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MORE CHARACTERIZATIONS ON P_p−COMPACT SPACES USING GRILLS

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Abstract. In this paper, we introduce a new class of compactness with grill such as $\mathcal{G} - P_p$-compact, $\mathcal{G}-$ strongly compact, $\mathcal{G} - \theta$ compact and $\mathcal{G} - P_S$-compact spaces. Some of their properties and characterizations are obtained. Also, we define and study the concept of $\mathcal{G} - P_P$-compactness spaces under continuous functions.

Keywords: $\mathcal{G} - P_P$ compact space; $\mathcal{G} - P_P$-compact subspaces.

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1. Introduction and Preliminaries

Recently, the characteristics of compactness play a big role in the various applications of topology in various fields Mashhour et al.[4] show and present sets and precontinuous functions. In 2014, Khalaf and Mershkhan [2] inserted P_P-open sets, which are more comprehensive preopen sets, for the purpose of create a profile for P_P-continuous functions. Jafari [16] present the imagine for $\theta-$compact spaces. Mashhour et al. [5] give the imagine for comprehensive compact spaces. Category P_P-compact spaces strictly falls between categories of heavily compact space and $\theta-$compact space, but not balance the compact space. A (Ω, τ) and $(\mathfrak{Y}, \sigma)$

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represent topological spaces $\mathcal{TS}$s without separation axioms are presumably unless otherwise. $\mu \subseteq \Omega$ is called preopen [4] (resp., semi-open [13] and $\alpha-$open [15]) if $\mu \subseteq Int(Cl(\mu))$ (resp., $\mu \subseteq Cl(Int(\mu))$ and $\mu \subseteq Int(Cl(Int(\mu))))$. Supplements for these groups are found in these references. $\mu \subseteq \Omega$ is preclopen[3]. Moreover $\mu \subseteq \Omega$ is $\theta-$open [14] if $\forall \epsilon \in \mu$, $\exists$ an open set μ : $\epsilon \in \mu \subseteq Cl(\mu) \subseteq \mu$. A preopen subset μ of Ω is P_p-open [2] (resp., P_S-open [1]) if $\forall \epsilon \in \mu$, $\exists$ a preclosed (resp., semi-closed) set $\eta : \epsilon \in \eta \subseteq \mu$. The supplementing to of a P_p-open set is a P_p-closed. A $\mu \subseteq \Omega$ is pre-regularopen [9] if $\mu = pInt(pCl(\mu))$. The comprehensive set of all preopen (resp., pre-regularopen, $\theta-$open, P_p-open and P_S-open) of Ω referred to.

In this paper, the main purpose is to present new types of compactness with grill such as $\mathcal{G} - P_p$ compact, $\mathcal{G}-$ strongly compact, $\mathcal{G} - \theta$ compact and $\mathcal{G} - P_S$ compact spaces and some of their characterizations are obtained. Also, the concept of $\mathcal{G} - P_P$ compactness spaces under continuous functions are discussed.

Definition 1.1. [18] *Let $\mathbb{U} \subseteq \Omega$ and $\ell \in \Omega$. Then, $\mathbb{U}$ is called a pre-neighbourhood (pre-nbd, for short) of ℓ in Ω if there exists $\mu \in PO(\Omega)$ such that $\ell \in \mu \subseteq \mathbb{U}$.*

Definition 1.2. [10] *A nonempty subcollection $\mathcal{G}$ of $\mathcal{S}$ which carries a topology τ is called a grill on $\mathcal{S}$ if the following are satisfied:*

(1) $\phi \notin \mathcal{G}$,
(2) *If $\xi \in \mathcal{G}$ and $\xi \subseteq \upsilon \subseteq \mathcal{S}$, then $\upsilon \in \mathcal{G}$,*
(3) *If $\xi \cup \upsilon \in \mathcal{G}$ for $\xi, \upsilon \subseteq \mathcal{S}$, then $\xi \in \mathcal{G}$ or $\upsilon \in \mathcal{G}$.*

Grill depends on the two functions Φ and Ψ which are generated a unique a grill topological structure (briefly, $\mathcal{GTS}$) that is finer than τ on $\mathcal{S}$. It is denoted by $\tau_{\mathcal{G}}$ and is discussed in [7, 8].

Definition 1.3. [7] *Let $(\Omega, \Im)$ be a $\mathcal{TS}$ and $\mathcal{G} \subseteq \Omega$. A function $\Phi : \mathcal{P}(\Omega) \rightarrow \mathcal{P}(\Omega)$, where $\mathcal{P}(\Omega)$ is the power set of Ω, is defined by $\Phi(\mu) = \Phi_{\mathcal{G}}(\mu, \Im) = \{\ell \in \Omega : \mu \cap \mathbb{U} \in \mathcal{G}\} \ \forall \ \mathbb{U} \in \Im(\Omega)$ and $\mu \in \mathcal{P}(\Omega)$. Φ is called the operator associated with $\mathcal{G}$ and $\Im$.*

Definition 1.4. [11] *Let $\mathcal{G}$ be define on a $\mathcal{TS}$ $(\Omega, \Im)$. $\exists \ \Im_{\mathcal{G}}$ on Ω is given by $\Im_{\mathcal{G}} = \{\mathbb{U} \subseteq \Omega :$ $\Psi(\Omega \setminus \mathbb{U}) = \Omega \setminus \mathbb{U}\}, \forall \ \mu \subseteq \Omega, \Psi(\mu) = \mu \cup \Phi(\mu)$.*

Theorem 1.5. [7] *Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two grills on (Ω, Γ). Then,*

(1) *If* $\mathcal{G}_1 \subseteq \mathcal{G}_2$, *then* $\Gamma_{\mathcal{G}_1} \subseteq \Gamma_{\mathcal{G}_2}$.

(2) *If* $\mathcal{G} \subseteq (\Omega, \Gamma)$ *and* $\mathcal{B} \notin \mathcal{G}$, *then* $\mathcal{B}$ *is closed in* $(\Omega, \Gamma_{\mathcal{G}})$.

(3) *For any subset* $\mathcal{A} \subseteq (\Omega, \Gamma)$ *and any* $\mathcal{G}$ *on* Ω, *then* $\Phi(\mathcal{A})$ *is* $\Gamma_{\mathcal{G}}$*-closed.*

Remark 1.6. [11] *Let* (Θ, Γ) *be a* $\mathcal{TS}$. *Then,* $\beta(\mathcal{G}, \Gamma) = \{\mathbb{U} \setminus \mathcal{A} : \mathbb{U} \in \Gamma$ *and* $\mathcal{A} \notin \mathcal{G}\}$ *is obviously an open base for* $\Gamma_{\mathcal{G}}$.

Corollary 1.7. [11] *For any grill* $\mathcal{G}$ *on a* $\mathcal{TS}$ $(\Theta, \mathfrak{I})$, $\mathfrak{I} \subseteq \beta(\mathcal{G}, \mathfrak{I}) \subseteq \mathfrak{I}_{\mathcal{G}}$.

Definition 1.8 ([8, 12, 17])**.** *A subset* ζ *of a space* Ω *which carries topology* τ *with grill* $\mathcal{G}$ *is said to be:*

(1) $\mathcal{G}-$*open or* $\Phi-$*open, if* $\zeta \subseteq int(\Phi(\zeta))$,

(2) $\mathcal{G}-$*regular if* $Int(\Psi(\zeta)) = \zeta$,

(3) $\mathcal{G}-$*regular open if* $Int(\Psi(\zeta)) = Int(\zeta)$,

(4) $\mathcal{G} - \alpha$*-open, if* $\zeta \subseteq int(\Psi(int(\zeta)))$,

(5) $\mathcal{G}-$*preopen, if* $\zeta \subseteq int(\Psi(\zeta))$,

(6) $\mathcal{G}-$*semiopen, if* $\zeta \subseteq \Psi(int(\zeta))$,

(7) $\mathcal{G} - \beta$*-open, if* $\zeta \subseteq cl(int(\Psi(\zeta)))$.

The family of all $\mathcal{G}-$*open (resp.* $\mathcal{G} - \alpha$*-open,* $\mathcal{G}-$*preopen,* $\mathcal{G}-$*semiopen,* $\mathcal{G} - \beta$*-open) sets in a* $\mathcal{GTS}$ (Ω, τ, G) *is denoted by* $\mathcal{G}O(\Omega)$ *(rep.* $\mathcal{G}\alpha O(\Omega)$, $\mathcal{G}PO(\Omega)$, $\mathcal{G}SO(\Omega)$, $\mathcal{G}\beta O(\Omega)$*).*

Proposition 1.9. [8] *Every* $\mathcal{G}-$*open or* $\Phi-$*open set* $\mathcal{A}$ *is* $\mathcal{G}-$*preopen.*

Definition 1.10. [6] *Let* $\mathcal{G}$ *on a* $(\mathcal{X}, \tau)$ *be a cover* $\{\zeta_\gamma : \gamma \in \Delta\}$ *of* $\mathcal{X}$. *Then* $\mathcal{X}$ *is said to be a* $\mathcal{G}-$*cover if* $\exists$ *a finite subset* $\Delta_0 : \Delta_0 \subseteq \Delta$, $\mathcal{X} \setminus \bigcup_{\gamma \in \Delta_0} \zeta_\gamma \notin \mathcal{G}$. *A cover which is not a* $\mathcal{G}-$*cover of* $\mathcal{X}$ *is named a* $\mathcal{G}^\star-$*cover.*

Definition 1.11. [6] *A* $\mathcal{GTS}$ $(\Omega, \Gamma, \mathcal{G})$ *is* $\mathcal{G}-$*compact if* $\forall$ *open cover of* Ω *is a* $\mathcal{G}-$*cover*

2. $\mathcal{G} - P_P$ Compact Space and Some Types of $\mathcal{G}-$Compacts

Definition 2.1. *Let* $(\Omega, \mathfrak{I}, \mathcal{G})$ *be* $\mathcal{GTS}$ *and* $\mathcal{A} \subseteq \Omega$. *Then,* $\mathcal{A}$ *is called:*

(1) $\mathcal{G}-$*pre-neighbourhood (*$\mathcal{G}-$*pre-nbd for short) of* x *in* Ω *if* $\exists$ $\mathcal{B} \in \mathcal{G}PO(\Omega) : x \in \mathcal{B} \subseteq \mathcal{A}$.

(2) $\mathcal{G}-$*pre regularopen if* $\mathcal{A} = Pint(P\Psi(\mathcal{A}))$, *such that* $P\Psi(\mathcal{A}) = \cap\{\mu \supseteq \mathcal{A} : \mu \supseteq \Psi(int(\mu))\}$.

(3) $\mathcal{G}-\theta$ *open if* $\forall x \in \mathcal{A}$, $\exists$ *an* $\mathcal{G}-$*open set* $\mu : x \in \mu \subseteq \Psi(\mu) \subseteq \mathcal{A}$.

(4) $\mathcal{G}-P_p$ *open if* $\forall x \in \mathcal{A} \in \mathcal{G}PO(\Omega)$ $\exists$ *a* $\mathcal{G}-$*preclosed set* λ *in* Ω *such that* $x \in \lambda \subseteq \mathcal{A}$. *The complement of a* $\mathcal{G}-P_p$ *open set is a* $\mathcal{G}-P_p$ *closed.*

(5) $\mathcal{G}-P_S$ *open if* $\forall x \in \mathcal{A} \in \mathcal{G}PO(\Omega)$ $\exists$ $\mathcal{G}-$*semiclosed set* λ *in* Ω *such that* $x \in \lambda \subseteq \mathcal{A}$.

(6) $\mathcal{G}-$*preclopen if* $\mathcal{A}$ *is both* $\mathcal{G}-$*preopen and* $\mathcal{G}-$*preclosed. The class of all* $\mathcal{G}-$*preopen (resp.,* $\mathcal{G}-$*pre-regularopen,* $\mathcal{G}-\theta$ *open,* $\mathcal{G}-P_p$ *open,* $\mathcal{G}-P_S$ *open,* $\mathcal{G}-$*semiclosed and* $\mathcal{G}-$*preclosed of* Ω *is denoted by* $\mathcal{G}PO(\Omega)$*(resp.,*$\mathcal{G}PRO(\Omega)$, $\mathcal{G}\theta O(\Omega)$, $\mathcal{G}P_PO(\Omega)$, $\mathcal{G}P_SO(\Omega)$, $\mathcal{G}SC(\Omega)$ *and* $\mathcal{G}PC(\Omega)$).

Remark 2.2.

(1) *Each* $\mathcal{G}-$ *regularopen set is* $\mathcal{G}-P_S$ *open.*

(2) *Each* $\mathcal{G}-\theta$ *open set is* $\mathcal{G}-P_S$ *open.*

From Definition 2.1 and Remark 2.2 we have the following implication diagram holds, where no other implication than those displayed, is true in general.

$$\begin{array}{ccccc} \mathcal{G}-P_SO(\Omega) & \Longleftarrow & \mathcal{G}-\theta O(\Omega) & \Longrightarrow & \mathcal{G}-P_PO(\Omega) \\ & & \Downarrow & & \Downarrow \\ & & \mathcal{G}-O(\Omega) & \Longrightarrow & \mathcal{G}-PO(\Omega) \end{array}$$

The reverses of the above implication are not verified, in general. These can be shown in the following examples.

Example 2.3. *Let* $\Omega = \{\imath, \jmath, \ell\}$ *with* $\tau = \{X, \phi, \{\imath\}, \{\ell\}, \{\imath, \jmath\}, \{\imath, \ell\}\}$. *If* $\mathcal{G}$ *is grill on* Ω *such that* $\mathcal{G} = \{\Omega, \{\imath\}, \{\imath, \jmath\}\}$. *Then,* $\mathcal{G}O(\Omega) = \{\Omega, \phi, \{\imath\}, \{\imath, \jmath\}\}$, $\mathcal{G}PO(\Omega) = \{\Omega, \phi, \{\imath\}, \{\ell\}, \{\imath, \jmath\}, \{\imath, \ell\}\}$, $\mathcal{G}PC(\Omega) = \{\Omega, \phi, \{\ell\}, \{\jmath\}, \{\imath, \jmath\}, \{\jmath, \ell\}, \{\imath, \ell\}\}$ *and* $\mathcal{G}P_PO(\Omega) = \{\Omega, \phi, \{\ell\}, \{\imath, \jmath\}, \{\imath, \ell\}\}$. *Then* $\{\imath\} \in \mathcal{G}PO(\Omega)$, *but* $\{\imath\} \notin \mathcal{G}P_PO(\Omega)$. *Also* $\{\ell\}, \{\imath, \ell\} \in \mathcal{G}PO(\Omega)$ *but* $\{\ell\}, \{\imath, \ell\} \notin \mathcal{G}O(\Omega)$.

Example 2.4. *From Example 2.3.* $\mathcal{G}O(\Omega) = \{\Omega, \phi, \{\imath\}, \{\imath, \jmath\}\}$, *then* $\mathcal{G}\theta O(\Omega) = \{\Omega, \phi, \{\imath, \jmath\}\}$, *also* $\mathcal{G}P_PO(\Omega) = \{\Omega, \phi, \{\ell\}, \{\imath, \jmath\}, \{\imath, \ell\}\}$. *Hence* $\{\ell\}, \{\imath, \ell\} \in \mathcal{G}P_PO(\Omega)$ *but* $\{\ell\}, \{\imath, \ell\} \notin \mathcal{G}\theta O(\Omega)$. *Also* $\{\imath\} \in \mathcal{G}O(\Omega)$ *but* $\{\imath\} \notin \mathcal{G}\theta O(\Omega)$.

Example 2.5. *From Example 2.3.* $\mathcal{G}SC(\Omega)) = P(\Omega)$ *and* $\mathcal{G}PO(\Omega) = \{\Omega, \phi, \{\imath\}, \{\ell\}, \{\imath, \jmath\}, \{\imath, \ell\}\}$, *implies that* $\mathcal{G}P_SO(\Omega) = \{\Omega, \phi, \{\imath\}, \{\ell\}, \{\imath, \jmath\}\ , \{\imath, \ell\}\}$ *and* $\mathcal{G}\theta O(\Omega) = \{\Omega, \phi, \{\imath, \jmath\}\}$. *Hence* $\{\imath\}, \{\ell\}, \{\imath, \ell\} \in \mathcal{G}P_SO(\Omega)$ *but* $\{\imath\}, \{\ell\}, \{\imath, \ell\} \notin \mathcal{G}\theta O(\Omega)$.

Definition 2.6. *Let* $(\Omega, \Gamma, \mathcal{G})$ *be* $\mathcal{GTS}$. *Then, a space* Ω *is called:*

(1) *Grill locally indiscrete space (*$\mathcal{G}LI_D$ *space, for short) if* $\forall\mathcal{G}-$*open subset of* Ω *is* $\mathcal{G}-$*closed.*
(2) *Pre-*$\mathcal{G}T_1$ *space if* $\forall x \neq y \in \Omega, \exists$ *two* $\mathcal{G}-$*preopen sets* $\eta, \rho : x \in \eta, y \notin \eta$ *and* $y \in \rho, x \notin \rho$.
(3) *Grill preregular space(*$\mathcal{G}PR$ *space for short) if* $\forall$ $\mathcal{G}-$*preclosed* ω *and* $\forall x \notin \omega, \exists$ *disjoint* $\mathcal{G}-$*preopen sets* η, ρ *and* $\eta \cap \rho = \phi : x \in \eta$ *and* $\omega \subseteq \rho$.

Lemma 2.7. *A* $(\mathcal{X}, \Im, \mathcal{G})$ *is* $\mathcal{G}PR$ *space iff* $\forall x \in \mathcal{X}$ *and* $\forall\ \mu \in \mathcal{G}PO(\mathcal{X})\exists\ \eta \in \mathcal{G}PO(\mathcal{X})$ *such that* $x \in \eta \subseteq P\Psi(\eta) \subseteq \mu$.

Proof. From Definition 2.6(3). □

Theorem 2.8. *Let* $(\Omega, \tau, \mathcal{G})$ *be* $\mathcal{GTS}$. *Then, a space* Ω *is Pre-*$\mathcal{G}T_1$, *iff the singleton set* $\{\ell\}$ *is* $\mathcal{G}-$*preclosed* $\forall\ \ell \in \Omega$.

Proof. $(\Rightarrow)$: Let a $\mathcal{GTS}$ $(\Omega, \Im, \mathcal{G})$ be Pre-$\mathcal{G}T_1$ and $\{\ell\}$ be $\mathcal{G}-$preclosed set, $\forall\ell \in \Omega$ implies that $\Omega \setminus \{\ell\}$ is a $\mathcal{G}-$pre-nbd of each of its points, $y \in \Omega \setminus \{\ell\}$ and by Definition 2.6(2) for each $\ell \neq y \in \Omega \exists$ a $\mathcal{G}-$preopen set $\mu : y \in \mu$ and $\ell \notin \mu$, then $y \in \mu \subseteq \Omega \setminus \{\ell\}$, this leads us to $\Omega \setminus \{\ell\}$ is a $\mathcal{G}-$pre-nbd of y, it follows that $\Omega \setminus \{\ell\}$ is $\mathcal{G}-$preopen set in Ω and hence $\{\ell\}$ is preclosed.
$(\Leftarrow)$: Let $\{\ell\}$ be $\mathcal{G}-$preclosed set, for each $\ell \in \Omega$, $y \neq z \in \Omega$. Then, $\{y\}$ is $\mathcal{G}-$preclosed set also in Ω it follows that $\Omega \setminus \{y\}$ is $\mathcal{G}-$preopen set and which contains z but not y. Also $\{z\}$ is $\mathcal{G}-$preclosed set in Ω and $\Omega \setminus \{z\}$ is $\mathcal{G}-$preopen set in Ω which contains y but not z. This implies that the space Ω is Pre-$\mathcal{G}T_1$. □

Proposition 2.9. *If* $(\Omega, \Im, \mathcal{G})$ *is* $\mathcal{GTS}$, *then the following statements are correct if* Ω *is*

(1) *Pre-*$\mathcal{G}T_1$ *space, then* $\mathcal{G}PO(\Omega) = \mathcal{G}P_PO(\Omega)$.
(2) $\mathcal{G}PR$ *space, then* $\mathcal{G}O(\Omega) \subseteq \mathcal{G}P_PO(\Omega)$.

Proof.

(1) Since Ω is Pre-$\mathcal{G}T_1$, then by Theorem 2.8 every singleton $\{x\}$ is $\mathcal{G}-$preclosed set. Also for each $x \in \mathcal{A}$, $\forall$ $\mathcal{G}-$preopen set $\mathcal{A}$ in Ω, implies $x \in \{x\} \subseteq \mathcal{A}$ and $\mathcal{A} \in \mathcal{G}P_pO(\Omega)$. Then $\mathcal{G}PO(\Omega) = \mathcal{G}P_pO(\Omega)$.

(2) Let μ be $\mathcal{G}-$open subset of a space Ω. Then, μ is $\mathcal{G}-$preopen. If Ω is $\mathcal{G}PR$ space, then by Lemma 2.7, $\forall x \in \mu \subseteq \Omega$, $\exists$ a $\mathcal{G}-$preopen set η such that $x \in \eta \subseteq P\psi(\eta) \subseteq \mu$. Hence $\mathcal{G}O(\Omega) \subseteq \mathcal{G}P_PO(\Omega)$. □

Lemma 2.10. *A* $(\Omega, \tau, \mathcal{G})$ *is* $\mathcal{GTS}$ *and* $\mu \subseteq \xi \subseteq \Omega$. *If* $\mu \in \mathcal{G}P_PO(\xi)$ *and* ξ *is* $\mathcal{G}-$*preclopen or* $\xi \in \mathcal{G}PRO(\Omega)$, *then* $\mu \in \mathcal{G}P_PO(\Omega)$.

Proof. If $\mu \in \mathcal{G}P_PO(\xi)$, then $\mu \in \mathcal{G}PO(\xi)$, since ξ is $\mathcal{G}-$preclopen then $\xi \in \mathcal{G}PO(\Omega)$ implies $\mu \in \mathcal{G}PO(\Omega)$, $\forall$ $x \in \mu$, $\exists$ a $\mathcal{G}-$preclosed set λ in ξ : $x \in \lambda \subseteq \mu$ implies $\mu \in \mathcal{G}P_PO(\Omega)$. On other hand since $\forall$ $x \in \mu$, $\exists$ a $\mathcal{G}-$preclosed set λ in ξ such that $x \in \lambda \subseteq \mu$ and ξ is $\mathcal{G}-$preclopen implies ξ is $\mathcal{G}-$preclosed set in Ω. Since λ is $\mathcal{G}-$preclosed set in ξ, λ is $\mathcal{G}-$preclosed set in Ω. Hence $\mu \in \mathcal{G}P_PO(\Omega)$. □

Definition 2.11. *If* $(\Omega, \Im, \mathcal{G})$ *is* $\mathcal{GTS}$ *then* Ω *is called:*

(1) $\mathcal{G} - P_p$ *compact* ($\mathcal{G} - P_p$*CMP, for short) if* $\forall$ P_p *open cover* $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ *of* Ω, $\exists$ *a finite subset* $\Delta_0 \subseteq \Delta : \Omega \setminus \bigcup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\} \notin \mathcal{G}$.

(2) $\mathcal{G} - \theta$ *compact* ($\mathcal{G} - \theta$*CMP, for short) if* $\forall$ θ *open cover* $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ *of* $\mathcal{X}$, *has* $\Delta_0 \subseteq \Delta$: $\Omega \setminus \bigcup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\} \notin \mathcal{G}$.

(3) $\mathcal{G} - P_S$ *compact* ($\mathcal{G} - P_S$*CMP, for short) if* $\forall$ P_S *open cover* $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ *of* Ω, $\exists$ $\Delta_0 \subseteq \Delta$: $\Omega \setminus \bigcup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\} \notin \mathcal{G}$.

(4) $\mathcal{G} - p$ *closed if* $\forall$ *preopen cover* $\{\mathbb{U}_\alpha : \alpha \in \Delta\}$, $\exists$ $\Delta_0 \subseteq \Delta : \Omega \setminus \bigcup_{\alpha\in\Delta_0}\{pcl(\mathbb{U}_\alpha)\} \notin \mathcal{G}$.

Lemma 2.12. *Each* P_p-*CMP space* $(\Omega, \Im, \mathcal{G})$ *is* $\mathcal{G} - P_p$*CMP* $\forall$ $\mathcal{G}$ *on* Ω.

Proof. Let $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ be any P_p open cover of Ω of an P_pCMP space $(\Omega, \Im, \mathcal{G})$, then $\exists$ a finite subcover $\{\mathbb{V}_\alpha : \alpha \in \Delta_0\}$ of Ω. Since $\Omega \setminus \bigcup_{\alpha\in\Delta_0} \mathbb{V}_\alpha \notin \mathcal{G}$, then $(\Omega, \Im, \mathcal{G})$ is $\mathcal{G} - P_p$CMP. □

Proposition 2.13. *Let* $\mathcal{G} = P(\Omega) \setminus \phi$ *be a grill on a* $(\Omega, \Im)$ *and space* $(\Omega, \Im_{\mathcal{G}})$ *be* $\mathcal{G} - P_p$*CMP. Then,* $(\Omega, \Im, \mathcal{G})$ *is* $\mathcal{G} - P_p$*CMP.*

Proof. Let $\{\zeta_\alpha : \alpha \in \Delta\}$ be any $\Im - P_p$ open cover of Ω. Since $\Im \subseteq \Im_{\mathcal{G}}$, then $\{\zeta_\alpha : \alpha \in \Delta\}$ is $\Im_{\mathcal{G}} - P_p$ open cover of Ω. Since $(\Omega, \Im_{\mathcal{G}})$ is $\mathcal{G} - P_p$CMP, then $\exists\, \Delta_0 \subseteq \Delta$ such that $\Omega \setminus \bigcup_{\alpha\in\Delta_0} \zeta_\alpha \notin \mathcal{G}$, but $\mathcal{G} = P(\Omega) \setminus \phi$ then $\Omega \setminus \bigcup_{\alpha\in\Delta_0} \zeta_\alpha = \phi$. Hence $(\Omega, \Im, \mathcal{G})$ is P_pCMP and by Lemma 2.12, $(\Omega, \Im, \mathcal{G})$ is $\mathcal{G} - P_p$CMP. □

Theorem 2.14. *A $\mathcal{GTS}$ $(\Theta, \Gamma, \mathcal{G})$ is $\mathcal{G} - P_p$CMP iff $(\Theta, \Gamma_{\mathcal{G}})$ is $\mathcal{G} - P_p$CMP.*

Proof. $(\Rightarrow)$: If $\Gamma \subseteq \Gamma_{\mathcal{G}}$ it follows that $(\Theta, \Gamma, \mathcal{G})$ is $\mathcal{G} - P_p$CMP if $(\Theta, \Gamma_{\mathcal{G}})$ is $\mathcal{G} - P_p$CMP.
$(\Leftarrow)$: let $(\Theta, \Gamma, \mathcal{G})$ be $\mathcal{G} - P_p$CMP and $\{\xi_\jmath : \jmath \in \Delta\}$ be a P_p open cover of Θ. Then $\forall$ $\jmath \in \Delta$, $\xi_\jmath = \mathbb{U}_\jmath \setminus \mathcal{B}_\jmath$ where $\mathbb{U}_\jmath \in P_pO(\Theta)$ and $\mathcal{B}_\jmath \notin \mathcal{G}$. Then, $\{\mathbb{U}_\jmath : \jmath \in \Delta\}$ is a P_p−open cover of Θ. Hence by $\mathcal{G}$−CMP of $(\Theta, \Gamma, \mathcal{G})$, $\exists\, \Delta_0 \subseteq \Delta$ such that $\Theta \setminus \bigcup_{\jmath\in\Delta_0} \mathbb{U}_\jmath \notin \mathcal{G}$. But, $\Theta \setminus \bigcup_{\jmath\in\Delta_0} \xi_\jmath = \Theta \setminus \bigcup_{\jmath\in\Delta_0}(\mathbb{U}_\jmath \setminus \mathcal{B}_\jmath) \subseteq (\Theta \setminus \bigcup_{\jmath\in\Delta_0} \mathbb{U}_\jmath) \cup (\Theta \setminus \bigcup_{\jmath\in\Delta_0} \mathcal{B}_\jmath) \notin \mathcal{G}\ \forall \mathcal{B}_\jmath \notin \mathcal{G}, \jmath \in \Delta_0$. Then $(\Theta, \Gamma_{\mathcal{G}})$ is $\mathcal{G} - P_p$CMP. □

From Lemma 2.12, Theorem 2.14 we have the following implication diagram holds.

$$\begin{array}{ccc} (\Omega, \Im, \mathcal{G}) \textbf{ is } P_p-\textbf{CMP} & \Longrightarrow & (\Omega, \Im_{\mathcal{G}}) \textbf{ is } P_p-\textbf{CMP} \\ \Downarrow & & \Downarrow \\ (\Omega, \Im, \mathcal{G}) \textbf{ is } \mathcal{G} - P_p\textbf{CMP} & \Longleftrightarrow & (\Omega, \Im_{\mathcal{G}}) \textbf{ is } \mathcal{G} - P_p\textbf{CMP} \end{array}$$

Proposition 2.15. *If preclosed cover $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ of a space Ω has a finite subcover $\{\mathbb{V}_\alpha : \alpha \in \Delta_0\} : \Delta_0 \subseteq \Delta$, then $\Omega \setminus \bigcup_{\alpha\in\Delta_0} \mathbb{V}_\alpha \notin \mathcal{G}$ and Ω is $\mathcal{G} - P_p$CMP.*

Proof. If $\mathbb{U}$ is P_p open, then $\forall\ x \in \Omega$, $\exists$ preclosed set $\mathbb{V} : x \in \mathbb{V} \subseteq \mathbb{U}$ and $\forall\ \alpha \in \Delta$ so $x_\alpha \in \mathbb{V}_\alpha \subseteq \mathbb{U}_\alpha$ and $x_\alpha \in \{\mathbb{V}_\alpha : \alpha \in \Delta\} \subseteq \{\mathbb{U}_\alpha : \alpha \in \Delta\}$. Since $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ is preclosed cover of a space Ω. Then $\exists\, \Delta_0 \subseteq \Delta$ is $\alpha \in \Delta_0 \subseteq \Delta$ and $x \in \mathbb{V}_{\alpha(x)}$, $\Omega = \{\mathbb{V}_{\alpha(x_i)} : i = 1, 2, 3, \ldots.., n\} \subseteq \{\mathbb{U}_{\alpha(x_i)} : i = 1, 2, 3, \ldots.., n\}$. Hence, $\Omega = \{\mathbb{U}_{\alpha(x_i)} : i = 1, 2, 3, \ldots.., n\}$ is P_p cover of Ω and Ω is $\mathcal{G} - P_p$CMP. □

Definition 2.16. *Let $(\Omega, \Im, \mathcal{G})$ be $\mathcal{GTS}$. Then, $(\Omega, \Im, \mathcal{G})$ is $\mathcal{G}$−strongly compact ($\mathcal{G}$ − SCMP, for short) if each cover of Ω by preopen sets has a finite subcover $\Delta_0 \subseteq \Delta : \Omega \setminus \bigcup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\} \notin \mathcal{G}$..*

Lemma 2.17.

(1) *Each* Ω *is* $\mathcal{G}-SCMP$ *is* $\mathcal{G}-P_pCMP$*.*

(2) *Each* $\mathcal{G}-SCMP$ *is* $\mathcal{G}-CMP$*.*

Proof. It is clearly because each $\mathcal{G}-P_p$ open set is $\mathcal{G}-$preopen and each $\mathcal{G}-$open set is $\mathcal{G}-$preopen □

Lemma 2.18. *Every* $\mathcal{G}-P_pCMP$ *space is* $\mathcal{G}-\theta CMP$*.*

Proof. Clear because each $\mathcal{G}-\theta$ open set is $\mathcal{G}-P_p$open. □

From Lemma 2.17 and Lemma 2.18 is established in the below diagram.

$$\begin{array}{ccc} \mathcal{G}-S\textbf{CMP} & \Longrightarrow & \mathcal{G}-P_p\textbf{CMP} \\ \Downarrow & & \Downarrow \\ \mathcal{G}-\textbf{CMP} & \Longrightarrow & \mathcal{G}-\theta\textbf{CMP} \end{array}$$

The reverses of the above implication are not verified, in general.

Lemma 2.19. *Let* Ω *be a* $\mathcal{G}PR$ *space. If* Ω *is* $\mathcal{G}-P_pCMP$*, then* Ω *is* $\mathcal{G}-CMP$*.*

Proof. It is clearly from Proposition 2.9(2). □

Theorem 2.20. *A* $\mathcal{GTS}$ $(\Upsilon, \Im, \mathcal{G})$ *hence every Pre-*$\mathcal{G}T_1$ *and* $\mathcal{G}-P_pCMP$ *space is* $\mathcal{G}-SCMP$*.*

Proof. Let Υ be a Pre-$\mathcal{G}T_1$, $\mathcal{G}-P_p$CMP space and $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ be any preopen cover of Υ. Hence, $\forall\ \ell \in \Upsilon$, $\exists\ \alpha(\ell) \in \Delta : \ell \in \mathbb{V}_{\alpha(\ell)}$. Since Υ is Pre-$\mathcal{G}T_1$ and by Proposition 2.9(1), the family $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ is a P_p open cover of Υ. Since Υ is $\mathcal{G}-P_p$CMP, then $\exists$ a $\Delta_0 \subseteq \Delta$ of Υ : $\Upsilon \setminus \bigcup_{\alpha:\alpha\in\Delta_0}\{\mathbb{V}_\alpha\} \notin \mathcal{G}$. Thus, Υ is $\mathcal{G}-S$CMP. □

Proposition 2.21. *A* $\mathcal{GTS}$ $(\Theta, \Gamma, \mathcal{G})$ *is a* $\mathcal{G}PR$ *space and* $\mathcal{G}-P$ *closed space, then* Θ *is* $\mathcal{G}-P_pCMP$*.*

Proof. Let $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ is a $\mathcal{G}-P_p$ open cover of Θ, $\mathbb{V}_\alpha$ is $\mathcal{G}-$preopen $\forall\ \alpha \in \Delta$. Since Θ, is a $\mathcal{G}PR$ space, by Lemma 2.7, $\forall\ \jmath \in \Theta$ and $\mathbb{V}_{\alpha(\jmath)}$ $\exists$ a $\mathcal{G}-$preopen set $\mu_\jmath : \jmath \in \mu_\jmath \subseteq P\Psi(\mu_\jmath) \subseteq \mathbb{V}_{\alpha(\jmath)}$. Hence $\{\mu_\jmath : \jmath \in \Theta\}$ is a $\mathcal{G}-$preopen cover of Θ. Since Θ is a $\mathcal{G}-P$ closed space, then

$\exists$ a subfamily $\{\mu_{J_i} : i = 1, 2, ..., n\} : \Theta = \cup_{i=1}^{n} pcl(\mu_{J_i}) \subseteq \cup_{i=1}^{n} \mathbb{V}_\alpha(J_i)$. Thus Θ is $\mathcal{G} - P_p$CMP. □

Theorem 2.22. *Let $(\Theta, \mathfrak{I}, \mathcal{G})$ be $\mathcal{GTS}$. Then, the following conditions are identical:*

(1) *Θ is $\mathcal{G} - P_p$CMP,*

(2) *Each P_p cover $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ of Θ, $\exists\, \Delta_0 \subseteq \Delta : \Theta \setminus (\cup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\}) \notin \mathcal{G}$,*

(3) *$\forall$ family $\{\lambda_\alpha : \alpha \in \Delta\}$ of $\mathcal{G} - P_p$closed subsets of Θ : $\bigcap\{\lambda_\alpha : \alpha \in \Delta\} = \phi$, $\exists\, \Delta_0 \subseteq \Delta$ of Θ : $\Theta = \bigcap\{\lambda_\alpha : \alpha \in \Delta_0\}$.*

Proof. (1) $\Rightarrow$ (2) from Definition 2.11(1)

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (1) Let $\{\mathbb{V}_\alpha : \alpha \in \Delta\}$ be a P_p open cover of Θ. Then, $\{\Theta \setminus \mathbb{V}_\alpha : \alpha \in \Delta\}$ is a family of P_p closed subsets of $\Theta : \cap\{\Theta \setminus \mathbb{V}_\alpha : \alpha \in \Delta\} = \phi$. Sine $\exists\, \Delta_0 \subseteq \Delta$ such that $\Theta \setminus (\cup_{\alpha\in\Delta_0}\{\mathbb{V}_\alpha\}) \notin \mathcal{G}$, then $\Theta = \cup\{\mathbb{V}_\alpha : \alpha \in \Delta_0\}$. This shows that Θ is $\mathcal{G} - P_p$CMP. □

3. $\mathcal{G} - P_P$ Compact Subspaces

Definition 3.1. *Let $\mathcal{GTS}$ $(\Omega, \mathfrak{I}, \mathcal{G})$ and $\varphi \subseteq \Omega$. Then, φ is said to be:*
$\mathcal{G} - P_p$CMP subspace of φ if for each cover $\{\mu_\alpha : \alpha \in \Delta\}$ of φ by P_p open subset of φ has a finite subcover $\Delta_0 \subseteq \Delta$ such that $\varphi \setminus \{\bigcup_{\alpha\in\Delta_0}\{\mu_\alpha\} \notin \mathcal{G}$.

Lemma 3.2. *If $(\Omega, \mathfrak{I}, \mathcal{G})$ is $\mathcal{GTS}$ and $\mathcal{A} \subseteq \Omega$, then $\mathcal{A}$ is $\mathcal{G} - P_p$CMP subspace iff each P_p open cover $\{\mu_\alpha : \alpha \in \Delta\}$ of $\mathcal{A}$ has a finite subcover $\Delta_0 \subseteq \Delta$ such that $\mathcal{A} \setminus \{\bigcup_{\alpha\in\Delta_0}\{\mu_\alpha\} \notin \mathcal{G}$.*

Proof. Clearly from Definition 3.1. □

Theorem 3.3. *A $(\Theta, \mathfrak{I}, \mathcal{G})$ is $\mathcal{GTS}$ and $\eta \subseteq (\Theta, \mathfrak{I}, \mathcal{G})$, hence every cover of η by $\mathcal{G}-$preclosed subsets of η has $\Delta_0 : \Delta_0 \subseteq \Delta$, then η is a $\mathcal{G} - P_p$CMP subspace.*

Proof. It is similar to Proposition 2.15. □

Theorem 3.4. *Let $(\Theta, \mathfrak{I}, \mathcal{G})$ be $\mathcal{GTS}$. Then, these conditions are identical:*

(1) *υ is $\mathcal{G} - P_p$CMP subspace,*

(2) *$\forall$ $\{\lambda_\alpha : \alpha \in \Delta\}$ of $\mathcal{G} - P_p$closed subsets of Θ, such that $(\bigcap\{\lambda_\alpha : \alpha \in \Delta\}) \cap \upsilon = \phi$, $\exists\, \Delta_0 \subseteq \Delta$ such that $(\bigcap\{\lambda_\alpha : \alpha \in \Delta_0\}) \cap \upsilon = \phi$.*

Proof. It is similar to Theorem 2.22 □

Theorem 3.5. *If a $\mathcal{GTS}$ $(\Omega, \Im, \mathcal{G})$ is $\mathcal{G} - P_p$CMP and μ is both $\mathcal{G}-$preclopen and $\mathcal{G} - P_p$ closed subset of Ω, then μ is a $\mathcal{G} - P_p$CMP subspace.*

Proof. Since μ is $\mathcal{G}-$preclopen, then by Lemma 2.10, $\mu_\alpha \in \mathcal{G}P_PO(\Omega)$ $\forall$ $\alpha \in \Delta$ also μ is $\mathcal{G} - P_p$ closed subset of Ω, then $\Omega \setminus \mu \in \mathcal{G}P_PO(\Omega)$, if $\{\upsilon_\alpha : \alpha \in \Delta\}$ is cover of μ implies $\Omega = \mu_\alpha \cup \Omega \setminus \mu = \{\upsilon_\alpha : \alpha \in \Delta\} \cup \Omega \setminus \mu$ is P_p- cover of Ω. Since Ω is $\mathcal{G} - P_p$ compact, $\exists$ $\Delta_0 \subseteq \Delta$ such that $\Omega = \cup\{\upsilon_\alpha : \alpha \in \Delta_0\} \cup \Omega \setminus \mu$. Hence, $\mu = \cup\{\upsilon_\alpha : \alpha \in \Delta_0\}$ and μ is a $\mathcal{G} - P_p$CMP subspace. □

Lemma 3.6. *Let a $\mathcal{GTS}$ $(\Upsilon, \Gamma, \mathcal{G})$ be $\mathcal{G} - P_p$CMP and ϱ be both $\mathcal{G}-$pre regularopen and $\mathcal{G} - P_p$ closed subset of Υ. Then, ϱ is $\mathcal{G} - P_p$CMP subspace.*

Proof. Clear from Theorem 3.5 and Lemma 2.10. □

Corollary 3.7. *Let $(\Omega, \Gamma, \mathcal{G})$ be a $\mathcal{GTS}$. Then, the condition is hold:*
The finite union of a $\mathcal{G} - P_p$ CMP subspace of Ω is a $\mathcal{G} - P_p$ CMP subspace.

Proof. Let $\mathcal{A}_i$ is $\mathcal{G} - P_p$ CMP subspace of Ω by $\mathcal{G} - P_p$open sets of $\mathcal{A}_i$ $\forall$ $i \in \Delta$. Then each cover $\{\mu_{\alpha i} : \alpha i \in \Delta\}$ of $\mathcal{A}_i$ by $\mathcal{G} - P_p$ open subset of $\mathcal{A}_i$, $\exists$ $\Delta_{0i} \subseteq \Delta : \mathcal{A}_i \setminus \bigcup_{\alpha i \in \Delta_{0i}}\{\mu_{\alpha i}\} \notin \mathcal{G}$. Therefore $\bigcup \mathcal{A}_i$ is has cover $\{\bigcup \mu_{\alpha i} : \alpha i \in \Delta\}$ of $\bigcup \mathcal{A}_i$ by $\mathcal{G} - P_p$ open subset of $\bigcup \mathcal{A}_i$, $\exists$ $\Delta_{0i} \subseteq \Delta : \bigcup \mathcal{A}_i \setminus \bigcup(\bigcup_{\alpha i \in \Delta_{0i}}\{\mu_{\alpha i}\}) = \bigcup \mathcal{A}_i \setminus \bigcup_{\alpha i \in \Delta_{0i}}\{\mu_{\alpha i}\} \notin \mathcal{G}$. Hence $\bigcup \mathcal{A}_i$ of a $\mathcal{G} - P_p$ CMP subspace of $\bigcup \mathcal{A}_i$ is a $\mathcal{G} - P_p$ CMP subspace. □

4. $\mathcal{G} - P_P$ Compactness Spaces Under Continuous Functions

Definition 4.1. *Let two $(\Omega, \Im, \mathcal{G}_1)$ and $(\xi, \sigma, \mathcal{G}_2)$ be $\mathcal{GTS}$s. Then a function $f : (\Omega, \Im, \mathcal{G}_1) \to (\xi, \sigma, \mathcal{G}_2)$ is called:*

(1) *$\mathcal{G}-$pre-continuous at a point $x \in \Omega$ if $\forall$ $\mathcal{G}-$open set λ of ξ, $f(x) \in \lambda$ $\exists$ a $\mathcal{G}-$preopen set μ of Ω, $x \in \mu$ such that $f(\mu) \subseteq \lambda$.*

(2) *$\mathcal{G} - P_P$ continuous at a point $x \in \Omega$ if $\forall$ $\mathcal{G}-$open set λ of ξ, $f(x) \in \lambda$ $\exists$ a $\mathcal{G} - P_P$ open set μ of Ω, $x \in \mu$ such that $f(\mu) \subseteq \lambda$.*

(3) *Almost $\mathcal{G}-$pre-continuous at a point $x \in \Omega$ if $\forall$ $\mathcal{G}-$open set λ of ξ, $f(x) \in \lambda$ $\exists$ a $\mathcal{G}-$preopen set μ of Ω, $x \in \mu$ such that $f(\mu) \subseteq Int(Cl(\lambda))$.*

(4) *Almost $\mathcal{G} - P_P$ continuous at a point $x \in \Omega$ if $\forall$ $\mathcal{G}-$open set λ of ξ, $f(x) \in \lambda$ $\exists$ a $\mathcal{G} - P_P$ open set μ of Ω, $x \in \mu$ such that $f(\mu) \subseteq Int(Cl(\lambda))$.*

Theorem 4.2. *If $g : (\Omega, \Im, \mathcal{G}_1) \to (\xi, \sigma, \mathcal{G}_2)$, g is a grill continuous, open function and λ is a $\mathcal{G} - P_P$ open set of ξ, then $f^{-1}(\lambda)$ is a $\mathcal{G} - P_P$ open set of Ω.*

Proof. Let λ be a $\mathcal{G} - P_P$ open set of ξ. Then λ is a $\mathcal{G}-$preopen set of ξ, $\forall$ $y \in \lambda$, then $\exists$ $\mathcal{G}-$open η in ξ such that $\lambda \subseteq \eta \subseteq \Psi(\lambda)$. Hence $g^{-1}(\lambda) \subseteq g^{-1}(\eta) \subseteq g^{-1}(\Psi(\lambda)) \subseteq \Psi(g^{-1}(\lambda))$, this implies $g^{-1}(\lambda)$ is $\mathcal{G}-$preopen set of Ω and let $x \in g^{-1}(\lambda)$, then $g(x) \in \lambda$. So $\exists$ a $\mathcal{G}-$preclosed set ρ_x of $\Omega : g(\rho_x) \subseteq \lambda$, implies $x \in \rho_x \subseteq g^{-1}(\lambda)$. Hence $g^{-1}(\lambda)$ is a $\mathcal{G} - P_P$ open set of Ω. □

Corollary 4.3. *If $g : (\Omega, \Im, \mathcal{G}_1) \to (\zeta, \sigma, \mathcal{G}_2)$ is $\mathcal{G} - P_P$ continuous surjection function and Ω is a $\mathcal{G} - P_P$CMP, then ζ is $\mathcal{G}-$CMP.*

Proof. Let $\{\mathbb{U}_i : i \in \Delta\}$ be any cover of $g(\mu)$ by $\mathcal{G} - P_P$ open sets of ζ $\forall$ $x \in \mu$, $\exists$ $i(x) \in \Delta$: $g(x) \in \mathbb{U}_{i(x)}$. Since g is $\mathcal{G} - P_P$ continuous $\exists$ a $\mathcal{G} - P_P$ open set $\mathbb{V}_x$ of Ω containing x such that $g(\mathbb{V}_x) \subseteq \mathbb{U}_{i(x)}$. Then $\{\mathbb{V}_{i(x)} : x \in \mu\}$ is a $\mathcal{G} - P_P$ open cover of μ $\exists$ finite subset μ_0 of μ, then $\mu \subseteq \cup\{\mathbb{V}_{i(x)} : x \in \mu_0\}$ implies $g(\mu) \subseteq \cup\{\mathbb{U}_{i(x)} : x \in \mu_0\}$. Then, $g(\mu)$ is $\mathcal{G}-$CMP relative to ζ. □

Proposition 4.4. *If $f : (\Theta, \Gamma, \mathcal{G}_1) \to (\xi, \sigma, \mathcal{G}_2)$ is a $\mathcal{G}-$pre-continuous surjection function, Θ is a pre-$\mathcal{G}T_1$ and $\mathcal{G} - P_P$CMP space, then ξ is $\mathcal{G}-$CMP.*

Proof. By use Corollary 4.3, 2.20, and Lemma 2.12 □

Proposition 4.5. *Let function $g : (\Omega, \Im, \mathcal{G}_1) \to (\xi, \sigma, \mathcal{G}_2)$ be a $\mathcal{G}-$continuous surjection and Ω is a $\mathcal{G}PR$ space and $\mathcal{G} - P_P$CMP space, then ξ is $\mathcal{G}-$CMP.*

Proof. Clear in Corollary 4.3 and Lemma 2.19. □

Conclusion and Future Work

This paper aims to introduce some new types of compactness in terms of grill theory. In future, $\mathcal{G}-P_p$ compact, $\mathcal{G}-$ strongly compact, $\mathcal{G}-\theta$ compact and $\mathcal{G}-P_S$ compact spaces can be applied in many directions and solve some real life problems as in [19, 20, 21, 22, 23, 24, 25, 26].

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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