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TWO WAREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS WITH FIXED SHELF-LIFE STOCK-DEPENDENT DEMAND AND PARTIAL BACKLOGGING

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Abstract: In inventory management self life expiration date has a unique role. In the practice there are many goods and services such as food, medication remain safe and suitable for human consumption until it exist their shelf life. In this paper, we implemented a fixed shelf life of a deterministic inventory model for decaying items in two warehouses system with partial backlogging. The demand rate is considered stock dependent means that demand inclined by display of the stock level. In two warehouse system we are considering one own warehouse and second warehouse is on rented basis. The preservation facility is good in rented warehouse than own warehouse. Due to various preserve conditions, deterioration rate in two warehouses may be differs. In addition, backlogging rate is time dependent which is inversely proportional to the waiting time for the next cycle. The model is also justified by the numerical examples under two cases and also sensitivity analysis is carried out with various parameters by using MATLAB R16b.

Keywords: inventory model with two-warehouse facility; demand depends on stock level; partial backlogging; fixed shelf life.

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1. INTRODUCTION AND PRELIMINARIES

Due to growing in costs, decrement in resources level, undersized life cycle of the goods and various other reasons are responsible for incompetence in today's world wide market. Therefore it is necessary to take various measures to make the business profitable. It is usually assumed that larger the display of the goods more goods will be purchased. From last few decades, many researchers led their attention towards the situation where demand rate is stock dependent and results implies that holding larger amount of goods will be cost-effective for the retailers. Gupta and Vrat (1986) established an inventory model with stock dependent consumption rate. Further, Baker and Urban (1988) modified this model by considering demand as a function of instantaneous inventory level. Mandal and Phaujdar (1989) studied an inventory model for decaying items taken consumption rate as stock dependent. Later, Vrat and Padmanabhan (1990) developed further model with stock dependent demand rate having its effects on inflation. This paper was then modified by Su et al.(1996) by projecting an inventory model where inventories dependent on demand rate under exponential decay.

A model discussing various inventory problems of decaying items with shortages having stock dependent as demand rate by Datta et al. (1990). Further Padmanabhan and Vrat (1995) investigated with three models i.e. with different backorders for deteriorating items. Karabi et al. (1996) purposed an inventory model with shortages and two-component demand rate. Some other models were done by Zhou and Yang (2005), Maiti et al. (2006), Goyal et al. (2009), Yang et al. (2010), Bhunia et al. (2014), Kumar et al. (2016), M Pal (2016), S Tiwari et al. (2017) Mishra et al. (2017), AA Shaikh et al.(2017), Abu Hashan Md Mashuda et al (2018), A Mashud et al. (2018), NH Shah et al. (2018), AA Shaikh et al. (2019), G C Panda (2019), V Dhaka(2020), LE Cárdenas-Barrón et al. (2020).

From many decades, the problems on inventory have been studied with variety of different scenarios. Inventory is a stock of perishable and non- perishable goods. So we can easily see the importance of fixed shelf life. It is something which can be relatable with the expiration date. It means up to a certain point of time the goods are suitable for consumption and after that deterioration starts. There are large number perishable goods which fall in this category such as radioactive substances, fruits, food grains, vegetables, pharmaceuticals, etc. Tal Avinadav, Teijo Arponen(2009) proposed an EOQ model with declining in demand rate which is based on time to expiry technical note with the effect of fixed shelf-life. Kai-Wayne (2015) discussed an inventory model with ramp type demand with fixed shelf life under shortages. Other authors are Shirajul Islam Ukil et al. (2015), Cinzia Muriana (2016), etc.

In last few decades two warehouse inventory system was discussed by various researchers. Such a system was first proposed by Hartley (1976). He discussed about two warehouses i.e. own warehouse (OW) and rented warehouse (RW). It is assumed that holding cost for RW is always greater than OW. Therefore, the stock of RW was used to meet the demand of the customers until it drops to zero, then stock of OW is used. It manages the bulk purchase of the goods and retailer can get discount on it. Large pile of goods attracts more customers and proves to be more beneficial. Hence, models in inventory should be comprehensive to several warehouses.

Hartley (1976), introduced two warehouse inventory model by assuming that the holding cost of RW is larger than OW then Further, Sarma (1987) extended two warehouse model with an infinite replenishment rate. Another extension of non-deteriorating goods was purposed by Goswami and Chaudhuri (1992) with shortages and demand rate as time dependent. The transportation cost was also taken into consideration. Later, Pakkala and Achary (1992) introduced two warehouse systems with deteriorating goods. Two warehouse systems with effect of inflation and shortages were taken into consideration by Yang (2004). Later, Chung and Haung (2004) presented another model for non- deteriorating goods with no shortages under permissible delay in payments. Zhou and Yang (2005) established another type of model having demand rate as stock dependent combination with two warehouse inventory system. CC Lee, SL Hsu (2009) presented inventorymodel under the title, “A two warehouse production model for deteriorating inventory items with time-dependent demands.” BK Sett, B Sarkar, A Goswami (2012) presented an inventory model of two-warehouse with increasing demand and time changing decaying. Two warehouse inventory system with the price dependent demand and decaying under partial backlogging was established by M Rastogi et al. (2017). Some other authors, namely, Chun Chen Lee (2009), B.Kumar Sett et al. (2012), Bhunia AK et al. (2013), Bhunia AK et al. (2014), M Rastogi et al (2017), SK Roy et al. (2018), R Chakrabarty et al. (2018) Gobinda Chandra Panda et al. (2019), Md. Al-Amin Khan (2019), GC Panda et al. (2019), Boina Anil Kumar et al. (2020) done excellent work on it.

In this present paper, we developed the deterministic inventory model for decaying items in two warehouse system with shelf life under partial backlogging and demand rate of the inventory is inclined by quantity of stock level in display and we assumed backlogging rate during the stock out stage which inversely proportional to the for the future time of the next replenishment. This paper having following arrangement section 2 consists the assumptions and notations and section 3 deliberated the mathematical model while section 4, explores the optimal solution of the model with numerical example to exemplify the model and the sensitivity analysis with the graphical representation by applying different parameters and followed by conclusion.

2. CIPHERS AND SUPPOSITION

The model of inventory with two warehouse system is developed with the following suppositions:

2.1 Ciphers

$R_I(t)$	Inventory level in rented warehouse(RW) for time t
$O_I(t)$	Inventory level in owed warehouse (OW) for time t
Z	Ordering cost for inventory, Rupees/ order
W	Capacity level of the own warehouse (OW)
S	Capacity level of the rented warehouse (RW)
H_1	Cost of holding, Rupees / unit time in rented warehouse (RW)
H_2	Cost of holding, Rupees / unit time in own warehouse (OW)
C	Cost of shortage, Rupee/ unit
L	Cost of lost sales, Rupee /per unit
D_1	Cost of deterioration, Rupee / per unit time in rented warehouse (RW)

D_2	Cost of deterioration, Rupee / per unit time in own warehouse (OW)
γ	Rate of deterioration in own warehouse(OW), value of γ lies in (0,1)
θ	Rate of deterioration in rented warehouse (RW), value of θ lies in (0,1)
t_a	Point of time for the inventory level goes down to zero in rented warehouse (RW)
t_b	Point of time for the inventory level goes down to zero in own warehouse (OW)
T_{ab}	Total sum of time of an inventory cycle, hence $T_{ab} = t_a + t_b$
TC_1	Total sum of cost for the inventory cycle in case A
TC_2	Total sum of cost for the inventory cycle in case B
TVC_1	Total sum of cost per unit time for an inventory cycle in case A
TVC_2	Total sum of cost per unit time for an inventory cycle in case B

2.2 Suppositions

1. The function used for demand rate D(t) is called function of instantaneous stock-level I(t) and considered as deterministic. It is defined as:

   $$D(t) = \begin{cases} \eta + \phi I(t), & 0 \le t \le \mu \\ \eta, & \mu \le t \le T \end{cases} \qquad \text{here } \eta, \text{ and } \phi > 0.$$

2. The time horizon of the system is taken as infinite.
3. Both the warehouses i.e. own warehouse (OW) and rented warehouse (RW) has a predetermined capacity of W units and S units respectively.
4. After consumptions of the goods kept in rented (RW) then only the goods of own warehouse (OW) are consumed.
5. By assuming the maximum decaying quantity for times in own warehouse (OW), it guarantees the existence of optimal solution exists.
6. Shortages are allowed and demand is backlogged which is unsatisfied, and the shortages backordered is given by $(1 + \Delta (T_{ab} - t)^{-1}$, where Δ is taken as constant which is of positive nature.

3. MATHEMATICAL FORMULATION

From the Figure (a) below, we can see the intervals of time individually as [0, t_a], [t_a, t_b] and [t_b, T_{ab}]. In the course of interval of time [0, t_a], the levels of inventory are (> 0) for both rented warehouse (RW) and own warehouse (OW). At rented warehouse (RW), the level of inventory is decreased because of the mutual outcome of demand and goods which are deteriorated. Till point of time f_1 (fixed shelf-life) there will not be any deterioration. Therefore, the differential equations leading to the different levels of inventory for rented warehouse (RW) and own warehouse (OW) are given below.

For CASE (1): When $0 \le f_1 \le t_a$

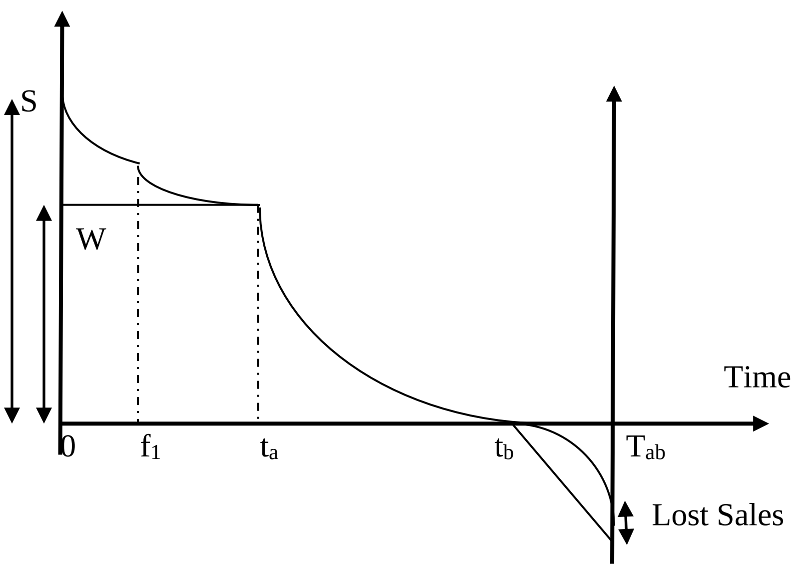


Fig 1: inventory level of two warehouse system

Differential equations for rented warehouse for different time intervals

$$\frac{dR_I(t)}{dt} = -(\eta + \phi I_r(t)) \qquad 0 < t < f_1 \tag{3.1}$$

$$\frac{dR_I(t)}{dt} = -(\eta + \phi R_I(t)) - \theta R_I(t) \qquad f_1 < t < t_a \tag{3.2}$$

Solving equation (3.1) with boundary condition $R_I(f_1) = S$

$$R_I(t) = -\frac{\eta}{\phi} + \left(\frac{\eta}{\phi} + S\right) e^{\phi(f_1 - t)} \tag{3.3}$$

Solving equation (3.2) with boundary condition $R_I(t_a) = 0$

$$R_I(t) = \frac{\eta}{\phi+\theta}(e^{(\phi+\theta)(t_a-t)}-1) \tag{3.4}$$

Differential equations for own warehouse for different time intervals

$$O_I(t) = W \qquad 0 < t < f_1 \tag{3.5}$$

$$\frac{dO_I(t)}{dt} = -O_I\gamma(t) \qquad f_1 < t < t_a \tag{3.6}$$

Solving equation (3.6) using boundary condition $O_I(0) = W$

$$O_I(t) = We^{-\gamma t} \tag{7}$$

In between the interval [t_a, t_b], the level of inventory in own warehouse (OW) is spent because of the effects of deterioration as well as demand. Hence, the level of inventory at own warehouse (OW) is given below by the differential equation given below:

$$\frac{dO_I(t)}{dt} = -(\eta+\phi O_I(t)) - \gamma O_I(t) \qquad t_a < t < tb \tag{3.8}$$

Solving equation (3.8) using boundary condition $O_I(t_2) = 0$

$$O_I(t) = \frac{\eta}{\phi+\gamma}(e^{(\phi+\gamma)(t_a-t)}-1) \tag{3.9}$$

Moreover, at time t_b, the level of inventory go downs to zero in own warehouse (OW) and shortages occur. In between the time interval [t_b,T_{ab}], the level of inventory depends on demand, and some fraction of demand got vanished but some fraction

$$\frac{1}{(1+\Delta(T_{ab}-t))} \tag{3.10}$$

is backlogged and where $t \in [t_b, T_{ab}]$. The given below differential equation gives the level of inventory.

$$\frac{dO_I(t)}{dt} = \frac{\eta}{1+\Delta(T_{ab}-t)} \tag{3.11}$$

With the use of boundary condition $O_I(t_b) = 0$. We get the following level of inventory as:

$$O_I(t) = -\frac{\eta}{\Delta}\log\left[1+\Delta(T_{ab}-t_b)\right]-\log\left[1+\Delta(T_{ab}-t)\right] \tag{3.12}$$

The total costs per cycle composed of certain constituents are given below:

1. **Cost of ordering per cycle**

OA= Z (3.13)

2. **Cost of holding goods per cycle in rented warehouse (RW)**

$$HC_{RW} = H_1\left\{\int_0^{f_1} R_I(t)dt + \int_{f_1}^{t_a} R_I(t)dt\right\}$$

$$= H_1\left[\frac{\eta}{(\phi+\theta)^2}\left\{e^{(t_a-f_1)(\phi+\theta)}-(\phi+\theta)(t_a-f_1)-1\right\}-\frac{S}{\phi}\left(e^{\phi f_1}-1\right)+\frac{\eta}{\phi}\left\{e^{\phi f_1}-\frac{1}{\phi}-f_1\right\}\right] \tag{3.14}$$

3. **Cost of holding goods per cycle in own warehouse (OW)**

$$HC_{OW} = H_2\left\{\int_0^{f_1} O_I(t)dt + \int_{f_1}^{t_a} O_I(t)dt + \int_{t_a}^{t_b} O_I(t)dt\right\}$$

$$= -H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_a}}{\gamma}-1\right)+\frac{\eta}{(\phi+\gamma)^2}(1-e^{(\phi+\gamma)(t_a-t_b)}+(\phi+\gamma)(t_b-t_a))\right] \tag{3.15}$$

4. **Cost of shortages per cycle**

$$SC = -C\left\{\int_{t_B}^{T_{ab}} O_I(t)dt\right\} = \frac{C\eta}{\Delta^2}\left\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\right\} \tag{3.16}$$

5. **Lost sales per cycle**

$$LS = L\eta\int_{t_b}^{T_{ab}}\left\{1-\frac{1}{[1+\Delta(T_{ab}-t)]}\right\}dt = \frac{L\eta}{\Delta}\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\} \tag{3.17}$$

6. **Cost of deterioration per cycle in rented warehouse (RW)**

$$DC_{RW} = D_1\left\{\int_{f_1}^{t_a}\theta R_I(t)dt\right\} = \frac{D_1\eta\theta}{(\phi+\theta)^2}(1-e^{(\phi+\theta)(t_a-f_1)}+(\phi+\theta)(t_a-f_1)) \tag{3.18}$$

7. **Cost of deterioration per cycle in own warehouse (OW)**

$$DC_{OW} = D_2\left\{\int_{f_1}^{t_a}\theta O_I(t)dt + \int_{t_a}^{t_b}\theta O_I(t)dt\right\}$$

$$= D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_a}-e^{-\gamma t_b})+\frac{\eta\theta}{(\phi+\gamma)^2}\left\{(e^{(\phi+\gamma)(t_b-t_a)}-1)-(\phi+\gamma)(t_b-t_a)\right\}\right] \tag{3.19}$$

Total Cost for time interval [0, T_{ab}]

TC_1 (0, T_{ab}) = $OA+HC_{RW}+HC_{OW}+SC+LS+DC_{RW}+DC_{OW}$

$$\begin{aligned}= Z &+ H_1\left[\frac{\eta}{(\phi+\theta)^2}\left\{e^{(t_a-f_1)(\phi+\theta)}-(\phi+\theta)(t_a-f_1)-1\right\}-\frac{S}{\phi}\left(e^{\phi f_1}-1\right)+\frac{\eta}{\phi}\left\{e^{\phi f_1}-\frac{1}{\phi}-f_1\right\}\right]\\ &- H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_a}}{\gamma}-1\right)+\frac{\eta}{(\phi+\gamma)^2}(1-e^{(\phi+\gamma)(t_a-t_b)}+(\phi+\gamma)(t_b-t_a))\right]\\ &+\frac{C\eta}{\Delta^2}\left\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\right\}+\frac{L\eta}{\Delta}\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\}\\ &+\frac{D_1\eta\theta}{(\phi+\theta)^2}(1-e^{(\phi+\theta)(t_a-f_1)}+(\phi+\theta)(t_a-f_1))\\ &+D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_a}-e^{-\gamma t_b})+\frac{\eta\theta}{(\phi+\gamma)^2}\left\{(e^{(\phi+\gamma)(t_b-t_a)}-1)-(\phi+\gamma)(t_b-t_a)\right\}\right]\end{aligned} \tag{3.20}$$

Total Cost per unit time

TVC_1 (0, T_{ab}) = $\dfrac{TC_1(0,T_{ab})}{T}$

$$\frac{1}{T_{ab}}\left[\begin{aligned}&Z + H_1\left[\frac{\eta}{(\phi+\theta)^2}\left\{e^{(t_a-f_1)(\phi+\theta)}-(\phi+\theta)(t_a-f_1)-1\right\}-\frac{S}{\phi}\left(e^{\phi f_1}-1\right)+\frac{\eta}{\phi}\left\{e^{\phi f_1}-\frac{1}{\phi}-f_1\right\}\right]\\ &- H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_a}}{\gamma}-1\right)+\frac{\eta}{(\phi+\gamma)^2}(1-e^{(\phi+\gamma)(t_a-t_b)}+(\phi+\gamma)(t_b-t_a))\right]\\ &+\frac{C\eta}{\Delta^2}\left\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\right\}+\frac{L\eta}{\Delta}\{\Delta(T_{ab}-t_b)-\log[1+\Delta(T_{ab}-t_b)]\}\\ &+\frac{D_1\eta\theta}{(\phi+\theta)^2}(1-e^{(\phi+\theta)(t_a-f_1)}+(\phi+\theta)(t_a-f_1))\\ &+D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_a}-e^{-\gamma t_b})+\frac{\eta\theta}{(\phi+\gamma)^2}\left\{(e^{(\phi+\gamma)(t_b-t_a)}-1)-(\phi+\gamma)(t_b-t_a)\right\}\right]\end{aligned}\right] \tag{3.21}$$

FOR CASE (2): When $t_a \leq f_1 \leq t_b$

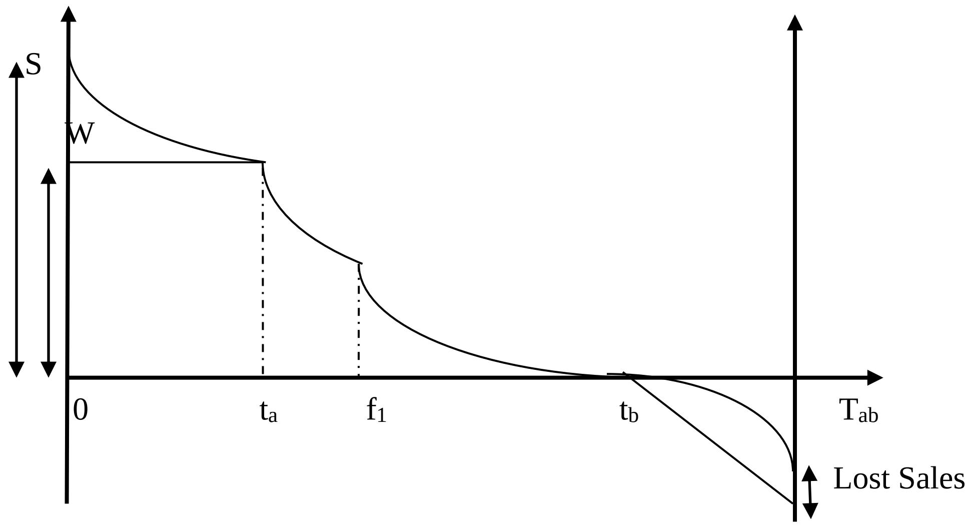


Fig: (2) Inventory level of two warehouse system

Differential equations for rented warehouse for different time intervals

$$\frac{dR_I(t)}{dt} = -(\eta + \phi R_I(t)) \qquad 0 < t < t_a \qquad (3.22)$$

Solving equation (3.22) using boundary condition $R_I(0) = S$

$$R_I(t) = \frac{\eta}{\phi}\left(\frac{1}{e^{\phi t}} - 1\right) + \frac{S}{e^{\phi t}} \qquad (3.23)$$

Differential equations for rented warehouse for different time intervals

$$\frac{dO_I(t)}{dt} = -(\eta + \phi O_I(t)) \qquad t_a < t < f_1 \qquad (3.24)$$

$$\frac{dO_I(t)}{dt} = -(\eta + \phi O_I(t)) - \gamma O_I(t) \qquad f_1 < t < t_b \qquad (3.25)$$

Solving differential equation (3.24) using boundary condition $O_I(t_a) = W$

$$O_I(t) = \frac{\eta}{\phi}(e^{\phi(t_a - t)} - 1) + We^{\phi(t_a - t)} \qquad (3.26)$$

Solving differential equation (3.25) using boundary condition $O_I(t_b) = 0$

$$O_I(t) = e^{(\phi+\gamma)(t_b - t)}\left(\frac{\eta}{\phi+\gamma} - 1\right) \qquad (3.27)$$

Moreover, at time t_b, the level of inventory goes down to zero in own warehouse (OW) and shortages occur. In between the time interval $[t_b, T_{ab}]$, the level of inventory depends on demand, and some fraction of demand got vanished but some fraction

$$\frac{1}{(1+\Delta(T_{ab}-t))} \tag{3.28}$$

is backlogged and where $t \in [t_b, T_{ab}]$. The given below differential equation gives the level of inventory.

$$\frac{dO_I(t)}{dt} = \frac{\eta}{1+\Delta(T_{ab}-t)} \tag{3.29}$$

Using the boundary condition $O_I(t_b) = 0$. We get the following level of inventory as:

$$O_I(t) = -\frac{\eta}{\Delta}\log\left[1+\Delta(T_{ab}-t_b)\right] - \log\left[1+\Delta(T_{ab}-t)\right] \tag{3.30}$$

The total cost per cycle composed of certain constituent is given below:

1. **Ordering cost per cycle**

   OA= Z (3.31)

2. **Cost of holding goods per cycle in rented warehouse (RW)**

$$HC_{RW} = H_1\left\{\int_0^{t_a} R_I(t)dt\right\} = H_1\left[\frac{\eta+\phi W}{\phi^2}(1-e^{-\phi t_a}) - t_a\left(\frac{\eta}{\phi}\right)\right] \tag{3.32}$$

3. **Cost of holding goods per cycle in own warehouse (OW)**

$$HC_{OW} = H_2\left\{\int_0^{t_a} O_I(t)dt + \int_{t_a}^{f_1} O_I(t)dt + \int_{f_1}^{t_b} O_I(t)dt\right\}$$

$$= -H_2\left[W + \frac{\eta+\phi W}{\phi^2}\left(1-e^{\phi(t_a-f_1)}\right) + (1-e^{(\phi+\gamma)(t_b-f_1)})\left(\frac{\phi+\gamma-\eta}{(\phi+\gamma)^2}\right)\right] \tag{3.33}$$

4. **Cost of shortages per cycle**

$$SC = -C\left\{\int_{t_b}^{T_{ab}} O_I(t)dt\right\} = \frac{C\eta}{\Delta^2}\left\{\Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)]\right\} \tag{3.34}$$

5. **Lost sales per cycle**

$$LS = L\eta \int_{t_b}^{T_{ab}} \left\{ 1 - \frac{1}{[1+\Delta(T_{ab}-t)]} \right\} dt = \frac{L\eta}{\Delta} \{ \Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)] \} \tag{3.35}$$

6. **Cost of deterioration per cycle in own warehouse (OW)**

$$DC_{OW} = D_2 \left\{ \int_{f_1}^{t_b} \theta O_I(t) dt \right\} = D_2 \left[\frac{\theta(1-\eta)}{\phi+\gamma} (1 - e^{(\phi+\gamma)(t_b - f_1)}) \right] \tag{3.36}$$

Total Cost for time interval [0, T_{ab}]

TC_2 (0, T_{ab}) = OA+ $HC_{RW} + HC_{OW} + SC + LS + DC_{OW}$

$$\begin{aligned} &= Z + H_1 \left[\frac{\eta+\phi W}{\phi^2} (1 - e^{-\phi t_a}) - t_a \left(\frac{\eta}{\phi} \right) \right] \\ &\quad - H_2 \left[W + \frac{\eta+\phi W}{\phi^2} \left(1 - e^{\phi(t_a - f_1)} \right) + (1 - e^{(\phi+\gamma)(t_b - f_1)}) \left(\frac{\phi+\gamma-\eta}{(\phi+\gamma)^2} \right) \right] \\ &\quad + \frac{C\eta}{\Delta^2} \{ \Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)] \} + \frac{L\eta}{\Delta} \{ \Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)] \} \\ &\quad + D_2 \left[\frac{\theta(1-\eta)}{\phi+\gamma} (1 - e^{(\phi+\gamma)(t_b - f_1)}) \right] \end{aligned} \tag{3.37}$$

Total Cost per unit time

$$TVC_2 (t_1, t_2) = \frac{TC_2(t_1, t_2)}{T}$$

$$= \frac{1}{T_{ab}} \begin{bmatrix} Z + H_1 \left[\frac{\eta+\phi W}{\phi^2} (1 - e^{-\phi t_a}) - t_a \left(\frac{\eta}{\phi} \right) \right] \\ -H_2 \left[W + \frac{\eta+\phi W}{\phi^2} \left(1 - e^{\phi(t_a - f_1)} \right) + (1 - e^{(\phi+\gamma)(t_b - f_1)}) \left(\frac{\phi+\gamma-\eta}{(\phi+\gamma)^2} \right) \right] \\ + \frac{C\eta}{\Delta^2} \{ \Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)] \} + \frac{L\eta}{\Delta} \{ \Delta(T_{ab}-t_b) - \log[1+\Delta(T_{ab}-t_b)] \} \\ +D_2 \left[\frac{\theta(1-\eta)}{\phi+\gamma} (1 - e^{(\phi+\gamma)(t_b - f_1)}) \right] \end{bmatrix} \tag{3.38}$$

4. NUMERICAL EXAMPLES

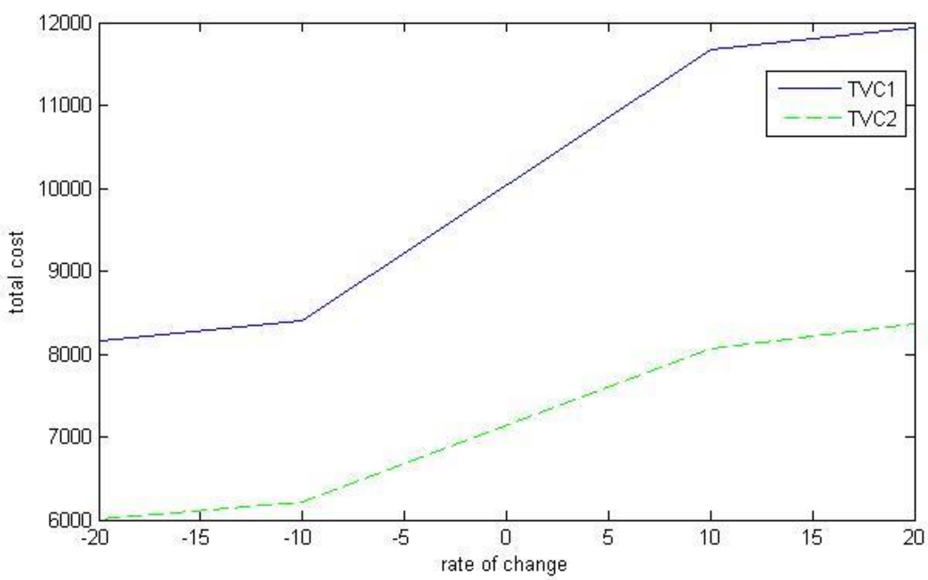
For CASE (1): When $0 \le f_1 \le t_a$

We assume the values as $\eta = 570$, $\phi = 22.8$, Z = 2000, $H_2 = 12$, W= 200, $\gamma = 0.06$, $\theta = 0.08$, s = 30, $D_1 = D_2 = 200$, $\Delta = 0.08$, L = 15, $t_a = 0.2228$, $t_b = 0.8818$, $f_1 = 0.1912$, $T_{ab} = 1.1047$ yrs. After calculation the optimum value of $TVC_1 = 10093.4542$.

Sensitive analysis of holding cost for own warehouse (H_1)

Table 1

Parameter change in %age	TVC_1`	Change in total cost in %age	TVC_2	Change in total cost in %age
-20	8160.1542	-1.9333	6016.5236	-1.1673
-10	8407.7542	-1.6857	6207.5236	-0.9763
10	11664.0542	1.5706	8067.0236	0.8832
20	11926.5542	1.8331	8360.4236	1.1766



Observations from Table (1):

1. From table (1) it can be stated that cost per unit time changes (increases/decreases) with the change (increases/decreases) in holding cost of the own warehouse.

2. From the graph it is clearly shown that the cost per unit time increase gradually in the above (both) cases with the increase in the value of holding cost of the own warehouse.
3. The value of TVC_1 is always larger than the value of TVC_2 for all the cases whether the holding cost increases or decreases.
4. It's clearly observed that it is profitable to have larger extend of the fixed self-life.

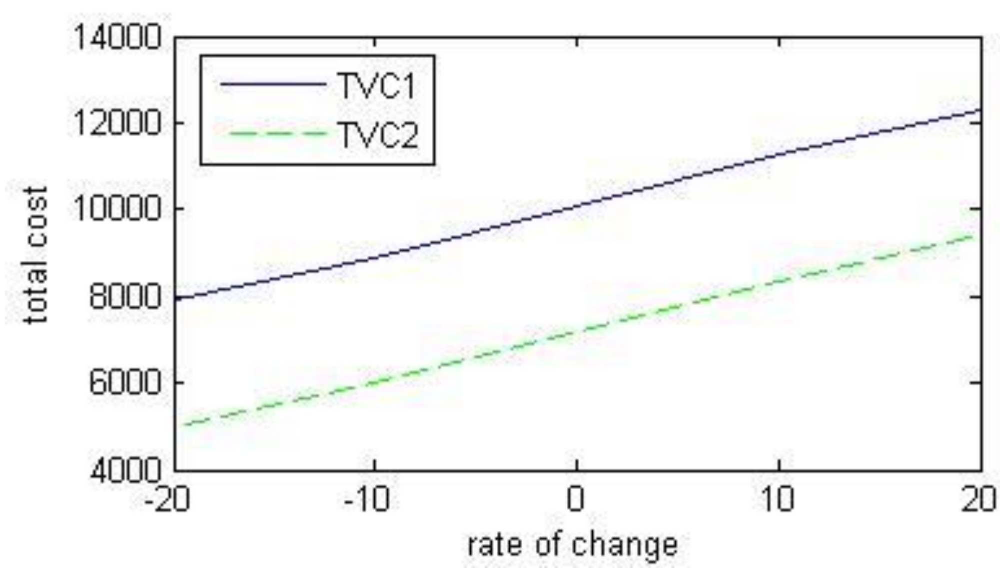
For CASE (2): When $t_a \leq f_1 \leq t_b$

We assume the values as $\eta = 570$, $\phi = 22.8$, Z = 2000, $H_2 = 12$, W= 200, $\gamma = 0.06$, $\theta = 0.08$, s = 30, $D_1 = D_2 = 200$, $\Delta = 0.08$, L = 15, $t_a = 0.1673$, $t_b = 0.8818$, $f_1 = 0.3567$, $T_{ab} = 1.1047$ yrs. After calculation the optimal value of $TVC_2 = 7183.8236$.

Sensitive analysis of holding cost for rented warehouse (H_2)

Table 2

Parameter change in %age	TVC_1`	Change in total cost in %age	TVC_2	Change in total cost in %age
-20	7888.9542	-2.2045	4991.5236	-2.1923
-10	8909.9542	-1.1835	5978.4236	-1.2054
10	11250.6542	1.1572	8363.6236	1.1798
20	12290.1542	2.1967	9400.5236	2.2167



Observations from Table (2):

1. From table (2) it can be stated that the cost per unit time changes (increases/decreases) with the change (increases/decreases) in holding cost of the rented warehouse.
2. From the graph it is clearly shown that the cost per unit time increase gradually in the above cases with the increase in the value of holding cost of the rented warehouse.
3. The value of TVC_1 is always greater than the value of TVC_2 for all the cases whether the value of holding cost of the rented warehouse increases or decreases.
4. It's clearly observed that it is profitable to have larger extend of the fixed self-life.

CONCLUSION

In this paper, we considered an inventory system with two storage houses i.e. own warehouse (OW) and rented warehouse (RW), namely. We calculate the effect of shelf life and holding costs of both the warehouses and observe their effect on the inventory model. Shelf life is the recommendation of time period up to which the product remains acceptable under expected conditions of distribution, storage and display. Such known time period is very much useful in consumption of commodities like fruits, vegetables, medicines, etc. From above numerical examples we can say that longer the shelf life period the cost decreases. The optimal cost hence obtained is the one we get when the shelf life time period in longer.

The effect of the modification in the value of cost of cost of own warehouse (OW) is shown in the table1 followed by the graph and its observation is made. Similarly, the effect of the modification in the value of holding cost of rented warehouse is shown in table2 followed by the graph and them its observations. By these observations we can easily say that with the increase of the holding costs of the warehouses the optimal cost increases and vice versa.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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