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A LIFESAVING TOOL FOR COVID PATIENTS USING BPF SOFT TOPOLOGY: DECISION MAKING IN OXYGEN CONCENTRATOR

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Abstract: In this paper, we introduced the notion of bipolar Pythagorean fuzzy soft topology and proved some of its basic properties. We defined bipolar Pythagorean fuzzy regular generalized soft sets. We presented some basic operations on bipolar Pythagorean fuzzy soft topology. We gave an application of bipolar Pythagorean fuzzy soft open sets which are bipolar Pythagorean fuzzy regular generalized closed sets taken from bipolar Pythagorean fuzzy topology into a decision-making problem.

Keywords: bipolar Pythagorean fuzzy soft set; bipolar Pythagorean fuzzy soft topology; bipolar Pythagorean fuzzy soft subsets; bipolar Pythagorean fuzzy soft open sets; bipolar Pythagorean fuzzy soft regular generalized closed sets; bipolar Pythagorean fuzzy topological spaces.

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1. INTRODUCTION

The theory of fuzzy sets, intuitionistic fuzzy sets, theory of bipolar fuzzy sets, bipolar

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Pythagorean fuzzy sets, and theory of vague sets have been proposed for dealing with uncertainties in the field of decision making, medical diagnosis and in all areas of engineering in an efficient way to get the best solution. Fuzzy sets were introduced by Zadeh [36]. Fuzzy topology was introduced by Chang [8] by using fuzzy set. In Molodtsov [19] proposed the idea of soft sets in order to deal with unpredictability. Maji et al. [23] discussed some important operations of soft sets and its implementation into the decision-making. Ali et al. [3] proposed some modified operations of soft set theory. Ça gman et al. [7] Shabir and Naz [34] independently brought out the concept of soft topology. Kharal and Ahmad [16] presented the idea of mappings of soft classes. In 1998, Zhang [39] introduced Bipolar fuzzy sets. In a bipolar fuzzy set, positive information represents what is guaranteed to be possible, while negative information represents what is impossible or forbidden, or surely false. Bipolar valued fuzzy set by Lee [17] introduced another generalization of fuzzy sets in which membership degree is enlarge from interval [0,1] to [-1,1]. In bipolar fuzzy sets, membership degree 0 means that elements are irrelevant to corresponding property, membership degree belong to (0,1] indicate that somewhat elements are satisfy the corresponding property and membership degree belong to [-1,0) indicate that somewhat elements are satisfy implicit counter property. It is well known that soft set theory as a tool for applications in both theoretical areas as well as a technique for laying the foundations. Nowadays, we come across many problems in taking decisions in uncertainty situations in the field of medical diagnosis, modeling, artificial intelligence, machine learning, share marketing and computing as seen in the following studies like decision making.

In this paper, we introduced a combination of Bipolar Pythagorean fuzzy soft set with topology. We introduced the notion of bipolar Pythagorean fuzzy soft topology and defined some basic properties. We presented basic operations on bipolar Pythagorean fuzzy soft topology. We defined Bipolar Pythagorean Fuzzy Soft Regular Generalized closed sets. We gave an application of Bipolar Pythagorean Fuzzy Soft Regular Generalized closed sets taken from Bipolar Pythagorean fuzzy topology into a decision-making problem and a general algorithm is given to solve this decision-making problem, particularly in choosing a best portable oxygen concentrator.

2. PRELIMINARIES

Definition 2.1: Let X be the non-empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

Definition 2.2: Let X be the non-empty universe of discourse. An Intuitionistic fuzzy set (IFS) A in X is given by A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in$ Xto the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

The degree of indeterminacy $I_A = 1 - (\mu_A(x) - \nu_A(x))$ for each $x \in X$.

Definition 2.3: Let X be the non-empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P = $\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non-membership of each element $x \in X$ to the set P, respectively, and $0 \leq \mu_P^2(x) + \nu_P^2(x) \leq 1$ for each $x \in X$.

The degree of indeterminacy $I_P = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

Definition 2.4: A bipolar fuzzy set A over U is an object having the form $A = \{(x, \mu_P(x), \ \mu_N(x)) : x \in U\}$, where $\mu_P(x), \ : U \to [0,1]$ and $\mu_N(x) : U \to [-1, 0]$. It is noted that $\mu_P(x)$ indicate positive information and $\mu_N(x)$indicate negative information.

Definition 2.5: Let X be a non-empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where $\mu_A^+ : X \to [0,1]$, $\nu_A^+ : X \to [0,1]$, $\mu_A^- : X \to [-1,0]$, $\nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leq (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leq 1$ and $-1 \leq -[(\mu_A^-(x))^2 + (\nu_A^-(x))^2] \leq 0$ where

$\mu_A^+(x)$ denote the positive membership degree.

$\nu_A^+(x)$ denote the positive non membership degree.

$\mu_A^-(x)$ denote the negative membership degree.

$\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.6: Let U be the initial universal set and E be the set of parameters. Let P (U) be the power set of U and $A \subset$ E. A pair (F, A) is called a soft set over U, where F is a mapping given by: F: $A \rightarrow$ P (U).

Definition 2.7: Let U be the universal set and E be the set of parameters. Let P (U) be the power set of U and $A \subset$ E, and P (U) is the collection of all fuzzy subsets of U, then (F, A) is called fuzzy soft set, where F: A $\rightarrow$ P (U).

Definition 2.8: Let U be a universal set, E be a set of parameters and $A \subset$ E. Define $F: A \rightarrow BF^U$, where BF^U is the collection of all bipolar fuzzy subsets of U. Then (F, A) is called bipolar fuzzy soft set over U, and is denoted by (F, A) = $F(\alpha_i)$ and defined by:

$$F(\alpha_i) = [\left(x_i, \mu_p(x_i), \mu_n(x_i)\right): \forall x_i \in \mathrm{U}, \forall \alpha_i \in \mathrm{A}\}.$$

Definition 2.9: Let U be a set of parameters and $E \subseteq$ A. Define $\mathcal{K} : A \Rightarrow BIF_S^U$ where BIF_S^U is the collection of all Bipolar Intuitionistic Fuzzy Soft subsets of U by BIF_S^U. Then $(\mathcal{K}, A)$ is called Bipolar Intuitionistic Fuzzy soft set over U, and is denoted by $(\mathcal{K}, A)$ and defined by:

$\mathcal{K}, A) = \left\{\left(x_i, \mu_A^+(x_i), \mu_A^-(x_i), \nu_A^+(x_i), \nu_A^-(x_i)\right) : x_i \in U\right\}$ where

$\mu_A^+ : U \rightarrow [0,1]$, $\nu_A^+ : U \rightarrow [0,1]$, $\mu_A^- : U \rightarrow [-1,0]$, $\nu_A^- : U \rightarrow [-1,0]$ are the mappings such that $0 \leq \mu_A^+(x) + \nu_A^+(x) \leq 1$ and -1 $\leq \mu_A^-(x) + \nu_A^-(x) \leq 0$ where

$\mu_A^+(x)$ denote the positive membership degree.

$\nu_A^+(x)$ denote the positive non membership degree.

$\mu_A^-(x)$ denote the negative membership degree.

$\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.10: Let U be a set of parameters and $E \subseteq$ A. Define $\mathcal{K} : A \Rightarrow BPF_S^U$ where BPF_S^U is the collection of all Bipolar Pythagorean Fuzzy Soft subsets of U by BPF_S^U. Then $(\mathcal{K}, A)$ is called Bipolar Pythagorean Fuzzy soft set over U, and is denoted by $(\mathcal{K}, A)$ and defined by:

$(\mathcal{K}, A) = \left\{\left(x_i, \mu_A^+(x_i), \mu_A^-(x_i), \nu_A^+(x_i), \nu_A^-(x_i)\right) : x_i \in U\right\}$ where

$\mu_A^+ : U \rightarrow [0,1]$, $\nu_A^+ : U \rightarrow [0,1]$, $\mu_A^- : U \rightarrow [-1,0]$, $\nu_A^- : U \rightarrow [-1,0]$ are the mappings such that

$0 \leq \left(\mu_A^+(x)\right)^2 + \left(\nu_A^+(x)\right)^2 \leq 1$ and $-1 \leq -[\left(\mu_A^-(x)\right)^2 + \left(\nu_A^-(x)\right)^2] \leq 0$ where

$\mu_A^+(x)$ denote the positive membership degree.

$\nu_A^+(x)$ denote the positive non membership degree.

$\mu_A^-(x)$ denote the negative membership degree.

$\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.11: If (F, A) and (G, B) are two Bipolar Pythagorean Fuzzy Soft sets over a common universe U, then the union of $(F, A)$and (G, B) is defined to be the Bipolar Pythagorean Fuzzy $Soft$ set (H, C) satisfying the following conditions:

$(i)\ C = A \cup B\ (ii)$ for all $c \in C, H(c) = \begin{cases} F_C \ if\ c \in A\backslash B \\ G_C \ if\ c \in B\backslash A \\ F_C \cup G_C \ if\ c \in A \cap B \end{cases}$.

This relation is denoted by (H, C) = (F, A) ∪(G, B).

Definition 2.12: Let U be a universal set and E be the set of parameters. Then, (U, E) is the set of all Bipolar Pythagorean Fuzzy sets on U from the attributes E and is said to be Bipolar Pythagorean Fuzzy Soft class.

Definition 2.13: Let $(\mathcal{K}, A) \in BPF_S^U$. Then, $(\mathcal{K}, A)$ is called null (or) void Bipolar Pythagorean Fuzzy Soft set denoted by Φ, if for all $b \in A, \mathcal{K}(b) = \varphi$, where φ is an empty set.

Definition 2.14: Let $(\mathcal{K}, A) \in BPF_S^U$. Then, $(\mathcal{K}, A)$ is called an absolute Bipolar Pythagorean Fuzzy Soft set, if for all $b \in A, \mathcal{K}(b) \in BPF^U$.

Definition 2.15: Let $(\mathcal{K}, A) \in BPF_S^U$. Then, complement of $(\mathcal{K}, A)$ is denoted by $(\mathcal{K}, A)^c = (\mathcal{K}^c, A)$.

Proposition 2.16: Let $(\mathcal{K}_1, A_1) \in BPF_S^U$. Then,

a) $(\mathcal{K}_1, A_1) \cup (\mathcal{K}_1, A_1) = (\mathcal{K}_1, A_1), (\mathcal{K}_1, A_1) \cap (\mathcal{K}_1, A_1) = (\mathcal{K}_1, A_1)$

b) $(\mathcal{K}_1, A_1) \cup \varphi_E = (\mathcal{K}_1, A_1), (\mathcal{K}_1, A_1) \cap \varphi_E = \varphi_E$.

Proposition 2.17: Let$(\mathcal{K}_1, A_1)$, $(\mathcal{K}_2, A_2)$ and $(\mathcal{K}_3, A_3) \in BPF_S^U$ Then,

a) $(\mathcal{K}_1, A_1) \cap (\mathcal{K}_2, A_2) = (\mathcal{K}_2, A_2) \cap (\mathcal{K}_1, A_1)$

b) $(\mathcal{K}_1, A_1) \cup (\mathcal{K}_2, A_2) = (\mathcal{K}_2, A_2) \cup (\mathcal{K}_1, A_1)$

c) $(\mathcal{K}_1, A_1) \cup \left((\mathcal{K}_2, A_2) \cup (\mathcal{K}_3, A_3)\right) = ((\mathcal{K}_1, A_1) \cup (\mathcal{K}_2, A_2)) \cup (\mathcal{K}_3, A_3)$

d) $(\mathcal{K}_1, A_1) \cap \big((\mathcal{K}_2, A_2) \cap (\mathcal{K}_3, A_3)\big) = ((\mathcal{K}_1, A_1) \cap (\mathcal{K}_2, A_2)) \cap (\mathcal{K}_3, A_3)$

e) $(\mathcal{K}_1, A_1) \cup ((\mathcal{K}_2, A_2) \cap (\mathcal{K}_3, A_3)) = \big((\mathcal{K}_1, A_1) \cup (\mathcal{K}_2, A_2)\big) \cap ((\mathcal{K}_1, A_1) \cup (\mathcal{K}_3, A_3))$

f) $(\mathcal{K}_1, A_1) \cap ((\mathcal{K}_2, A_2) \cup (\mathcal{K}_3, A_3)) = \big((\mathcal{K}_1, A_1) \cap (\mathcal{K}_2, A_2)\big) \cup ((\mathcal{K}_1, A_1) \cap (\mathcal{K}_3, A_3))$

g) $((\mathcal{K}_1, A_1) \cup (\mathcal{K}_2, A_2))^c = ((\mathcal{K}_1, A_1))^c \cup (\mathcal{K}_2, A_2))^c$

h) $((\mathcal{K}_1, A_1) \cup (\mathcal{K}_2, A_2))^c = ((\mathcal{K}_1, A_1))^c \cap (\mathcal{K}_2, A_2))^c$

Definition 2.18: Let $(\mathcal{K}, A) \in BPF_S^U$ be Bipolar Pythagorean Fuzzy subsets of $(\mathcal{K}, B)$ and write $(\mathcal{K}, A) \subseteq (\mathcal{K}, B)$ if $\mu_A^+(x) \leq \mu_B^+(x)$, $\nu_A^+(x) \geq \nu_B^+(x)$, $\mu_A^-(x) \geq \mu_B^-(x)$, $\nu_A^-(x) \leq \nu_B^-(x)\rangle$ for each $x \in X$.

3.BIPOLAR PYTHAGOREAN FUZZY SOFT TOPOLOGICAL SPACES

Definition 3.1: Let $(U, E) \in BPF_S^{\mathrm{U}}$ be an absolute BPF Soft set. Let $\mathcal{C}(U, E)$ be the class of all BPF Soft sets of (U, E). Then, τ_p a family of BPF Soft -topology, If

T_1 $\phi, (U, E) \subseteq \tau_p$.

T_2 For any $(\mathcal{K}_1, \mathrm{E}), (\mathcal{K}_2, \mathrm{E}) \in \tau_p$, we have $(\mathcal{K}_1, \mathrm{E}) \cap (\mathcal{K}_2, \mathrm{E}) \in \tau_p$.

T_3 $\cup$ $(\mathcal{K}_n, \mathrm{E}) \in \tau_p$ for arbitrary family $\{ (\mathcal{K}_n, \mathrm{E})$ such that $n \in J \} \subseteq \tau_p$.

Then τ_p is called Bipolar Pythagorean Soft Topology on (U, E) and the trinity (X, τ_p, E) is said to be Bipolar Pythagorean Fuzzy Soft Topological space over (U, E). Each member of τ_p is called Bipolar Pythagorean Fuzzy Soft open set (BPF Soft OS). The complement of a Bipolar Pythagorean Fuzzy Soft open set is called a Bipolar Pythagorean fuzzy Soft Closed set (BPF Soft CS).

Example 3.2: Consider a universal set $U = \{a_1, a_2, a_3\}$ and the set of variables $E = \{b_1, b_2\}$.

Let $(\mathcal{K}_1, \mathrm{E}), (\mathcal{K}_2, \mathrm{E}),$, and $(\mathcal{K}_3, \mathrm{E}), \in \mathrm{C}(U, E)$ where

$$(\mathcal{K}_1, \mathrm{E}) = \begin{cases} (a_1, 04, 0.6, -0.3, -0.5), (a_2, 0.2, 0.6, -0.2, -0.5), (a_3, 0.6, 0.5, -0.5, -0.4) \\ (a_1, 04, 0.6, -0.3, -0.5), (a_2, 0.2, 0.6, -0.2, -0.5), (a_3, 0.6, 0.5, -0.5, -0.4) \end{cases}$$

$$(\mathcal{K}_2, \mathrm{E}) = \begin{cases} (a_1, 04, 0.6, -0.3, -0.5), (a_2, 0.6, 0.2, -0.5, -0.2), (a_3, 0.6, 0.5, -0.5, -0.4) \\ (a_1, 0.6, 0.2, -0.5, -0.2), (a_2, 0.2, 0.6, -0.2, -0.5), (a_3, 0.6, 0.5, -0.5, -0.4) \end{cases}$$

$$(\mathcal{K}_3, \mathrm{E}) = \begin{cases} (a_1, 04, 0.6, -0.3, -0.5), (a_2, 0.2, 0.6, -0.2, -0.5), (a_3, 0.6, 0.5, -0.5, -0.4) \\ (a_1, 0.6, 0.2, -0.5, -0.2), (a_2, 0.5, 0.1, -0.4, -0.1), (a_3, 0.7, 0.3, -0.6, -0.3) \end{cases}$$

Then, $\tau_p = \{\varphi_E, (\mathcal{K}_1, \mathrm{E}), (\mathcal{K}_2, \mathrm{E}), (\mathcal{K}_3, \mathrm{E}), (U, E)\}$ is a BPF Soft topology on (U, E).

Definition 3.3: The collection $\tau_p = \{\varphi_E, (U, E)\}$ of null or void BPFS-set and absolute BPFS-set is also, a BPF Soft-topology on (U, E) and it is called BPF Soft-indiscrete topology on (U, E). It is denoted by BPFS- $\tau_p$indiscrete. The BPFSoft-space (U, τ_p indiscrete, E) is called BPF Soft-indiscrete topological space over (U, E).

Definition 3.4: Let $\mathcal{C}(U, E)$ be collection of all BPFSoft-subsets of , (U, E), then $\tau_p = \mathcal{C}(U, E)$ is a BPFSoft-topology and is called BPF Soft-discrete topology on (U, E). It is denoted by BPF Soft-$\tau_p$discrete. The BPF Soft-space (U, $\tau_p$discrete, E) is called BPF Soft-discrete topological space over (U, E).

Definition 3.5: The family of all BPF Soft-sets BPF_S (U, E) is also a BPF Soft-topology on U.

Definition 3.6: Let (U, τ_p, E) be a BPF Soft topological space over (U, E), a BPF Soft set $(\mathcal{K}_1, \mathrm{E}) \in$ P (U, E) is called a BPF Soft closed in (U, τ_p, E), if its relative BPF Soft compliment $(\mathcal{K}_1, \mathrm{E})$c is a BPF Soft open set.

Definition 3.7: Let (U, τ_{p1}, E) and (U, τ_{p2}, E) two BPF Soft topological spaces, then the followings are valid.

1. Two BPF Soft-topologies τ_{p1}, and τ_{p2}, are BPF Soft comparable if one of them is BPF Soft subset of the other.

2. If τ_{p1}, is BPF Soft-subset of τ_{p2} then τ_{p1} is BPF Soft coarser, thenτ_{p2} and τ_{p2} is BPF Soft stronger then τ_{p1}.

Definition 3.8: Let (X, τ_p, E) be a BPF Soft topological space over X and $(\mathcal{K}, \mathrm{E}) = \{(x, \mu_A^+(x), \mu_A^-(\mathrm{x}), \nu_A^+(x), \nu_A^-(x)) : x \in X\}$ be a BPFSoft set over X. Then the Bipolar Pythagorean Fuzzy Soft Interior, Bipolar Pythagorean Fuzzy Soft Closure of P are defined by:

a) $BPF_S\ int\ (\mathcal{K}, \mathrm{E}) = \cup \{\ O : O$ is a BPF Soft open set in (X, τ_p, E) and $O \subseteq (\mathcal{K}, \mathrm{E})\}$.

b)$BPF_S\ cl\ (\mathcal{K}, \mathrm{E}) = \cap \{C : C$ is a BPF Soft Closed set in (X, τ_p, E) and $(\mathcal{K}, \mathrm{E}) \subseteq C\}$.

It is clear that

a) $BPF_S int$ $(\mathcal{K}, \mathrm{E})$ is the biggest Bipolar Pythagorean Fuzzy Soft Open set contained in $(\mathcal{K}, \mathrm{E})$.

b) $BPF_S int$ $(\mathcal{K}, \mathrm{E})$ is the smallest Bipolar Pythagorean Fuzzy Soft Closed set containing $(\mathcal{K}, \mathrm{E})$.

Definition 3.9: Bipolar Pythagorean Fuzzy Soft Regular Generalized closed set: If BPF Soft set $(\mathcal{K},E)$ of a BPFTS (X,τ_p,U) is a Bipolar Pythagorean Fuzzy Soft Regular Generalized closed set $(BPF_SRGC\mathcal{S})$, if $BPF_Scl(\mathcal{K},E)\subseteq U_R$ whenever $(\mathcal{K},E)\subseteq U_R$ and U_R is BPF Soft ROS in (X,τ_p,U).

(ii) If BPF Soft set $(\mathcal{K},E)$ of a BPFTS (X,τ_p,U) is a Bipolar Pythagorean Fuzzy Soft Regular Generalized open set $(BPF_SRGO\mathcal{S})$, if $(\mathcal{K},E)^c$ is a BPF Soft RGCS in (X,τ_p,U) .

Example 3.10: Let $\mathrm{U}=\{\mathrm{a},\mathrm{b}\}$ be the set of two bikes and $\mathrm{E}=\{\mathrm{e}_1,\mathrm{e}_2,\mathrm{e}_3\}$, $\mathrm{e}_1=\mathrm{costly},\mathrm{e}_2=\mathrm{milage},\mathrm{e}_3=\mathrm{beautiful}$ be the set of parameters, where $\mathrm{A}=\{\mathrm{e}_1,\mathrm{e}_2\}\subset\mathrm{E}$ and consider that $\tau_\mathrm{p}=\{\phi_\mathrm{A},(\mathrm{G},\mathrm{A}),(\mathrm{F},\mathrm{A}),(\mathrm{U},\mathrm{A})\}$ be a BPFSoft topology over (U,A) where (G,A) ,(F,A) and (H,A) are BPF soft sets over $(\mathrm{U},\tau_\mathrm{p},\mathrm{A})$ defined as follows:

$$(\mathrm{G},\mathrm{A})=\begin{Bmatrix}\{\mathrm{e}_1,\{(\mathrm{a},0.3,0.8,-0.4,-0.9)\},\{(\mathrm{b},0.6,0.6,-0.5,-0.7)\}\}\\\{\mathrm{e}_2,\{(\mathrm{a},0.4,0.7,-0.3,-0.9)\},\{(\mathrm{b},0.8,0.4,-0.4,-0.8)\}\}\end{Bmatrix}$$
$$(\mathrm{F},\mathrm{A})=\begin{Bmatrix}\{\mathrm{e}_1,\{(\mathrm{a},0.6,0.3,-0.7,-0.5)\},\{(\mathrm{b},0.7,0.4,-0.8,-0.5)\}\}\\\{\mathrm{e}_2,\{(\mathrm{a},0.7,0.4,-0.1,-0.9)\},\{(\mathrm{b},0.7,0.4,-0.7,-0.9)\}\}\end{Bmatrix}$$
$$(\mathrm{H},\mathrm{A})=\begin{Bmatrix}\{\mathrm{e}_1,\{(\mathrm{a},0.5,0.4,-0.6,-0.5)\},\{(\mathrm{b},0.6,0.6-0.7,-0.7)\}\}\\\{\mathrm{e}_2,\{(\mathrm{a},0.6,0.5,-0.2,-0.9)\},\{(\mathrm{b},0.7,0.5,-0.6,-0.9)\}\}\end{Bmatrix},$$

Here $BPF_Scl(\mathrm{H},\mathrm{A})\subseteq(\mathrm{U},\mathrm{A})=\begin{Bmatrix}\{\mathrm{e}_1,\{(\mathrm{a},1,0,-1,0)\},\{(\mathrm{b},1,0,-1,0)\}\}\\\{\mathrm{e}_2,\{(\mathrm{a},1,0,-1,0)\},\{(\mathrm{b},1,0,-1,0)\}\}\end{Bmatrix}\subseteq U_R$ whenever $(\mathrm{H},\mathrm{A})\subseteq U_R$ and U_R is BPF Soft ROS in $(\mathrm{U},\tau_\mathrm{p},\mathrm{A})$. Hence (H,A) is a $\mathrm{BPF_SRGCS}$ in $(\mathrm{U},\tau_\mathrm{p},\mathrm{A})$.

4. Decision Making in Portable Oxygen Concentrator, A Lifesaving Tool for COVID Patients Using BPF Soft Topology

Molodstov (1999) introduced the theory of soft sets which has many applications in game theory, Operations research, Riemann- integration, Probability, etc., with respect to applications in decision making in real life situations. The importance of decision-making problem in an imprecise environment is growing very significantly in recent years. Here, we see application of decision-making problem to find solution to real life problem. We used the concept of BPFSoft open sets which are BPF soft RG Closed Sets for modelling a decision-making problem and an algorithm for the choice of optimistic solution based on the available data and the problem is solved with the help of comparison technique.

Definition 4.1: (Comparison table) The comparison table of a Bipolar Pythagorean fuzzy soft set $(\mathcal{K}, E)$ a square table in which rows and columns are both labelled by the objects A_1, A_2, A_3 …………… A_n of the universe.

(1) The entries m_{ij}^{+} and indicates the number of parameters for which the positive membership value Ai greater than or equal to the positive membership value of A_j,

$$m_{ij}^{+} = |\{e_i \in A; F(e_i)(A_i) \geq F(e_i)(A_j)\}|$$

(2) p_{ij}^{+} indicates the number of parameters for which the positive non-membership value A_i less than or equal to the positive non- membership value of A_j,

$$p_{ij}^{+} = |\{e_i \in A; F(e_i)(A_i) \leq F(e_i)(A_j)\}|$$

(3) m_{ij}^{-}indicates the number of parameters for which the negative membership value A_i less than or equal to the negative membership value of A_j,

$$m_{ij}^{-} = |\{e_i \in A; F(e_i)(A_i) \leq F(e_i)(A_j)\}|$$

(4) p_{ij}^{-} indicates the number of parameters for which the negative non- membership value A_i less than or equal to the membership value of A_j,

$$p_{ij}^{-} = |\{e_i \in A; F(e_i)(A_i) \geq F(e_i)(A_j)\}|$$

Definition 4.2: Let positive (membership table's) row sum and column sum be $Row_i^1$and Col_i^1 of an object S_i calculating by the formula $Row_i^1 = \sum_{j=1}^{n} m_{ij}^{+}$ and $Col_i^1 = \sum_{j=1}^{n} m_{ij}^{+}$ .Let negative (membership table's) row sum and column sum be $Row_i^2$and Col_i^2 of an object S_i calculating by the formula $Row_i^2 = \sum_{j1}^{n} p_{ij}^{+}$ and $Col_i^2 = \sum_{j=1}^{n} p_{ij}^{+}$. Let positive (non-membership table's) row sum and column sum be Row_i^3 and Col_i^3 of an object S_i calculating by the formula $Row_i^1 = \sum_{j-1}^{n} m_{ij}^{-}$ and $Col_i^1 = \sum_{j-1}^{n} m_{ij}^{-}$. Let the negative (non-membership table's) row sum and column sum be $Row_i^4$and Col_i^4 of an object S_i calculating by the formula $Row_i^1 = \sum_{j-1}^{n} p_{ij}^{-}$ and $Col_i^1 = \sum_{j-1}^{n} p_{\bar{i}j}$.

Definition 4.3: Let positive membership score be $P_i^+$of an object h_i is given by $P_i^+ = Row_i^1 - Col_i^1$.Similarly negative membership score N_i^+ of an object S_i is given by $N_i^+ = Row_i^2 - Col_i^2$ and positive non-membership score be P_i^- of an object S_i is given by $P_i^- = Row_i^3 - Col_i^3$. Let negative non- membership score be N_i^- of an object S_i is given by by $N_i^- = Row_i^4 - Col_i^4$. The positive score S_P of an object S_i given by score $S_P = P_i^+ - P_i^-$and negative score S_N is given by $S_N = N_i^+ - N_i^-$. Thus, the final score $Score_i$ is given by $Score_i = S_P - S_N$ for all $j = 1, 2, , \dots n$. The optimum solution is obtained by adding all positive values and all negative values and finding the difference between positive and negative values $S = P - N$.

Portable Oxygen concentrators are available in various sizes, styles and models. Oxygen concentrators sucks the atmospheric air with 78% nitrogen, 21% of oxygen and the rest with other gases which filters out nitrogen and absorbs oxygen up to 95% of purity. Oxygen concentrators are used for the patients whose O_2 level is below 90% as directed by the doctors, with chronic obstructive pulmonary disease (COPD), hypoxemia and pulmonary edema. From 2019, we are thundered with severe COVID in India. Oxygen concentrators are most wanted for COVID patients whose O_2 level is below 90%. Some patients are advised to take O_2 in home with the proper guidance of a doctor. It is must to follow the doctor's approval for using oxygen concentrators with his guidance. Upto 5 liters of oxygen per minute are allowed for home usage. Many People are confused in purchasing Oxygen concentrators. Here we have got suggestions from decision experts and solved an uncertainty situation to choose the best oxygen concentrator for home usage.

Suppose Mr. R want to select a best oxygen concentrator from several companies. The experts decided the criteria for choosing the oxygen concentrator. The criteria are $\{ p, q, r, s, t\}$, where and the oxygen concentrators are denoted as $\{O_1, O_2, O_3, O_4\}$ oxygen concentrators taken into consideration. In order to evaluate the best oxygen concentrator, we utilized the method proposed to extract the five key attributes (ie) the decision variables $E = \{p, q, r, s, t\}$ described as follows:

(i) $p = flowrate,$

(ii) $q = powerconsumption,$

(iii) $r = oxygen\ concentration,$

(iv) $s = portability,$

(v) $t = cost$

They short list four Oxygen concentrators which are according to the requirements of Mr. R. Let us consider the initial universal set $X \quad = \{\, p, q, r, s, t\}$ consist of four Oxygen concentrators. It is obvious that the above five attributes are all benefit attributes. The decision makers choose the subsets $A = \{\, p, q, r, t\}$ and $B = \{\ p, r, t\}$ of X.

We consider Bipolar Pythagorean Fuzzy Soft sets as follows:

$$(\mathcal{K}_1, \mathrm{A}) = \begin{cases} \mathcal{K}_p^1 = \{(O_1, 0.61, 0.3, -0.5, , -0.2), (O_2, 0.4, 0.6, -0.5, , -0.4), (O_3, 0.8, 0.35, -0.3, -0.2), (O_4, 0.3,0.6, -0.32, -0.31)\} \\ \mathcal{K}_q^1 = \{(O_1, 0.7,0.5, -0.2, -0.4), (O_2, 0.7,0.4, -0.15, -0.4), (O_3, 0.62, 0.5, -0.4, -0.2), (O_4, 0.5,0.4, -0.4, -0.1)\} \\ \mathcal{K}_r^1 = \{(O_1, 0.2,0.41, -0.5, -0.3), (O_2, 0.6,0.4, -0.56, -0.5)(O_3, 0.6,0.5, -0.41, -0.36), (O_4, 0.3, 0.6, -0.24, -0.5)\} \\ \mathcal{K}_t^1 = \{(O_1, 0.4, 0.3, -0.6, -0.5), (O_2, 0.2,0.2, -0.6, -0.5)(O_3, 0.6, 0.5, -0.5, -0.4), (O_4, 0.6, 0.5, -0.2, -0.2)\} \end{cases}$$

$$(\mathcal{K}_2, \mathrm{B}) = \begin{cases} \mathcal{K}_p^2 = \{(O_1, 0.3,0.6, -0.4, -0.8), (O_2, 0.4,0.7, -0.5, -0.4), (O_3, 0.3,0.8, -0.4, -0.8), (O_4, 0.5,0.6, -0.5, -0.7)\} \\ \mathcal{K}_r^2 = \{(O_1, 0.6,0.6, -0.7, -0.7), (O_2, 0.4,0.4, -0.5, -0.5), (O_3, 0.6,0.3, -0.8, -0.4), (O_4, 0.8,0.1, -0.6, -0.7)\} \\ \mathcal{K}_t^2 = \{(O_1, 0.8, 0.0, -0.2, -0.1), (O_2, 0.1,0.4, -\ 0.4, -0.8)(O_3, 0.9, 0.3, -0.4, -0.8), (O_4, 0.2, 0.7, -0.1, -0.6)\} \end{cases}$$

Algorithm:

Step 1: *Input the BPF soft open sets* $(\mathcal{K}_1, A)$ *and* $(\mathcal{K}_2, B\ )$ *according to positive and negative feedback ratio.*

Step 2: *Construct BPF Soft -topology* $\left(X, \tau_p, U\right)$ *such that* $(\mathcal{K}_1, A)$ *and* $(\mathcal{K}_2, B)$ *which are BPFSoft RG Closed Sets in* $\left(X, \tau_p, U\right)$.

Step 3: *Write tabular forms of BPF soft open sets* $(\mathcal{K}_1, A)$ *and* $(\mathcal{K}_2, B\ )$.

Step 4: *Construct the comparison table of positive membership sum* m_{ij}^{+}, *negative membership sums* p_{ij}^{+} *positive non-membership sum* m_{ij}^{-} *and negative non-membership sum* p_{ij}^{-} *of the BPFRG Soft sets*$(\mathcal{K}_1, A)$ *and* $(\mathcal{K}_2, B)$.

Step 5: *Compute the positive membership score* P_i^{+} *negative membership score* N_i^{+} *, positive non-membership score* P_i^{-} *and negative non-membership score* N_i^{-}.

Step 6: *Compute the final score by subtracting positive score from negative score.*

Step 7: *Optimistic choice is the maximum score of* $Score_i$ *by using the formula* $Score_i = S_P - S_N$.

As can be seen that these sets are open sets and define a BPFRG soft topology $\tau_p = \{\varphi_E, (U,E), (\mathcal{K}_1, \mathrm{A}), (\mathcal{K}_2, \mathrm{B})\}$, where φ_E and (U,E) are null BPFS-set and absolute BPFS-set respectively. This shows the harmony between opinion of decision-makers which is essential for a sound and correct decision (i.e., if we take union or intersection of decision-makers opinion, then we do not obtain a new set, we obtain a set inside the space, which keep safe original collected information.)

By applying the above algorithm,

Step 1: Input the BPF soft open sets $(\mathcal{K}_1, A)$ and $(\mathcal{K}_2, B)$ according to positive and negative feedback ratio.

Step 2: Construct BPF Soft -topology (X, τ_p, U) such that $(\mathcal{K}_1, A)$ and $(\mathcal{K}_2, B)$ are BPF Soft RG Closed Sets in (X, τ_p, U) .

Step 3: Write tabular forms of BPF soft open sets $(\mathcal{K}_1, A)$ and $(\mathcal{K}_2, B)$.

	p	q	r	t
O1	(0.61,0.3, -0.5, -0.2)	(0.4, 0.6, -0.5, -0.5)	(0.8,0.35, -0.3, -0.2)	(0.3,0.6, -0.32, -0.31)
O2	(0.7, 0.5, -0.2, -0.4)	(0.7, 0.4, -0.15, -0.4)	(0.62,0.5, -0.4, -0.2)	(0.5, 0.4, -0.4, -0.1)
O3	(0.2,0.41, -0.5, -0.3)	(0.6,0.4, -0.56, -0.5)	(0.6,0.5, -0.41, -0.36)	(0.3,0.6, -0.24, -0.5)
O4	(0.4, 0.3, -0.6, -0.5)	(0.2, 0.2, -0.6, -0.5)	(0.6, 0.5, -0.5, -0.4)	(0.6, 0.5, -0.2, -0.2)

	p	r	t
O1	(0.3, 0.6, -0.4, -0.8)	(0.6, 0.6, -0.7, -0.7)	(0.8, 0.0, -0.2, -0.7)
O2	(0.4, 0.7, -0.5, -0.4)	(0.4, 0.4, -0.5, -0.5)	(0.1, 0.4, -0.4, -0.8)
O3	(0.4, 0.7, -0.5, -0.4)	(0.6, 0.3, -0.8, -0.4)	(0.9, 0.4, -0.1, -0.8)
O4	(0.5, 0.6, -0.5, -0.7)	(0.8, 0.1, -0.6, -0.7)	(0.2, 0.7, -0.1, -0.6)

Step 4: Construct the comparison table of positive membership sum m_{ij}^{+}, negative membership sums p_{ij}^{+} positive non-membership sum m_{ij}^{-} and negative non-membership sum p_{ij}^{-} of the bipolar Pythagorean fuzzy soft sets $(\mathcal{K}_1, \mathrm{A})$ and $(\mathcal{K}_2, \mathrm{B})$.

Step 5: Compute the positive membership score P_i^{+} negative membership score N_i^{+} , positive non-membership score P_i^{-} and negative non-membership score N_i^{-}.

Tabular representation of Positive membership function$(\mathcal{K}_1, \mathrm{A})$ and $(\mathcal{K}_2, \mathrm{B})$

Tabular representation of Positive membership function of $(\mathcal{K}_1, \mathrm{A})$						Comparison table for positive membership function m_{ij}^{+} of $(\mathcal{K}_1, \mathrm{A})$				
	p	q	r	t			O1	O2	O3	O4
O1	0.61	0.4	0.8	0.3		O1	4	1	2	2
O2	0.7	0.7	0.62	0.5		O2	3	4	3	3
O3	0.8	0.62	0.6	0.3		O3	3	1	4	3
O4	0.3	0.5	0.6	0.6		O4	2	1	2	4

Tabular representation of Positive membership function of $(\mathcal{K}_2, \mathrm{B})$					Comparison table for positive membership function m_{ij}^{+} of $(\mathcal{K}_2, \mathrm{B})$				
	p	r	t			O1	O2	O3	O4
O1	0.3	0.6	0.8		O1	3	2	2	1
O2	0.4	0.4	0.1		O2	1	3	1	0
O3	0.3	0.6	0.9		O3	3	2	3	1
O4	0.5	0.8	0.2		O4	2	3	2	3

Table for Positive membership Score of $(\mathcal{K}_1, \mathrm{A})$

	Row	Column	Positive membership of $(\mathcal{K}_1, \mathrm{A})$
	Sum (Row_i^1)	Sum (Col_i^1)	Score $(\boldsymbol{P_i^+})$
O1	9	12	-3
O2	13	7	6
O3	11	11	0
O4	9	12	3

Table for Positive membership Score P_i^+ of $(\mathcal{K}_2, \mathrm{B})$

	Row	Column	Positive membership P_i^+ of $(\mathcal{K}_2, \mathrm{B})$
	Sum (Row_i^1)	Sum (Col_i^1)	Score $(\boldsymbol{P_i^+})$
O1	8	9	-1
O2	5	10	-5
O3	9	9	0
O4	10	10	0

Tabular representation of Positive non-membership functions $(\mathcal{K}_1, \mathrm{A})$ and $(\mathcal{K}_2, \mathrm{B})$

Tabular representation of Positive non-membership function of $(\mathcal{K}_1, \mathrm{A})$						Comparison table for Positive non - membership function p_{ij}^+ of $(\mathcal{K}_1, \mathrm{A})$				
	p	q	r	t			O1	O2	O3	O4
O1	0.3	0.6	0.3	0.6		O1	4	2	2	3
O2	0.5	0.4	0.5	0.4		O2	2	4	3	4
O3	0.41	0.4	0.5	0.6		O3	3	3	4	4
O4	0.3	0.2	0.5	0.5		O4	2	3	4	4

Tabular representation of Positive non-membership function of $(\mathcal{K}_2, \mathrm{B})$					Comparison table for positive non-membership function p_{ij}^{+} of $(\mathcal{K}_2, \mathrm{B})$				
	p	r	t			O1	O2	O3	O4
O1	0.6	0.6	0.0		O1	3	2	2	2
O2	0.7	0.4	0.4		O2	1	3	2	1
O3	0.8	0.3	0.4		O3	1	2	3	1
O4	0.6	0.1	0.7		O4	2	2	2	3

Table for Negative membership Score of $(\mathcal{K}_1, \mathrm{A})$

	Row	Column	Negative membership of $(\mathcal{K}_1, \mathrm{A})$
	Sum (Row_i^2)	Sum (Col_i^2)	Score $(\boldsymbol{N_i^+})$
O1	11	11	0
O2	13	12	0
O3	14	13	1
O4	13	15	-2

Table for Negative membership Score of $(\mathcal{K}_2, \mathrm{B})$

	Row	Column	Negative membership of $(\mathcal{K}_2, \mathrm{B})$
	Sum (Row_i^2)	Sum (Col_i^2)	Score $(\boldsymbol{N_i^+})$
O1	9	7	2
O2	7	9	-2
O3	7	9	-2
O4	9	7	2

Tabular representation of Negative membership function of $(\mathcal{K}_1, \mathrm{A})$

Tabular representation of Negative membership function of $(\mathcal{K}_1, \mathrm{A})$						Comparison table for Negative membership function m_{ij}^{-} of $(\mathcal{K}_1, \mathrm{A})$				
	p	q	r	t			O1	O2	O3	O4
O1	-0.5	-0.5	-0.3	-0.32		O1	4	2	3	1
O2	-0.2	-0.15	-0.4	-0.4		O2	2	4	1	1
O3	-0.5	-0.56	-0.41	-0.24		O3	3	3	4	1
O4	-0.6	-0.6	-0.5	-0.2		O4	3	4	3	4

Tabular representation of Negative membership of $(\mathcal{K}_2, \mathrm{B})$					Comparison table for Negative membership function m_{ij}^{-} of $(\mathcal{K}_2, \mathrm{B})$				
	p	r	t			O1	O2	O3	O4
O1	-0.4	-0.7	-0.2		O1	3	1	2	2
O2	-0.5	-0.5	-0.4		O2	2	3	2	2
O3	-0.4	-0.8	-0.1		O3	2	1	3	2
O4	-0.5	-0.6	-0.1		O4	1	2	2	3

Table for Positive non-membership Score of $(\mathcal{K}_1, \mathrm{A})$

	Row	Column	Negative membership of $(\mathcal{K}_2, \mathrm{B})$
	Sum (Row_i^3)	Sum (Col_i^3)	Score $(\boldsymbol{P_i^-})$
O1	10	12	-2
O2	8	13	-5
O3	11	11	0
O4	14	7	7

Table for Positive non-membership Score of $(\mathcal{K}_2, \mathrm{B})$

	Row Sum (Row_i^3)	Column Sum (Col_i^3)	Negative membership of $(\mathcal{K}_2, \mathrm{B})$ Score $(\boldsymbol{P_i^-})$
O1	8	8	0
O2	9	7	2
O3	8	9	-1
O4	8	9	-1

Tabular representation of Negative non-membership function

Tabular representation of Negative non-membership function of $(\mathcal{K}_1, \mathrm{A})$

	C1	C2	C3	C4
O1	-0.2	-0.5	-0.2	-0.31
O2	-0.4	-0.4	-0.2	-0.1
O3	-0.3	-0.5	-0.36	-0.5
O4	-0.5	-0.5	-0.4	-0.2

Comparison table for Negative non - membership function p_{ij}^- of $(\mathcal{K}_1, \mathrm{A})$

	S1	S2	S3	S4
O1	4	2	4	3
O2	3	4	3	4
O3	1	1	4	3
O4	2	0	2	4

Tabular representation Negative non-membership function of $(\mathcal{K}_2, \mathrm{B})$

	p	r	t
O1	-0.8	-0.7	-0.7
O2	-0.4	-0.5	-0.8
O3	-0.4	-0.4	-0.8
O4	-0.7	-0.7	-0.6

Comparison table for Negative non-membership function p_{ij}^- of $(\mathcal{K}_2, \mathrm{B})$

	O1	O2	O3	O4
O1	3	1	1	1
O2	2	3	2	2
O3	2	3	3	2
O4	3	1	1	3

Table for Negative non-membership Score of $(\mathcal{K}_1, \text{A})$

	Row	Column	Negative non- membership of $(\mathcal{K}_1, \text{A})$
	Sum (Row_i^4)	Sum (Col_i^3))	Score $(\boldsymbol{N_i^-})$
O1	13	10	3
O2	14	7	7
O3	9	13	-4
O4	8	14	-6

Table for Positive non-membership Score of $(\mathcal{K}_2, \text{B})$

	Row	Column	Negative non- membership of $(\mathcal{K}_2, \text{B})$
	Sum (Row_i^4)	Sum (Col_i^4)	Score $(\boldsymbol{N_i^-})$
O1	6	10	-4
O2	9	8	1
O3	10	7	3
O4	8	8	0

Step 6: Compute the final score by subtracting positive score from negative score.

Score Table of $(\mathcal{K}_1, \text{A})$.

	Score (S_P= $P_i^+ - P_i^-$) of $(\mathcal{K}_1, \text{A})$	Score ($S_N = N_i^+ - N_i^-$)of $(\mathcal{K}_1, \text{A})$	Score Si = $(S_P - S_N)$ of $(\mathcal{K}_1, \text{A})$
O1	-1	-3	2
O2	11	-7	18
O3	0	5	-5
O4	-4	4	-8

Score Table of $(\mathcal{K}_2, \mathrm{B})$

	Score ($S_P = P_i^+ - P_i^-$) of $(\mathcal{K}_2, \mathrm{B})$	Score ($S_N = N_i^+ - N_i^-$)of $(\mathcal{K}_2, \mathrm{B})$	Score $S_i = (S_P - S_N)$ of $(\mathcal{K}_2, \mathrm{B})$
O1	1	6	-5
O2	0	-3	3
O3	0	-5	5
O4	-7	2	-9

Step 7: Optimistic choice is the maximum score of $\mathrm{Score_i}$ by using the formula $\mathrm{Score_i} = \mathrm{S_P} - \mathrm{S_N}$.

	P= S_P of $(\mathcal{K}_1, \mathrm{A})$+ S_P of $(\mathcal{K}_2, \mathrm{B})$	N= S_N of $(\mathcal{K}_1, \mathrm{A})$+ S_N of $(\mathcal{K}_2, \mathrm{B})$	S=P-N
O1	0	3	-3
O2	11	-10	21
O3	0	0	0
O4	3	6	-3

It is clear that the maximum score value is 21 obtained by O2.

Decision: The choice of Mr. R will be O2.

5. CONCLUSION

Here we merge the concept of Bipolar Pythagorean fuzzy soft set and topology to introduced the concept of Bipolar Pythagorean fuzzy soft Topological Spaces. We introduced some operations on Bipolar Pythagorean fuzzy Soft Topological spaces. We defined of Bipolar Pythagorean Fuzzy Soft Regular Generalized closed sets from Bipolar Pythagorean fuzzy soft topological spaces and gave an application in the field of decision making to make the best choice of portable oxygen concentrator. In future, we will focus on the following problems Bipolar Pythagorean fuzzy soft matrix, Bipolar Pythagorean fuzzy soft functions and its applications in machine learning and deep learning.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986), 87-96.

[2] M. Akram, A. Adeel, Representation of labeling tree based on m-polar fuzzy sets, Ann. Fuzzy Math. Inform. 13 (2017), 189-197.

[3] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Computers Math. Appl. 57 (2009), 1547–1553.

[4] D. Coker, An introduction to intuitionistic topological spaces, Busefal, 81 (2000), 51-56.

[5] S. Abdullah, M. Aslam, K. Ullah, Bipolar fuzzy soft sets and its applications in decision making problem, J. Intell. Fuzzy Syst. 27 (2014), 729–742.

[6] N. Çağman, F. Çıtak, S. Enginoğlu, Fuzzy parameterized fuzzy soft set theory and its applications. Turk. J. Fuzzy Syst. 1 (2010), 21-35.

[7] S. EngiNoğlu, N. Çağman, S. Karataş, T. Aydin, On Soft Topology, El-Cezeri Fen ve Mühendislik Dergisi. 2 (2015), 23-38.

[8] N. Cagman, S. Enginoglu, F. Citak, Fuzzy soft set theory and its applications. Iran. J. Fuzzy Syst. 8 (2011), 137-147.

[9] C.L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182-190.

[10] S. Das, S.K. Samanta, Soft real sets, soft real numbers and their properties. J. Fuzzy Math. 20 (2012), 551–576.

[11] S. Das, S.K. Samanta, Soft metric, Ann Fuzzy Math. Inf. 6 (2013), 77–94.

[12] F. Feng, Y.B. Jun, X. Zhao, Soft semirings, Comput Math Appl. 56 (2008), 2621–2628.

[13] F. Feng, Y.B. Jun, X. Liu, L. Li, An adjustable approach to fuzzy soft set-based decision making, J. Comput. Appl. Math. 234(2010a), 10–20.

[14] F. Feng, C. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach. Soft Comput. 14(9) (2010b), 899– 911.

[15] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Jun YB Soft sets and soft rough sets. Inf. Sci. 181 (2011), 1125–1137.

[16] A. Kharal, B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. 7(3) (2011), 471–482.

[17] K.M. Lee, Bipolar-valued fuzzy sets and their basic operations, In: Proceeding International Conference, Bangkok, Thailand, (2000), 307-317.

[18] K.M. Lee, Bipolar-valued fuzzy sets and their basic operations. In: Proceeding international conference, Bangkok, Thailand (2000), pp. 307–317.

[19] D. Molodtsov, Soft set theory first results. Computer Math. Appl. 37(4-5) (1999), 19–31.

[20] D. Molodtsov, The theory of soft sets, URSS Publishers, Moscow, 2004 (in Russian).

[21] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001), 589–602.

[22] P.K. Maji, R. Biswas, A.R. Roy, An application of soft sets in decision making problem. Comput. Math. Appl. 44 (2002), 1077–1083.

[23] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory. Comput. Math. Appl. 45 (2003), 555–562.

[24] P. Majumdar, S.K. Samanta, On soft mappings, Comput. Math. Appl. 60(9) (2010), 2666–2672.

[25] N. Malik, M. Shabir, Rough fuzzy bipolar soft sets and application in decision-making problems, Soft. Comput. 23 (2019), 1603–1614.

[26] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (1999), 19–31.

[27] M. Riaz, S.T. Tehrim, Certain properties of bipolar fuzzy soft topology via Q-neighborhood. Punjab Univ. J. Math. 51(3) (2019a), 113–131.

[28] M. Riaz, S.T. Tehrim, Cubic bipolar fuzzy ordered weighted geometric aggregation operators and their application using internal and external cubic bipolar fuzzy data, Comput. Appl. Math. 38 (2019), 87.

[29] M. Riaz, Z. Fatima, Certain properties of soft metric spaces, J. Fuzzy Math. 25(3) (2017), 543–560.

[30] M. Riaz, S.T. Tehrim, Certain properties of bipolar fuzzy soft topology via Q-neighborhood. Punjab Univ. J. Math. 51(3) (2019a), 113–131.

[31] M. Riaz, S.T. Tehrim, Cubic bipolar fuzzy ordered weighted geometric aggregation operators and their application using internal and external cubic bipolar fuzzy data. Comput. Appl. Math. 38(87) (2019b), 1–25.

[32] M. Riaz, F. Smarandache, A. Firdous, A. Fakhar, On soft rough topology with multi-attribute group decision making, Mathematics. 7 (2019), 67.

[33] M. Riaz, B. Davvaz, A. Firdous, F. Fakhar, Novel concepts of soft rough set topology with applications. J. Intell. Fuzzy Syst. 36(4) (2019b), 3579–3590.

[34] M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011), 1786–1799.

[35] B.P. Varol, H. Aygun, Fuzzy soft topology, Hacet. J. Math. Stat. 41(3) (2012), 407–419.

[36] L.A. Zadeh, Fuzzy sets, Inform. Control, 8 (1965), 338-353.

[37] W.R. Zhang, Bipolar fuzzy sets. In Proc. of FUZZ-IEEE (1998), pp. 835-840.

[38] X. Zhang, Bipolar-value fuzzy soft lie subalgebras, IOP Conf. Ser.: Mater. Sci. Eng. 231 (2017), 012008.