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NEIGHBOR SUM DISTINGUISHING TOTAL CHOOSABILITY OF PLANAR GRAPHS WITHOUT 4-CYCLES ADJACENT TO 3-CYCLES

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Abstract. Let ϕ be a proper total coloring of a graph G with integers as colors. For a vertex v, let $w(v)$ denote the sum of colors assigned to edges incident to v and the color assigned to v. If $w(u) \neq w(v)$ whenever $uv \in E(G)$, then ϕ is called a *neighbor sum distinguishing total coloring*. A k-assignment L of G is a list assignment L of integers to vertices and edges with $|L(z)| = k$ for each $z \in V(G) \cup E(G)$. A *total-L-coloring* is a total coloring ϕ of G such that $\phi(v) \in L(v)$ whenever $v \in V(G)$ and $\phi(e) \in L(e)$ whenever $e \in E(G)$. The smallest integer k such that G has a neighbor sum distinguishing total-L-coloring for every k-assignment L is called the neighbor sum distinguishing total choosability of G and is denoted by $Ch''_{\Sigma}(G)$. Wang, Cai, and Ma [15] proved that every planar graph G without 4-cycles with $\Delta(G) \geq 7$ has $Ch''_{\Sigma}(G) \leq \Delta(G)+3$. In this work, we strengthen the result of Wang et al by proving that $Ch''_{\Sigma}(G) \leq \Delta(G)+3$ for every planar graph G without 4-cycles adjacent to 3-cycles with $\Delta(G) \geq 7$.

Keywords: coloring; discharging method; neighbor sum distinguishing total coloring.

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1. INTRODUCTION

We consider only simple, finite, and undirected graphs in this work. For a plane graph G, we use $V(G)$, $E(G), F(G), \delta(G)$, and $\Delta(G)$ to denote the vertex set, edge set, face set, minimum

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degree, and maximum degree of a graph G, respectively. We say that two faces are *adjacent* if their boundaries share an edge.

A k-vertex (face) is a vertex (face) of degree k, a k^+-vertex (face) is a vertex (face) of degree at least k, and a k^--vertex (face) is a vertex (face) of degree at most k. A $(d_1,d_2,\dots,d_k)$*-face* f is a face of degree k where vertices incident to f have degree $d_1,d_2,\dots,d_k$. A k-face f_1 with incident vertices $v_1,v_2\dots,v_k$ in a cyclic order is a *spacial k-face* of a 3-face f_2 if the boundaries of f_1 and f_2 share exactly two vertices v_i and v_{i+1} and at least one of edges $v_{i-1}v_i$ and $v_{i+1}v_{i+2}$ is not incident to a 3-face.

Let $\phi : V(G)\cup E(G) \longrightarrow \{1,\dots,k\}$ be a proper k-total coloring. We denote the sum of colors assigned to edges incident to v and the color on the vertex v by $w(v)$ (i.e., $w(v) = \sum_{uv\in E(G)} \phi(uv)+\phi(v)$). The total coloring ϕ of G is a *neighbor sum distinguishing total coloring* if $w(u)\neq w(v)$ for each edge $uv\in E(G)$. The smallest integer k such that G has a neighbor sum distinguishing total coloring is called the *neighbor sum distinguishing total chromatic number of* G, denoted by $\text{tndi}_\Sigma(G)$.

Pilśniak and Woźniak [7] introduced neighbor sum total coloring and obtained $\text{tndi}_\Sigma(G)$ for cycles, cubic graphs, bipartite graphs, and complete graphs. Furthermore, they posed the following conjecture.

Conjecture 1. [7] If G is a graph with at least two vertices, then $\text{tndi}_\Sigma(G)\leq \Delta(G)+3$.

The conjecture is verified for K_4-minor free graphs by Li, Liu, and Wang [6], for planar graphs with large maximum degrees by Li et al [5], and for triangle free planar graphs with maximum degree at least 7 by Wang, Ma, and Han [16]. The conjecture is also shown to be true for planar graphs with other conditions [2, 3, 4, 8, 13].

A k-assignment L of G is a list assignment L of integers to vertices and edges with $|L(z)|=k$ for each $z\in V(G)\cup E(G)$. A *total-L-coloring* is a total coloring ϕ of G such that $\phi(z)\in L(z)$ when $z\in V(G)\cup E(G)$. We call that G has a *neighbor sum distinguishing total-L-coloring* (or *nsd total-L-coloring*) if G has a total-L-coloring such that $w(u)\neq w(v)$ for each $uv\in E(G)$. The smallest integer k such that G has a neighbor sum distinguishing total-L-coloring for every k-assignment L, denoted by $Ch''_\Sigma(G)$, is called the *neighbor sum distinguishing total choosability* of G.

Qu et al [9] proved that $Ch''_{\Sigma}(G) \leq \Delta(G)+3$ for every planar graph G with $\Delta(G) \geq 13$. Yao et al [17] studied $Ch''_{\Sigma}(G)$ of d-degenerate graphs. More results about the neighbor sum distinguishing total choosability for planar graphs can be seen in [10, 11, 12, 14]. Wang, Cai, and Ma [15] studied the neighbor sum distinguishing total choosability for planar graphs without 4-cycles and proved the following theorem.

Theorem 1. *([15]). If G is a planar graph without 4-cycles with $\Delta(G) \geq 7$, then $Ch''_{\Sigma}(G) \leq \Delta(G)+3$.*

In this paper, we strengthen Theorem 1 by extending the result to planar graphs without 4-cycles adjacent to 3-cycles.

2. Helpful Lemmas

The first lemma is an easy observation about plane graphs without 4-cycles adjacent to 3-cycles.

Lemma 2. *If H is a plane graph without 4-cycles adjacent to 3-cycles, then a 3-face in H is adjacent to neither 4-face nor another 3-face. Consequently, if v is a k-vertex in H, then v is incident to at most $\lfloor \frac{k}{2} \rfloor$ 3-faces.*

The two following lemmas are required to prove the results about minimal counterexamples.

Lemma 3. *([9]). Suppose that m and n are positive integers such that $m \geq n$, and L_i is a set of at least n integers ($i = 1, \dots, m$). Let $T_m(L_1 \dots, L_m) = \{\sum_{i=1}^{m} x_i | x_i \in L_i, i \neq j \implies x_i \neq x_j\}$. Then $T_m(L_1 \dots, L_m) \geq mn - m^2 + 1$.*

Lemma 4. *([1]). Let $\mathbb{F}$ be a field, and let $P = P(x_1, \dots, x_n)$ be a polynomial in $\mathbb{F}[x_1 \dots, x_n]$. Suppose that the degree $\deg(P)$ of P equals $k_1 + \cdots + k_n$ where k_i is a nonnegative integer ($i = 1, \dots, n$), and the coefficient of $\prod_{i=1}^{n} x_i^{k_i}$ in P is nonzero. If $S_1, \dots, S_n$ are subsets of $\mathbb{F}$ with $|S_i| > k_i$, then there are $s_i \in S_i$ ($i = 1, \dots, n$) such that $P(s_1, \dots, s_n) \neq 0$.*

We also use the following helpful observation. For a 3^--vertex v, there are at most 3 adjacent vertices, 3 incident edges, and the sum at v must be different from at most 3 sums at adjacent neighbors. Since $|L(u)| \geq 10$, we may delete the color at u and recolor it later to have an appropriate coloring. Thus we will omit the recoloring of 3^--vertices in subsequent arguments.

Let G be a minimal non $(\Delta+3)$-choosable plane graph (with respect to $|V(G)|+|E(G)|$). Let H be the graph obtained by removing all the 2^--vertices from G. For a vertex v in G, we use $d_G(v)$ (or $d_H(v)$) to denote the degree of v in G (or in H.) We have that the graph H satisfies all the following lemmas regardless of conditions on cycles.

Lemma 5. *(*[15]*).*
(a) $\delta(H) \geq 3$.
(b) Each 4^-*-vertex in H is not adjacent to a* 3*-vertex.*
(c) Each 3*-face in H is either a* $(3,5^+,5^+)$*-face or a* $(4^+,4^+,5^+)$*-face.*

Lemma 6. *If a vertex u has* $d_H(u)=3$*, then* $d_G(u)=3$*.*

Proof. Suppose to the contrary that H has a vertex u with $d_H(u)=3$ but $d_G(u)\geq 4$. It follows that u is adjacent to three 3^+-vertices u_1,u_2,u_3, and t 2^--vertices $v_1,\ldots,v_t$ where $t=d_G(u)-d_H(u)\geq 1$. Let $G'=G-\{uv_1,\ldots,uv_t\}$. Let L be a $(\Delta(G)+3)$-assignment that G has no nsd total-L-coloring. By the minimality of G, there is an nsd total-L-coloring for G' where L is restricted to the graph G'.

(1) $t=1$.

First, we delete the colors on vertices u and v_1.

To extend an nsd total-L-coloring to G, a color for uv_1 must be different from the colors of edges incident to u and v_1. Let S_1 denote the set of legal colors that can be assigned to uv_1. Then we have $|S_1|\geq |L(vu_1)|-4=\Delta(G)-1\geq 6$. Similarly, a color for u must be different from the colors assigned to uu_i and u_i $(i=1,2,3)$. Let S_2 denote the set of legal colors that can be assigned to u. Then we have $|S_2|\geq |L(u)|-6=\Delta(G)-3\geq 4$.

Next, we aim to make the sum obtained at u distinct from the sums at u_1,u_2, and u_3. Let w_0 be the temporary sum at u and let w_i be the sum at u_i $(i=1,2,3)$. We use x_1 for a color assigned to uv_1 and use x_2 for a color assigned to u. Altogether, we want to find x_1 and x_2 such that the following polynomial is non-zero:

$$P(x_1,x_2)=(x_1-x_2)(x_1+x_2+w_0-w_1)(x_1+x_2+w_0-w_2)(x_1+x_2+w_0-w_3).$$

We have $\deg(P) = 4$ and the coefficient of $x_1^3 x_2$ is 2 (calculated by Scilab). By Lemma 4, there exist $x_1 \in S_1$ and $x_2 \in S_2$ such that $P(x_1, x_2) \neq 0$. Finally, we recolor the 2^--vertex v_1 to extend an nsd total-L-coloring to G which contradicts the choice of G.

(2) $t \geq 2$.

We delete the colors on vertices $v_1, \ldots, v_t$. To extend an nsd total-L-coloring to G, a color for uv_i must be different from the colors of edges incident to u and v_i, and from the color of u. Let S_i denote the set of legal colors that can be assigned to uv_i $(i = 1, \ldots, t)$. Then $|S_i| \geq |L(uv_i)| - 5 = \Delta(G) - 2 \geq 5$. It follows from Lemma 3 that $T_t(S_1, \ldots, S_t) \geq 2 \times 4 - 2^2 + 1 = 5$ when $t = 2$, and $T_t(S_1, \ldots, S_t) \geq 3 \times 4 - 3^2 + 1 = 4$ when $t = 3$. Note that $|S_i| \geq \Delta(G) - 2 \geq t + 1$ since $\Delta(G) \geq t + 3$. By Lemma 3, $T_t(S_1, \ldots, S_t) \geq t(t+1) - t^2 + 1 \geq 5$ when $t \geq 4$. Thus we can find $x_i \in S_i$ $(i = 1, \ldots, t)$ that are mutually distinct such that the sum at u is distinct from the sums at u_1, u_2, and u_3. Finally, we recolor the 2^--vertices $v_1, \ldots, v_t$ to extend an nsd total-L-coloring to G which contradicts the choice of G. □

Lemma 7. *Each* 5*-vertex in* H *is adjacent to at most one* 3*-vertex.*

Proof. Suppose to the contrary that H has a 5-vertex v adjacent to 3-vertices u_1 and u_2. Let v_1, v_2, and v_3 be the remaining neighbors of v in H, and let $w_1, \ldots, w_t$ be the 2^--neighbors of v in G where $t = d_G(v) - d_H(v)$.

(1) $t \leq 2$.

Let $G' = G - \{vu_1, vu_2, vw_1, \ldots, vw_t\}$. Let L be a $(\Delta(G)+3)$-assignment that G has no nsd total-L-coloring. By the minimality of G, there is an nsd total-L-coloring for G' where L is restricted to the graph G'.

We delete the colors on vertices $u_1, u_2, w_1, \ldots, w_t$. We use x_i for a color assigned to vu_i $(i = 1, 2)$ and use x_{2+j} for a color assigned to vw_j $(j = 1, \ldots, t)$. To extend an nsd total-L-coloring to G, a color for vu_i where $i = 1, 2$ must be different from the colors of edges vv_1, vv_2, vv_3 and the colors of edges incident to u_i, and the color of the vertex v. Let S_i denote the set of legal colors that can be assigned to vu_i. From Lemma 6, each of u_1 and u_2 has exactly three neighbors in G. Then we have $|S_i| \geq |L(uv)| - 6 = \Delta(G) - 3$. Similarly, a color for vw_j where $j = 1, \ldots, t$ must be different from the colors of edges vv_1, vv_2, vv_3 and the colors of edges incident to uw_j, and the color of the vertex v. Let S_{2+j} denote the set of legal colors that can be assigned to vw_j. Then we have $|S_{2+j}| \geq |L(u)| - 5 = \Delta(G) - 2$.

Next, we aim to make the sum obtained at v distinct from the sums at v_1, v_2, and v_3. Let w_0 be the temporary sum at v and let w_i be the sum at v_i $(i = 1,2,3)$. Altogether, we want to find $x_1, \ldots, x_{2+t}$ such that the following polynomial is non-zero:

$$P(x_1, \ldots, x_{2+t}) = \prod_{1 \leq i < j \leq 2+t} (x_i - x_j) \prod_{i=1}^{3} \left(\sum_{r=1}^{2+t} x_r + w_0 - w_i \right)$$

If $t = 0$, then we have $\deg(P) = 4$ and the coefficient of $x_1^3 x_2$ is 2 (calculated by Scilab). Note that $|S_1|, |S_2| \geq 4$. By Lemma 4, there exist $x_1 \in S_1$ and $x_2 \in S_2$ such that $P(x_1, x_2) \neq 0$.

If $t = 1$, then we have $\deg(P) = 6$ and the coefficient of $x_1^2 x_2 x_3^3$ is 1 (calculated by Scilab). Note that $|S_1|, |S_2| \geq 4$ and $|S_3| \geq 5$. By Lemma 4, there exist $x_1 \in S_1$, $x_2 \in S_2$, and $x_3 \in S_3$ such that $P(x_1, x_2, x_3) \neq 0$.

If $t = 2$, then we have $\deg(P) = 9$ and the coefficient of $x_1^2 x_3^4 x_4^3$ is 1 (calculated by Scilab). Note that $|S_1|, |S_2| \geq 4$ and $|S_3|, |S_4| \geq 5$. By Lemma 4, there exist $x_1 \in S_1$, $x_2 \in S_2$, $x_3 \in S_3$, and $x_4 \in S_4$ such that $P(x_1, x_2, x_3, x_4) \neq 0$.

Thus we can find $x_i \in S_i$ $(i = 1, \ldots, 2+t)$ that are mutually distinct such that the sum at v is distinct from the sums at v_1, v_2, and v_3. Finally, we recolor the 3^--vertices $u_1, u_2, w_1, \ldots, w_t$ to extend an nsd total-L-coloring to G which contradicts the choice of G.

(2) $t \geq 3$.

Let $G' = G - \{vw_1, \ldots, vw_t\}$. Let L be a $(\Delta(G)+3)$-assignment that G has no nsd total-L-coloring. By the minimality of G, there is an nsd total-L-coloring for G' where L is restricted to the graph G'.

We delete the colors on vertices $u_1, u_2, w_1, \ldots, w_t$. We use x_i for a color assigned to vw_i $(j = 1, \ldots, t)$. Let $i = 1, \ldots, t$. To extend an nsd total-L-coloring to G, a color for vw_i must be different from the colors of edges $vu_1, vu_2, vv_1, vv_2, vv_3$ and the colors of edges incident to w_i, and the color of the vertex v. Let S_i denote the set of legal colors that can be assigned to vw_i. Then we have $|S_i| \geq |L(uv)| - 7 = \Delta(G) - 4 \geq (t+5) - 4 = t+1$. By Lemma 3, $T_t(S_1, \ldots, S_t) \geq t(t+1) - t^2 + 1 \geq 4$ when $t \geq 3$. Thus we can find $x_i \in S_i$ $(i = 1, \ldots, t)$ that are mutually distinct such that the sum at u is distinct from the sums at v_1, v_2, and v_3. Finally, we recolor the 3^--vertices $u_1, u_2, w_1, \ldots, w_t$ to extend an nsd total-L-coloring to G which contradicts the choice of G.

□

3. Main Results

Theorem 2. *If G is a planar graph without 4-cycles adjacent to 3-cycles with $\Delta(G) \geq 7$, then $Ch''_{\Sigma}(G) \leq \Delta(G)+3$.*

Proof. Suppose to the contrary that G is a minimal counterexample with respect to $|V(G)| + |E(G)|$. Let the graph H be defined as in the previous section. The initial charge is defined to be $\mu(x) = d(x) - 4$ for each $x \in V(H) \cup F(H)$. Then by Euler's formula and by the Handshaking lemma, we have

$$\sum_{v \in V(H)} \mu(v) + \sum_{f \in F(H)} \mu(f) = -8.$$

Now, we derive a new charge $\mu^*(x)$ for each $x \in V(H) \cup F(H)$ by transferring charge from one element to another and the summation of new charge $\mu^*(x)$ remains -8. If we show that $\mu^*(x) \geq 0$ for each $x \in V(H) \cup F(H)$, then we obtain a contradiction and a counterexample does not exist.

The discharging rules are defined as follows: Let $w(x \to y)$ be the charge transferred from x to y where $x, y \in V(H) \cup F(H)$.

(R1) Let f be a 3-face incident to a vertex u and adjacent to a face g.

(R1.1) If u is a 5-vertex, then $w(u \to f) = \frac{1}{3}$.

(R1.2) If u is a 6^+-vertex, then

$$w(u \to f) = \begin{cases} \frac{1}{3}, & \text{when } f \text{ is a } (3,5^+,5^+)\text{-face,} \\ \frac{2}{3}, & \text{when } f \text{ is a } (4^+,4^+,5^+)\text{-face.} \end{cases}$$

(R1.3) If g is a 5^+-face, then

$$w(g \to f) = \begin{cases} \frac{3}{10}, & \text{when } g \text{ is a special face of } f, \\ \frac{1}{5}, & \text{when } g \text{ is not a special face of } f. \end{cases}$$

(R2) If u is a 5^+-vertex adjacent to a 3-vertex v, then $w(u \to v) = \frac{1}{3}$.

Now, it remains to show that after discharging, the new charge $\mu^*(x) \geq 0$ for all $x \in V(H) \cup$

$F(H)$.

Consider a 3-face f. It follows from Lemma 5(c) that f is a $(3,5^+,5^+)$-face or a $(4^+,4^+,5^+)$-face. Note that all adjacent faces of f are 5^+-faces by Lemma 2. If f be a $(3,5^+,5^+)$-face, then $\mu^*(f) \geq \mu(f) + (3\times\frac{1}{5}) + (2\times\frac{1}{3}) > 0$ by (R1). If f is a $(4^+,4^+,6^+)$-face or a $(4^+,5^+,5^+)$-face, then $\mu^*(f) \geq \mu(f) + (3\times\frac{1}{5}) + \frac{2}{3} = 0$ or $\mu^*(f) \geq \mu(f) + (3\times\frac{1}{5}) + (2\times\frac{1}{3}) > 0$ by (R1), respectively. Suppose that f is a $(4,4,5)$-face. Let v be a 5-vertex incident to f. Let f_1 and f_2 be faces adjacent to f and incident to v. Let a face $g_i \neq f$ $(i=1,2)$ be adjacent to f_i and incident to v. It follows from Lemma 2 that g_1 or g_2 is not a 3-face. Consequently, f_1 or f_2 is a special face of f. Thus $\mu^*(f) \geq \mu(f) + (2\times\frac{1}{5}) + \frac{3}{10} + \frac{1}{3} > 0$ by (R1).

If f is a 4-face, then it does not involve in a discharging process and thus $\mu^*(f) = \mu(f) = 0$.

Consider a k-face f where $k \geq 5$. Assume f is adjacent to the faces $f_1,\ldots,f_k$ in a cyclic order. To calculate $\mu^*(f)$, we redistribute $w(f\to f_i)$ as follows. Let $w(f\to f_i) = \frac{1}{5}$. If f_i is not a 3-face, then we transfer from f_i the charge $\frac{1}{10}$ to f_{i-1} and $\frac{1}{10}$ to f_{i+1} where all subscripts are taken modulo k. Thus if f_i is a 3-face, then it gains charge at least $\frac{1}{5}$, otherwise f_i gains charge at least $\frac{1}{5} - (2\times\frac{1}{10}) = 0$. Moreover if f is a special face k-face of f_i, then f_{i-1} or f_{i+1} is not a 3-face. By the rules of redistribution, f gains charge at least $\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$. Thus the new charge of f is at least $\mu(f) - (k\times\frac{1}{5}) = \frac{4k}{5} - 4 \geq 0$ while its adjacent faces receive charges not less than ones according to (R1.3). This implies that $\mu^*(f) \geq 0$ according to (R1.3).

Consider a vertex v. It follows from Lemma 5(a) that v is a 3^+-vertex. If v is a 3-vertex v, then it follows from Lemma 5(b) that each neighbor of v is a 5^+-vertex. Thus $\mu^*(v) \geq \mu(v) + (3\times\frac{1}{3}) = 0$ by (R2).

If v is a 4-vertex, then it does not involve in a discharging process and thus $\mu^*(v) = \mu(v) = 0$.

If v is a 5-vertex, then v is incident to at most two 3-faces and adjacent to at most one 3-vertex by Lemmas 2 and 7, respectively. Thus $\mu^*(v) \geq \mu(v) - (3\times\frac{1}{3}) = 0$ by (R1.1) and (R2).

Consider a k-vertex v where $k \geq 6$. Let $f_1,\ldots,f_k$ be incident faces of v in a cyclic order and $v_1,\ldots,v_k$ be adjacent vertices of v such that v_i and v_{i+1} are incident to f_i where $i=1,\ldots,k$ and all subscripts are taken modulo k. To calculate $\mu^*(v)$, we redistribute $w(v\to v_i)$ as follows. Let $w(v\to v_i) = \frac{1}{3}$. If v_i is not a 3-vertex but f_{i-1} or f_i is a 3-face, then we transfer $\frac{1}{3}$ from v_i to a 3-face f_{i-1} or a 3-face f_i. By Lemma 2, at most one of f_{i-1} and f_i is a 3-face. It follows that if

v_i is a 3-vertex, then it gains charge $\frac{1}{3}$, otherwise it gains charge at least $\frac{1}{3}-\frac{1}{3}=0$. Consider a 3-face f_i. It follows from Lemma 5(c) that f_i is a $(3,5^+,5^+)$-face or a $(4^+,4^+,5^+)$-face. If f_i is a $(3,5^+,5^+)$-face, then it gains charge $\frac{1}{3}$ from v_i or v_{i+1}. If f_i is a $(4^+,4^+,5^+)$-face, then it gains charge $2\times\frac{1}{3}=\frac{2}{3}$ from v_i and v_{i+1}. Thus the new charge of v is at least $\mu(v)-k\times\frac{1}{3}=\frac{2k}{5}-4\geq 0$ while its incident faces and adjacent vertices receive charges not less than ones according to (R1.2) and (R2). This implies that $\mu^*(v)\geq 0$ according to (R1.2) and (R2).

This completes the proof. □

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] N. Alon, Combinatorial Nullstellensatz, Comb. Probab. Comput. 8 (1999), 7–29.

[2] X. Cheng, D. Huang, G. Wang, J. Wu, Neighbor sum distinguishing total colorings of planar graphs with maximum degree Δ, Discrete Appl. Math. 190-191 (2015), 34-41.

[3] A. Dong, G. Wang, Neighbor sum distinguishing total colorings of graphs with bounded maximum average degree, Acta Math. Sin. 30 (2014), 703-709.

[4] S. Ge, J. Li, C. Xu, Neighbor sum distinguishing total coloring of planar graphs without 5-cycles. Theor. Comput. Sci. 689 (2017), 169-175.

[5] H. Li, L. Ding, B. Liu, G. Wang, Neighbor sum distinguishing total colorings of planar graphs, J. Comb. Optim. 30 (2015), 675-688.

[6] H. Li, B. Liu, G. Wang, Neighbor sum distinguishing total colorings of K_4-minor free graphs, Front. Math. China. 8 (2013), 1351-1366.

[7] M. Pilśniak, M. Woźniak, On the total-neighbor-distinguishing index by sums, Graphs Comb. 31 (2015), 771-782.

[8] C. Qu, G. Wang, J. Wu, X. Yu, On the neighbor sum distinguishing total coloring of planar graphs. Theor. Comput. Sci. 609 (2015), 162-170.

[9] C. Qu, G. Wang, G. Yan, X. Yu, Neighbor sum distinguishing total choosability of planar graphs. J. Comb. Optim. 32 (2016), 906-916.

[10] C. Song, X. Jin, C.Q. Xu, Neighbor sum distinguishing total coloring of IC-planar graphs with short cycle restrictions, Discrete Appl. Math. 279 (2020), 202-209.

[11] W. Song, L. Miao, Neighbor sum distinguishing total choosability of IC-planar graphs, Discuss. Math. Graph Theory, 40 (2020), 331-344.

[12] W. Song, L. Miao, J. Li, Y. Zhao, J. Pang, Neighbor sum distinguishing total coloring of sparse IC-planar graphs. Discrete Appl. Math. 239 (2018), 183-192.

[13] H. Song, W. Pan, X. Gong, C. Xu. A note on the neighbor sum distinguishing total coloring of planar graphs. Theoret. Comput. Sci. 640 (2016), 125-129.

[14] C. Song, C. Xu, Neighbor sum distinguishing total colorings of IC-planar graphs with maximum degree 13. J. Comb. Optim. 39 (2020), 293-303.

[15] J. Wang, J. Cai, Q. Ma, Neighbor sum distinguishing total choosability of planar graphs without 4-cycles. Discrete Appl. Math. 206 (2016), 215-219.

[16] J. Wang, Q. Ma, X. Han, Neighbor sum distinguishing total colorings of triangle free planar graphs. Acta Math. Sin. 31 (2015), 216-224.

[17] J. Yao, X. Yu, G. Wang, C. Xu, Neighbor sum (set) distinguishing total choosability of d-degenerate graphs. Graphs Combin. 32 (2016), 1611-1620.