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STOCHASTIC MODELING OF A REPAIRABLE WARM STANDBY SYSTEM IN THE PRESENCE OF REPAIR MACHINES

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Abstract: Most of the researches done on repairable systems assumed that when the system failed, it is repaired by repairman or repairmen. Little literature is found in the use of repair machines to repair the failed unit. In this study, stochastic analysis of a repairable 2-out-of-4 system is presented. The system comprises of two subsystems A and B arranged in series. Subsystems A and B are two units warm standby. The system is attended by two repair machines assigned to each subsystem to repair any failed unit. Explicit expressions for mean time to system failure (MTSF), the steady-state availability, busy period of repair machines, and profit function of the system are analyzed stochastically using Kolmogorov's forward equation method. Analytical and numerical results giving some particular values to the costs and other parameters have been obtained.

Keywords: Repair machines, availability, mean time to system failure, profit function.

2000 AMS Subject Classification: 90B25

1. Introduction

Reliability is vital for proper utilization and maintenance of any system. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. Studies on redundant system are becoming more and

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richer day by day due to the fact that numbers of researchers in the field of reliability of redundant system are making huge contributions. Models of redundant systems as well as methods of evaluating system reliability indices such as mean time to system failure (MTSF), system availability, busy period of repairman, profit analysis, etc have been studied in order to improve the system effectiveness (see for instant [1,2,3,4] and references therein). Example of such systems are 1-out-of-2, 2-out-of-3,2-out-of-4, or 3-out-of-4 redundant systems. These systems have wide application in the real world. The communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system.

Authors above have analyzed the system under the assumption that repairman is called to repair the failed units/system. However, there are situations where repair machines are employed to repair the failed unit. Example of such situations can be seen in nuclear reactor, marine equipments, etc, [7]. In this study, two repair machines are assigned to each subsystem to repair the failed unit.

In this paper, we construct redundant system and derived its corresponding mathematical models. Furthermore, we study reliability characteristics of the system involving two types of repair machines using Kolmogorov's forward equation method. We derived measures of system effectiveness like MTSF, availability, busy period of repair machines and profit function. Graphical studies of effect of failure rate on the measures mentioned above are also given.

2. Preliminaries

2.1 Notations and Assumptions

Notations

β_1, α_1: Failure and repair rate of unit A_1

β_2, α_2: Failure and repair rate of unit A_2

β_3, α_3: Failure and repair rate of warm standby unit B_2

A_1, A_2 : Operational and cold standby units in subsystem A

B_1, B_2 : Operational and warm standby units in subsystem B

M_1 : Repair Machine I

M_2 : Repair Machine II

Assumptions

1. The system is 2-out-of-4 system
2. The system can be in operation or fail state
3. The system suffer four types of failures
4. The system is down when number of units failure goes beyond one in each subsystem
5. Failure rates and repairs follow exponential
6. Failure rates and repair rates are constant
7. The system is attended by two repair machines
8. Repair Machine cannot fail either in operation or in idle state

2.2 Model descriptions and Formulation

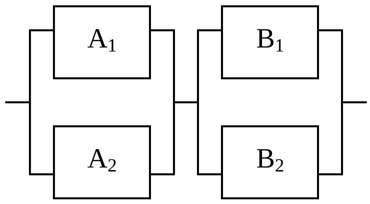

Fig. 1 Reliability block diagram of the system

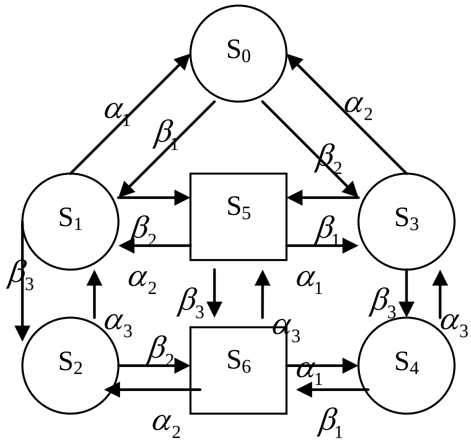

Fig. 2 Transition diagram of the System

State of the system:

S0: Unit A_1 is operational, A_2 is in standby, B_1 is operational, B_2 is in standby. M_1 and M_2 are idle. The system is operational

S1: Unit A_1 is under repair, A_2 is operational, B_1 is operational, B_2 is in standby. M_1 is busy and M_2 is idle. The system is operational

S2: Unit A_1 is under repair, A_2 is operational, B_1 is operational, B_2 is under repair. M_1 and M_2 are busy. The system is operational

S3: Unit A_1 is operational, A_2 is under repair, B_1 is operational, B_2 is in standby. M_1 is busy and M_2 is idle. The system is operational

S4: Unit A_1 is operational, A_2 is under repair, B_1 is operational, B_2 is under repair. M_1 and M_2 are busy. The system is operational

S5: Unit A_1 is under repair, A_2 is waiting for repair, B_1 is good, B_2 is in standby. M_1 is busy and M_2 is idle. The system failed

S6: Unit A_1 is under repair, A_2 is waiting for repair, B_1 is good, B_2 is under repair. M_1 i and M_2 is busy. The system failed

3. Main results

3.1 **Mean time to system failure analysis**

From Fig. 1 above, define $P_i(t)$ to be the probability that the system at time $t, (t \geq 0)$ is in state S_i. Let $P(t)$ be the probability row vector at time t, the initial condition for this paper are

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0, 0]$$

we obtain the following differential equations:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2)P_0(t) + \alpha_1 P_1(t) + \alpha_2 P_3(t)$$

$$\frac{dP_1(t)}{dt} = -(\alpha_1 + \beta_2 + \beta_3)P_1(t) + \beta_1 P_0(t) + \alpha_3 P_2(t) + \alpha_2 P_5(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha_3 + \beta_2)P_2(t) + \beta_3 P_1(t) + \alpha_2 P_6(t)$$

$$\frac{dP_3(t)}{dt} = -(\alpha_2 + \beta_1 + \beta_3)P_3(t) + \beta_2 P_0(t) + \alpha_3 P_4(t) + \alpha_1 P_5(t)$$

$$\frac{dP_4(t)}{dt} = -(\alpha_3 + \beta_1)P_4(t) + \beta_3 P_3(t) + \alpha_1 P_6(t)$$

$$\frac{dP_5(t)}{dt} = -(\alpha_1 + \alpha_2 + \beta_3)P_5(t) + \beta_2 P_1(t) + \beta_1 P_3(t) + \alpha_3 P_6(t)$$

$$\frac{dP_6(t)}{dt} = -(\alpha_1 + \alpha_2 + \alpha_3)P_6(t) + \beta_2 P_2(t) + \beta_1 P_4(t) + \beta_3 P_5(t) \qquad (1)$$

the system of ordinary differential equations can be written in matrix as form as

$$\dot{P} = AP \qquad (2)$$

where

$$A=\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0\\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0\\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2\\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0\\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1\\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3\\ 0 & 0 & \beta_2 & 0 & \beta_1 & \beta_3 & -(\alpha_1+\alpha_2+\alpha_3)\end{bmatrix}$$

It is difficult to evaluate the transient solutions hence following [1,4,5,6] we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q.

The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\int_0^\infty e^{Qt}dt \tag{3}$$

and

$$\int_0^\infty e^{Qt}dt=Q^{-1}\text{ , since } Q^{-1}<0 \tag{4}$$

For system explicit expression for the $MTSF$ is given by

$$E\left[T_{P(0)\to P(absorbing)}\right]=MTSF=P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} \tag{5}$$

Where

$$Q=\begin{bmatrix}-(\beta_1+\beta_2) & \beta_1 & 0 & \beta_2 & 0\\ \alpha_1 & -(\alpha_1+\beta_2+\beta_3) & \beta_3 & 0 & 0\\ 0 & \alpha_3 & -(\alpha_3+\beta_2) & 0 & 0\\ \alpha_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \beta_3\\ 0 & 0 & 0 & \alpha_3 & -(\alpha_3+\beta_1)\end{bmatrix}$$

$$MTSF = \frac{N}{D}$$

$$N = \alpha_1\alpha_2\alpha_3^2 + \alpha_1\alpha_2\alpha_3\beta_1 + \alpha_1\alpha_2\alpha_3\beta_1 + \alpha_1\alpha_2\beta_1\beta_2 + \alpha_1\alpha_3^2\beta_1 + \alpha_1\alpha_3\beta_1\beta_3 + \alpha_1\alpha_3\beta_1^2 + \alpha_1\alpha_3\beta_1\beta_2 + \alpha_1\beta_1\beta_2\beta_3 +$$
$$\alpha_1\beta_1^2\beta_2 + \alpha_2\alpha_3^2\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_2\alpha_3\beta_1\beta_2 + \alpha_2\alpha_3\beta_2^2 + \alpha_2\beta_1\beta_2\beta_3 + \alpha_2\beta_1\beta_2^2 + \alpha_3^2\beta_1\beta_2 + \alpha_3\beta_1^2\beta_2 + \alpha_3\beta_1\beta_2^2 +$$
$$2\alpha_3\beta_1\beta_2\beta_3 + \beta_1^2\beta_2\beta_3 + \beta_1\beta_2\beta_3^2 + \beta_1\beta_2^2\beta_3 + \beta_1^2\beta_2^2 + \beta_1(\alpha_2\alpha_3^2 + \alpha_3\alpha_2\beta_1 + \alpha_2\alpha_3\beta_2 + \alpha_2\beta_1\beta_2 + \alpha_3^2\beta_1 + \alpha_3\beta_1\beta_3 +$$
$$\alpha_3\beta_1^2 + \alpha_3\beta_1\beta_2 + \beta_1\beta_2\beta_3 + \beta_1^2\beta_2) + \beta_1(\alpha_3\beta_1 + \alpha_2\beta_1 + \alpha_2\alpha_3 + \beta_1\beta_3 + \beta_1^2) + \beta_2(\alpha_1\alpha_3^2 + \alpha_1\alpha_3\beta_1 + \alpha_1\alpha_3\beta_2 +$$
$$\alpha_1\beta_1\beta_2 + \alpha_3^2\beta_2 + \alpha_3\beta_2\beta_3 + \alpha_3\beta_1\beta_2 + \alpha_3\beta_2^2 + \beta_1\beta_2\beta_3 + \beta_1\beta_2^2) + \beta_2\beta_3(\alpha_1\alpha_3 + \alpha_1\beta_2 + \alpha_3\beta_2 + \beta_2^2 + \beta_2\beta_3)$$

$$D = 2\beta_1\beta_2\beta_3 + \alpha_2\alpha_3\beta_1 + \alpha_2\beta_1\beta_2 + \alpha_1\alpha_3\beta_1 + \alpha_1\alpha_3\beta_3 + \alpha_1\alpha_3^2 + \alpha_2\alpha_3^2 + 2\alpha_3\beta_1\beta_2 + 2\alpha_3\beta_2\beta_3 + 2\alpha_3\beta_1\beta_3 + \beta_2^2\beta_3$$
$$+\alpha_2\alpha_3\beta_2 + \alpha_3\beta_2^2 + \alpha_3^2\beta_2 + \beta_1\beta_2^2 + \alpha_3\beta_1^2 + \beta_1^2\beta_2 + \beta_1^2\beta_3 + \alpha_3^2\beta_1 + \alpha_1\beta_1\beta_2 + \alpha_1\alpha_3\beta_2 + \beta_1\beta_3^2 + \alpha_2\beta_1\beta_3 + \alpha_2\alpha_3\beta_3$$
$$\beta_2\beta_3^2 + \alpha_1\beta_2\beta_3$$

3.2 **Availability analysis**

For the analysis of availability case of Fig. 1 using the same initial conditions in subsection 3.1 for this problem as:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)]$$
$$= [1, 0, 0, 0, 0, 0, 0]$$

The differential equations can be expressed as

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \end{bmatrix} = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0 \\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2 \\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1 \\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3 \\ 0 & 0 & \beta_2 & 0 & \beta_1 & \beta_3 & -(\alpha_1+\alpha_2+\alpha_3) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix}$$

Following [1,5,6,7] the steady-state availability is given by

$$A(\infty) = 1 - \{P_5(\infty) + P_6(\infty)\} \tag{6}$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP = 0 \tag{7}$$

which in matrix form

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0 \\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2 \\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1 \\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3 \\ 0 & 0 & \beta_2 & 0 & \beta_1 & \beta_3 & -(\alpha_1+\alpha_2+\alpha_3) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) = 1 \tag{8}$$

We substitute (8) in any of the redundant rows in (7) to give

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0 \\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2 \\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1 \\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve for the system of linear equations in the matrix above to obtain the steady-state probabilities

The steady-state availability is given by

$$A(\infty)=\frac{\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_3\beta_2+\alpha_1\beta_2\beta_3+\alpha_2\alpha_3\beta_1+\alpha_2\beta_1\beta_3}{\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_3\beta_2+\alpha_1\beta_2\beta_3+\alpha_3\beta_1\beta_2+\alpha_2\alpha_3\beta_1+\beta_1\beta_2\beta_3+\alpha_2\beta_1\beta_3}$$

3.3 Busy period analysis

Using the same initial condition in subsection 3.1, the differential equations can be expressed as

$$AP=\dot{P} \tag{9}$$

which in matrix form

$$\begin{bmatrix}\dot{P}_0\\ \dot{P}_1\\ \dot{P}_2\\ \dot{P}_3\\ \dot{P}_4\\ \dot{P}_5\\ \dot{P}_6\end{bmatrix}=\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0\\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0\\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2\\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0\\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1\\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3\\ 0 & 0 & \beta_2 & 0 & \beta_1 & \beta_3 & -(\alpha_1+\alpha_2+\alpha_3)\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\end{bmatrix}$$

The steady-state busy period is given by

$$B(\infty)=1-P_0(\infty) \tag{10}$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP=0 \tag{11}$$

which in matrix form

$$\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0\\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0\\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2\\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0\\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1\\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3\\ 0 & 0 & \beta_2 & 0 & \beta_1 & \beta_3 & -(\alpha_1+\alpha_2+\alpha_3)\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

Using the following normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2\left(\infty\right)+P_3\left(\infty\right)+P_4\left(\infty\right)+P_5\left(\infty\right)+P_6\left(\infty\right)=1 \qquad (12)$$

We substitute (12) in any of the redundant rows in (11) to give

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_2+\beta_3) & \alpha_3 & 0 & 0 & \alpha_2 & 0 \\ 0 & \beta_3 & -(\alpha_3+\beta_2) & 0 & 0 & 0 & \alpha_2 \\ \beta_2 & 0 & 0 & -(\alpha_2+\beta_1+\beta_3) & \alpha_3 & \alpha_1 & 0 \\ 0 & 0 & 0 & \beta_3 & -(\alpha_3+\beta_1) & 0 & \alpha_1 \\ 0 & \beta_2 & 0 & \beta_1 & 0 & -(\alpha_1+\alpha_2+\beta_3) & \alpha_3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve for the system of equations in the matrix above to obtain the steady-state probabilities

$P_0(\infty)$

The steady-state busy period is given by

$$B(\infty)=1-P_0(\infty)$$

$$=\frac{\alpha_1\alpha_3\beta_2+\alpha_1\beta_2\beta_3+\alpha_3\beta_1\beta_2+\alpha_2\alpha_3\beta_1+\beta_1\beta_2\beta_3+\alpha_2\beta_1\beta_3}{\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_3\beta_2+\alpha_1\beta_2\beta_3+\alpha_3\beta_1\beta_2+\alpha_2\alpha_3\beta_1+\beta_1\beta_2\beta_3+\alpha_2\beta_1\beta_3}$$

3.4 Profit analysis

Following [1,4,5,6]the expected profit per unit time incurred to the system in the steady-state is given by:

Profit =total revenue from system using - total cost due to repair

$$PF=C_0A(\infty)-C_1B(\infty) \qquad (13)$$

Where PF: is the profit incurred to the system

C_0: is the revenue per unit up time of the system

C_1: is the cost per unit time which the system is under repair

3.5 Numerical Simulations

In this section, we numerically obtain the results for MTSF, availability and profit function for the developed models using the following set of parameter values:

(i) $\alpha_1 = 0.99, \alpha_2 = 0.94, \alpha_3 = 0.83, \beta_2 = 0.5, \beta_3 = 0.99$ for Fig. 3

(ii) $\alpha_1 = 0.2, \alpha_2 = 0.04, \alpha_3 = 0.03, \beta_2 = 0.3, \beta_3 = 0.6$ for Fig. 4

(iii) $\alpha_1 = 0.2, \alpha_2 = 0.04, \alpha_3 = 0.03, \beta_2 = 0.3, \beta_3 = 0.6, C_0 = 1000, C_1 = 100$ for Fig. 5

and vary β_1 for all the figures.

- Fig. 3: shows relation between β_1 and MTSF of the system
- Fig. 4: shows relation between β_1 and availability of the system
- Fig. 5: shows the relation between β_1 and profit function of the system

3.6 Discussion

Using numerical solution with Matlab, we obtained the results depicted in Fig. 3 to 5. Fig. 3 to 5 provides description of MTSF, system availability and profit with respect to β_1 (failure rate of unit A_1). These figures provide description on the effect of β_1 on various measures of system effectiveness such as MTSF, system availability and profit. Fig. 3-5 show that MTSF, system availability, and profit decrease as β_1 increases.

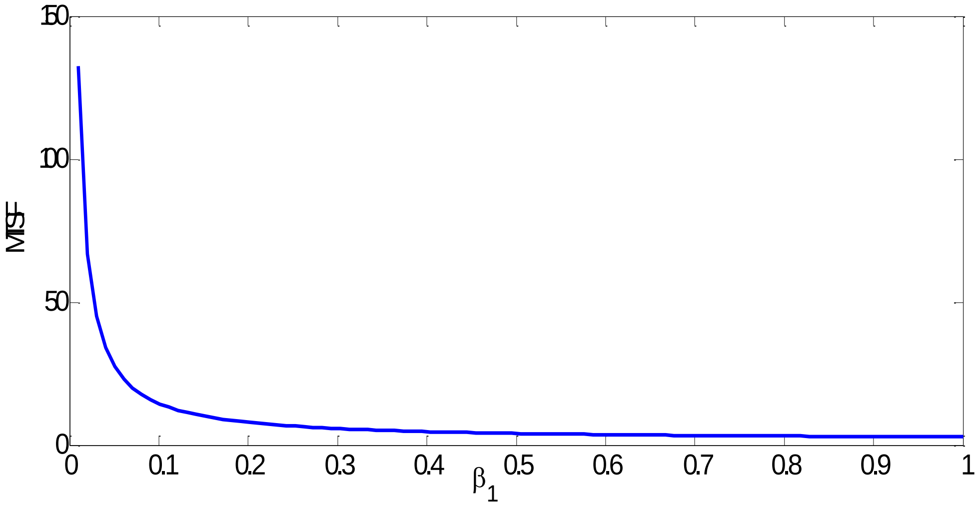

Fig. 3 effect of β_1 on MTSF

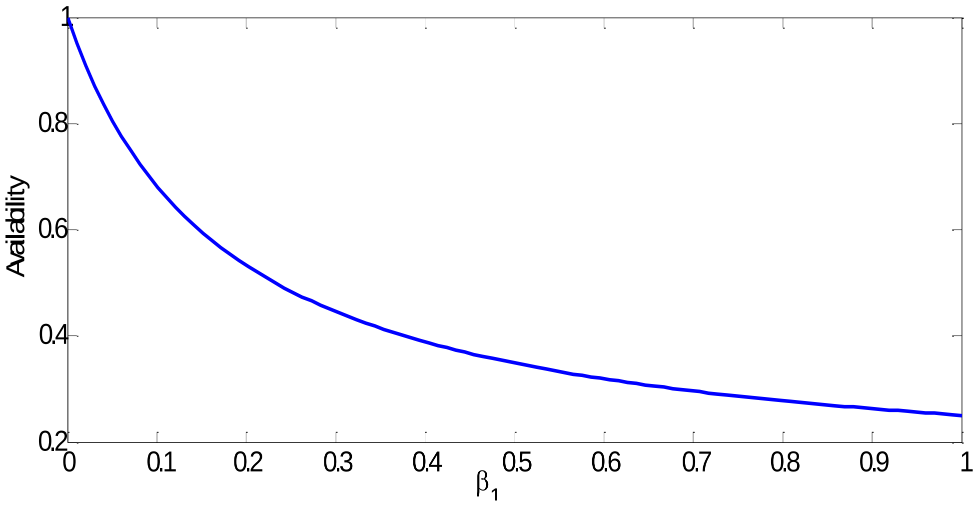

Fig. 4 effect of β_1 on system availability

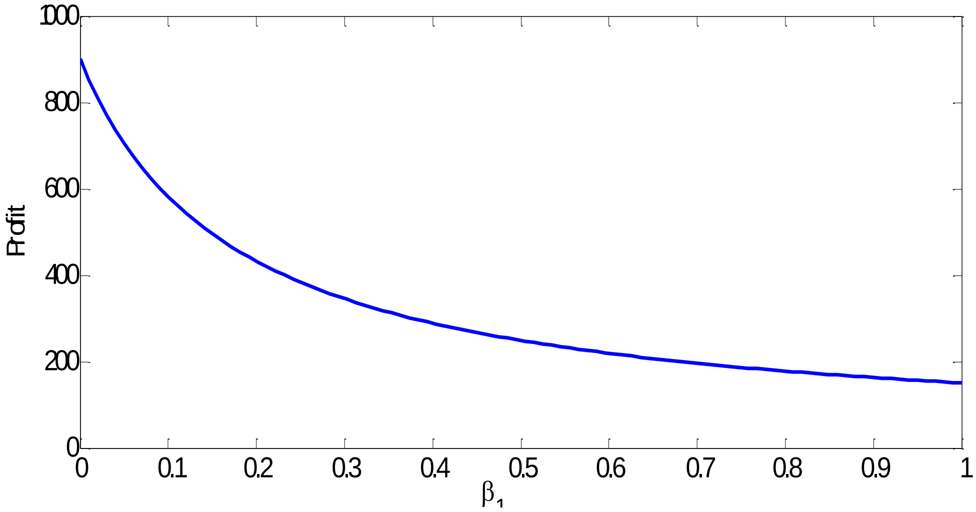

Fig. 5 effect of β_1 on profit

3.7 Conclusion

In this paper we constructed a series system with two subsystems each having two warm standby units. The system is attended by two repair machines each assigned to one subsystem to repair the failed unit. Explicit expressions for various measures such as MTSF, system availability and profit have been developed in the paper. Numerical simulations obtained provide description on the effect of failure rate β_1 (of unit A_1) on mean time to system failure (MTSF), system availability and profit. From the simulations, MTSF, system availability and profit decreases as β_1 increase.

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