)

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} \left(d_1 u_0^{3n+1} g^0 t^n - d_2 u_0^{3n} g^1 t^{n+1} + d_3 u_0^{3n-1} g^2 t^{n+2} - \cdots + d_\alpha u_0^0 g^{3n+1} t^{4n+1} \right).$$

Since the gravity force does not effect on the horizontal components, the gravity constant does not appear in the terms of the horizontal motion solution.

$$v(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} \left(d_1 u_0^{3n+1} t^n \right) \tag{46}$$

Now, the ability to get a general solution for any order of nonlinearity for the projectile motion equation shows how much the Laplace decomposition method is effective.

From Equations 22, 25, 42, and 44 for order γ of velocity, we can generalize the solution as:

(47)

$$v_\gamma(t) =$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} \left(\mu_1 u_0^{(\gamma-1)n+1} g^0 t^n - \mu_2 u_0^{(\gamma-1)n} g^1 t^{n+1} + \mu_3 u_0^{(\gamma-1)n-1} g^2 t^{n+2} - \cdots + \mu_\alpha u_0^0 g^{(\gamma-1)n+1} t^{\gamma n+1} \right)$$

where $\mu_1, \mu_2, \mu_3, \dots$ are related to γ.

Similarly, for the horizontal motion,

$$v_\gamma(t) = \sum_{n=0}^{\infty} \frac{(-1)^n k^n}{m^n} \left(\mu_1 u_0^{(\gamma-1)n+1} t^n \right). \tag{48}$$

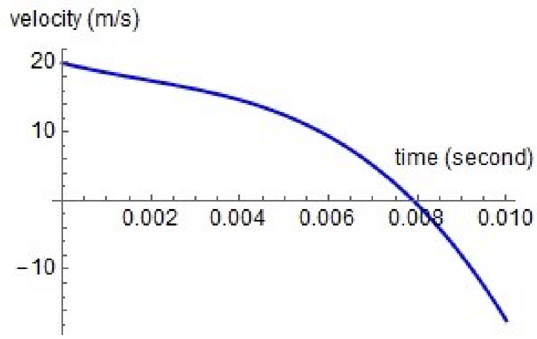

FIGURE 2. z-component of the velocity for mass $m = 10g$ with $k = 0.1$ and $v_0 = 20m/s$ when air resistance is proportional to v^3.

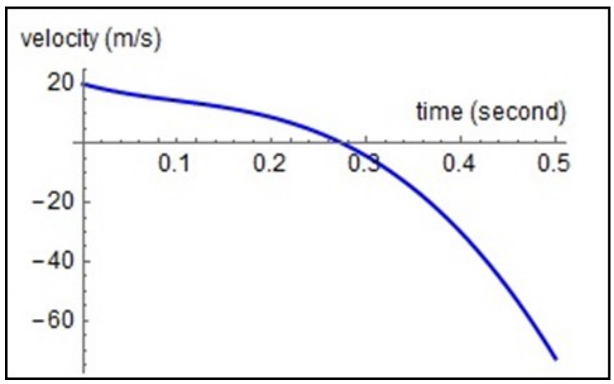

FIGURE 3. z-component of the velocity for previous projected mass when air resistance is proportional to v^4 .

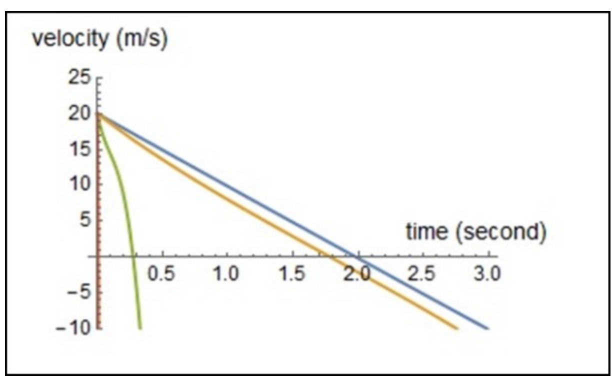

FIGURE 4. z-component of the velocity of $m = 10g$, $k = 0.1$ and initial speed $v_0 = 40m/s, \theta = 30$ for the four values of γ .

The first few terms of the solutions plotted to represent the behavior of the projectile. We assume that for all cases the object in launched with initial velocity equals to 20 $\hat{v}$ m/s with $k = 0.1s^{-1}$. Starts from $t = 0$, then the velocity in Equation 22 represented in Figure 4 where the maximum height occurs when $v = 0$. The maximum height can be evaluated by integrating Equation 22 with respect to time. It can be seen in the figures that the time needed to reach the maximum height is about 2 second. Since the power of force increases with increasing in, the air resistance impedes the motion of the object leading to decreasing in the maximum height that the mass object reaches and therefore the time needed to reach the maximum height will be decreased. Figures 4, 5, 6 show the effect of the power of velocity on the parameters of the motion. On the other hand, two values for γ were considered to plot the velocity of the projectile for several values of k. see Figures 5 and 6.

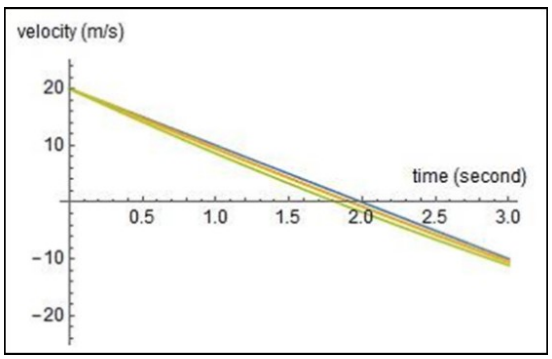

FIGURE 5. z-component of the velocity for $10g$ with $v_0 = 20m/s$ and $\gamma = 1$ for several values of k

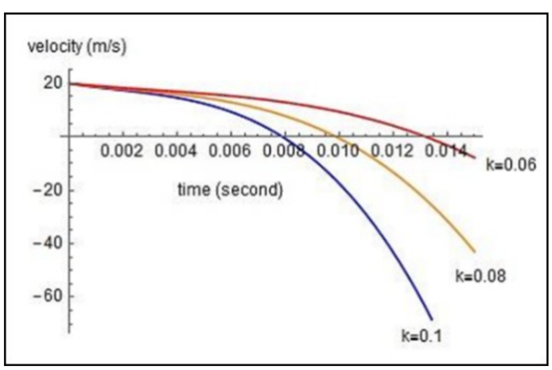

FIGURE 6. z-component of the velocity for $10g$ projected mass with $v_0 = 20m/s$ and $\gamma = 4$ for several values of k.

In fact, the effect of v^3, v^4 appears when the projectile mass is small such as; the bullet. Several values for m were substituted in the motion of equations in the two cases (v^3, v^4). Then, the first few terms of the solutions were plotted (See Figure 7 and 8 which represent the vertical velocity as a function of time for ($m = 8, 10, 12, 15g$). The Figures show how increasing the object mass lengthen the time for it to reach maximum height for the two cases (v^3, v^4).

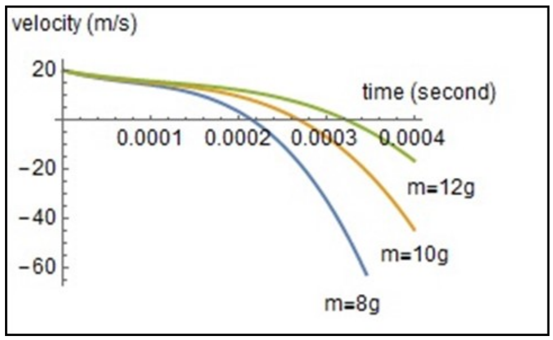

FIGURE 7. z-component of the velocity for several projected masses with $v_0 = 20m/s, \gamma = 3$ and $k = 0.1$.

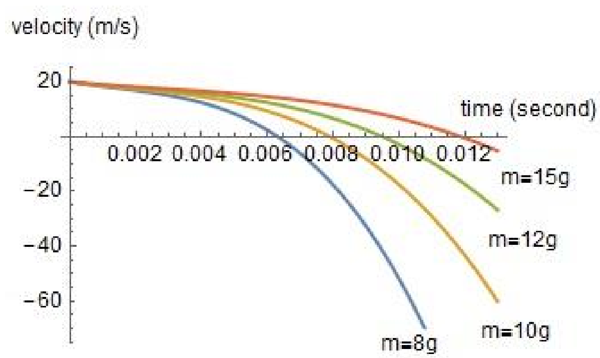

FIGURE 8. z-component of the velocity for several projected masses with $v_0 = 20m/s, \gamma = 4$ and $k = 0.1$

For comparison, we plot the solution of the projectile with quadratic air resistance solved in two methods. The method used in this paper and common methods used in others papers. Figure 9 shows how our technique leads to the same solution obtained by common methods to solve nonlinear differential equations. This comparison shows the effectiveness of Laplace decomposition method especially if we consider that the exact solution is the summation of infinite terms. The small errors, which may appear in the higher order, refers to neglecting the rest of the terms of the summation.

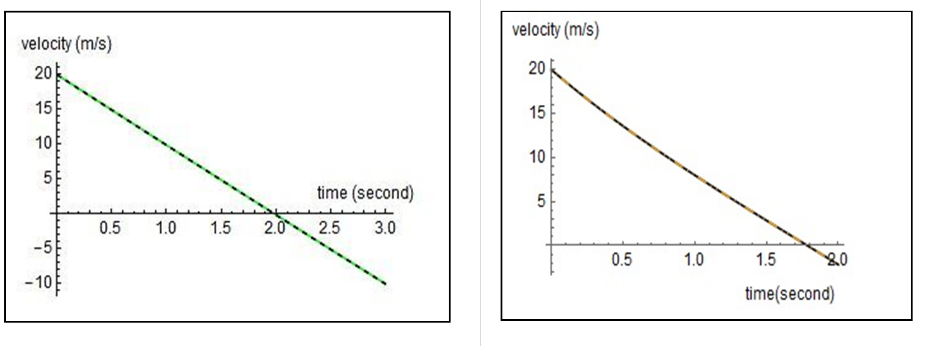

FIGURE 9. z-component of the velocity for projected $m = 10g, k = 0.1, v_0 = 20m/s$with (9.a) linear air resistance and (9.b) quadratic air resistance . The solid line represents the solution that obtained by LDM and the dashed one represent the solution that obtained by common method depending on integration with respect to time.

5. CONCLUSION

In summary, we have solved and discussed the projectile motion with nonlinear air resistance. The solution was obtained using LDM method, the method shows the simplicity to obtain the solutions successfully. The order of nonlinearity affected the general solution of the trajectory. We plotted the projectile velocity as a function of time and explained the efficiency of the used technique. We compared the solutions for quadratic air resistance with the solution obtained in the literature. The scheme described in this paper can be further employed to solve nonlinear problems in science.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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