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SUMUDU-ITERATION TRANSFORM METHOD FOR FRACTIONAL TELEGRAPH EQUATIONS

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Abstract. We suggested a suitable algorithm, the Sumudu-iterative transform method, in this research study (SITM). SITM illustrates space and time-fractional telegraph equations by combining the iterative approach and the Sumudu transform. Caputo sense derivatives were employed.

Keywords: Sumudu transform; Sumudu iterative transform method; fractional differential equations; telegraph equations.

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1. INTRODUCTION

In mathematics and fields such as physics and engineering technology, partial differential equations are more important. To solve partial differential equations like Telegraph equations, Fokker-Planck equations, fractional telegraph equations, fractional Fokker-plank equations are solved by Laplace transform or by iterative method etc. [1, 2, 3].

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Fractional differential equations can be solved using a variety of approaches, including fractional improved homotopy perturbation, fractional Laplace Adomain decomposition method, fractional wavelet method, and so on[4, 5, 6, 7, 8]. Daftardar-gejji and Jafari devised the iterative approach in 2006 to solve non-linear and linear fractional differential equations.[9, 10, 11, 2, 12].

In this article, we consider the space- time fractional telegraph equation:

$$(1)\qquad \begin{aligned} & D_{\xi}^{\alpha} y(\xi,\mu) = D_{\mu}^{k\delta} y(\xi,\mu) + a D_{\mu}^{n\delta} + b y(\xi,\mu) + g(\xi,\mu), \;\; 0<\xi\le 1, \mu>0 \\ & where, \;\; \delta = \frac{1}{m}, k,m,n \in N, 1<\alpha\le 2, 1<k\delta\le 2, 0<n\delta\le 1, \\ & D_{\mu}^{k}\delta \equiv D_{\mu}^{\delta} D_{\mu}^{\delta} ... D_{\mu}^{\delta} (k-times) \\ & D_{\mu}^{n}\delta \equiv D_{\mu}^{\delta} D_{\mu}^{\delta} ... D_{\mu}^{\delta} (n-times) \end{aligned}$$

$D_{\xi}^{\alpha}, D_{\mu}^{\delta}$- are capto fractional derivaties defined by eq.(2) a,b,c are constants and $g(\xi,\mu)$ is given function. The space-time fractional telegraph equation is reduced to the classical telegraph equation where $\alpha = 2, m = 1, k = 2, n = 1, g = 0$, To solve fractional telegraph equations, we now employ the SITM. It's a hybrid of two methods for solving non-linear fractional equations with exact solutions.

2. Basic Defination and Terminology

Definition 2.1. *Function $y(\xi,\mu)$ has a caputo fractional derivative defined as*

$$(2)\qquad D_{\xi}^{\alpha} y(\xi,\mu) = \frac{1}{\Gamma(j-\alpha)} \int_0^x (\xi-p)^{(j-\alpha-1)} y^{(j)}(p,\mu) dp, j-1<\alpha\le j, j\in N$$

$d^j \equiv \frac{d^j}{dx^j}$ *and* j_x^{α} *denote the Riemann-Lioville fractional integral operator of order* $\alpha > 0$ *defined as* $d^j \equiv \frac{d^j}{dx^j}$ *and* j_x^{α} *respectively.*

$$(3)\qquad J_{\xi}^{\alpha} y(\xi,\mu) = \frac{1}{\Gamma\alpha} \int_0^{\xi} (\xi-p)^{(\alpha-1)} y(p,\mu) dp, p>0, k-1<\alpha\le k, k\in N$$

Definition 2.2. *The sumudu trasform of a function $f(c), c>0$ is defined as*

$$(4)\qquad S[f(c)] = F(u) = \int_0^{\infty} e^{-c} f(uc) dc, u \in (-C_1, C_2) and f(c) \in A,$$

where

$$A=\left\{f(c)/\exists M,C_1,C_2>0,|f(c)|\leq Me^{\frac{|c|}{C_j}},if\ c\in(-1)^j\times[0,\infty)\right\} \tag{5}$$

Definition 2.3. *The order $\alpha\in\mathbb{C},Re(\alpha)>0$ Riemann Liouville fractional integral $I^{\alpha}_{p+}f$ is defined as*[13]

$$\left({}_pD_q^{-\alpha}f\right)(q)=\left(I^{\alpha}_{p+}f\right)(q)=\frac{1}{\Gamma(\alpha)}\int_p^q\frac{f(c)}{(q-c)^{1-\alpha}}dc,(q>p,Re(\alpha)>0 \tag{6}$$

Definition 2.4. *The Riemann Liouville fractional derivatives $\left({}_pD_q^{\alpha}y\right)(x)$ of order $\alpha\in\mathbb{C},Re(\alpha>0)$ is defined by*[5]

$$\begin{aligned}\left({}_pD_q^{\alpha}y\right)(c)&=\left(\frac{d}{dc}\right)^n\left(\left(I^{n-\alpha}_{p+}y\right)(c)\right)\\&=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dc}\right)^n\int_p^c\frac{y(q)dq}{(c-q)^{\alpha-n+1}},(n=Re(\alpha)+1;c>p)\end{aligned} \tag{7}$$

Definition 2.5. *The mittag-leffler function and its generalazation as*

$$E_{\alpha}(y)=\sum_{k=0}^{\infty}\frac{y^k}{\Gamma(\alpha^k+1)}(\alpha\in c,re(\alpha)>0) \tag{8}$$

$E_{\alpha,\beta}$ is Mittag-Leffler function in two parameters.

$$E_{\alpha,\beta}(y)=\sum_{k=0}^{\infty}\frac{y^k}{\Gamma(\alpha^k+\beta)}\alpha,\beta\in C,R(\alpha)>0,R(\beta)>0 \tag{9}$$

Theorem 2.6. *If $F(\mu)$ is the Sumudu Transform of the function $f(c)$, then $\left(I^{\alpha}_{a+}f\right)(c)$ is the Sumudu Transform of the Riemann Liouville fractional integral of $f(c)$ of order α* [14] .

$$S\left[\left({}_aD_c^{-\alpha}f\right)(c)\right](\mu)=S\left[\left(I^{\alpha}_{a+}f\right)(c)\right](\mu)=\mu^{\alpha}F(\mu)\ \ ,Re(\alpha)>0 \tag{10}$$

Theorem 2.7. [15, 16, 5] *Let $F(\mu)$ and $G(\mu)$ be the Sumudu transform of $f(c)andg(c)$ respectively. If*

$$h(c)=(f*g)(c)=\int_0^c f(C)g(c-C)dC \tag{11}$$

where $*$ denotes the convolution of f and g then the Sumudu transform of $h(c)$ is $S[h(c)]=\mu F(\mu)G(\mu)$

Theorem 2.8. *Let $F(\mu)$ be the Sumudu Transform of the function $f(c)$ and $m \geq 1$. The Sumudu transform of m^{th} derivative of f(c) denoted by*

$$S[f^m(c)](\mu) = F_m(\mu) = \frac{F(\mu)}{\mu^m} - \sum_{k=0}^{m-1} \frac{f^k(0)}{\mu^{m-k}} \tag{12}$$

Theorem 2.9. *Let $\alpha > 0$ be such that $m-1 \leq \alpha < m$ and $F(\mu)$ be the Sumudu transform of the function $f(c)$ and $m \in \mathbb{N}$ and then the Sumudu transform of the Riemann- Liouville fractional derivatives of $f(c)$ of order α is given by*

$$S[{}_0D_c^{\alpha} f(c)](\mu) = F_\alpha(\mu) = \frac{F(\mu)}{\mu^\alpha} - \sum_{k=0}^{m-1} \frac{{}_0D_c^{\alpha-k-1} f(0)}{\mu^{(k+1)}} \tag{13}$$

Theorem 2.10. *Let $p \in \mathbb{N}$ and $\alpha > 0$ be such that $p-1 < \alpha \leq p$ and $F(\mu)$ be the Sumudu transform of the function $f(c)$ then the Sumudu transform of the Caputo fractional derivatives of $f(c)$ of order α is given by*

$$S[{}_0^cD_c^{\alpha} f(c)](\mu) = F_\alpha^c(\mu) = \mu^{-\alpha}\left[F(\mu) - \sum_{z=0}^{p-1} \mu^z [f^z(0)]\right], -1 < p-1 < \alpha \leq p \tag{14}$$

3. Sumudu Iterative Transform Method

To illustrate this Sumudu Iterative Transform Method[10, 14, 9, 2, 17, 11, 9, 5] we consider a fractional non-linear ,non-homogenous partial differentail equationwith the initial conditions of the form:

$$D_\mu^\alpha y(\xi,\mu) + R(y(\xi,\mu)) + \mathcal{N}(y(\xi,\mu)) = g(\xi,\mu), \;\; m-1 < \alpha \leq m, m \in \mathbb{N} \tag{15}$$

$$y^{(j)}(\xi,0) = h_j(\xi), \quad j = 0,1,2,\ldots,m-1 \tag{16}$$

where $D_\mu^\alpha y(\xi,\mu)$ is the caputo fractional derivative of order α, $m-1 < \alpha \leq m$ defined by the equation (2), R is a linear operator which might include the other fractional derivatives of order less than α,$\mathcal{N}$ is the non linear operator which also might includes the fractional derivatives of order less than α and $g(\xi,\mu)$ is known as analytic function.

Applying the Sumudu transform to the equation eq .(16) we have,

$$S\left[D_\mu^\alpha y(\xi,\mu)\right] + S\left[R(y(\xi,\mu)) + \mathcal{N}(y(\xi,\mu))\right] = S\left[g(\xi,\mu)\right], \tag{17}$$

using the equation eq.(13), we get

$$S[y(\xi,\mu)] = \frac{1}{u^{-\alpha}}\sum_{j=0}^{n-1} u^{j-\alpha}\left[y^j(\xi,0)\right] - \frac{1}{u^{-\alpha}}S[R(y(\xi,\mu)) + \mathcal{N}(y(\xi,\mu))] + \frac{1}{u^{-\alpha}}S[g(\xi,\mu)] \tag{18}$$

Apply inverse Sumudu transform to the equation (14) we get,

$$\begin{aligned} y(\xi,\mu) = S^{-1}&\left[\frac{1}{u^{-\alpha}}\sum_{j=0}^{n-1} u^{j-\alpha}\left[y^j(\xi,0)\right] + \frac{1}{u^{-\alpha}}S[g(\xi,\mu)]\right] \\ &- S^{-1}\left[\frac{1}{u^{-\alpha}}S[R(y(\xi,\mu)) + \mathcal{N}(y(\xi,\mu))]\right] \end{aligned} \tag{19}$$

Now,we apply the Iterative method

$$y(\xi,\mu) = \sum_{i=0}^{\infty} y_i(\xi,\mu) \tag{20}$$

since $R$is linear operator

$$R(\sum_{i=0}^{\infty} y_i(\xi,\mu)) = \sum_{i=0}^{\infty} R(y_i(\xi,\mu)) \tag{21}$$

and the non-linear operator $\mathcal{N}$ is decomposed as

$$\mathcal{N}(\sum_{i=0}^{\infty} y_i(\xi,\mu)) = \mathcal{N}(y_0(\xi,\mu)) + \sum_{i=1}^{\infty}\left\{\mathcal{N}(\sum_{j=0}^{i}(y_0(\xi,\mu))) - \mathcal{N}(\sum_{j=0}^{i-1} y_k(\xi,\mu))\right\} \tag{22}$$

substituting equations (20,21,22) into equation (19) we get,

$$\begin{aligned} \sum_{i=0}^{\infty} y_i(\xi,\mu) &= S^{-1}\left[\frac{1}{u^{-\alpha}}\sum_{j=0}^{m-1} u^{j-\alpha}\left[y^j(0,\mu)\right] + \frac{1}{u^{-\alpha}}S[g(\xi,\mu)]\right] \\ &- S^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(\sum_{i=0}^{\infty} Ry_i(\xi,\mu)) + \mathcal{N}(y_0(\xi,\mu)) + \sum_{i=1}^{\infty}\left\{\mathcal{N}(\sum_{j=0}^{i}(y_j(\xi,\mu))) - \mathcal{N}(\sum_{j=0}^{i-1} y_j(\xi,\mu))\right\}\right]\right] \end{aligned} \tag{23}$$

we define the recurrence relation as

$$\begin{aligned} y_0(\xi,\mu) &= S^{-1}\left[\frac{1}{u^{-\alpha}}\sum_{j=0}^{m-1} u^{j-\alpha}\left[y^j(0,\mu)\right] + \frac{1}{u^{-\alpha}}S[g(\xi,\mu)]\right] \\ y_1(\xi,\mu) &= -S^{-1}\left[\frac{1}{u^{-\alpha}}S[R(y_0(\xi,\mu)) + \mathcal{N}(y_0(\xi,\mu))]\right] \\ y_{m+1}(\xi,\mu) &= -S^{-1}\left[\frac{1}{u^{-\alpha}}S\left[R(y_0(\xi,\mu)) - \left\{\mathcal{N}(\sum_{j=0}^{m}(y_j(\xi,\mu))) - \mathcal{N}(\sum_{j=0}^{m-1} y_j(\xi,\mu))\right\}\right]\right], m \geq 1 \end{aligned} \tag{24}$$

Therefore ,the m-th term approximation solution in series form is given by,

$$y(\xi,\mu) \equiv y_1(\xi,\mu) + y_2(\xi,\mu) + \ldots + y_m(\xi,\mu), m = 1,2,\ldots \tag{25}$$

4. Implication of Method

In this part, we solve the homogeneous and nonhomogeneous fractional telegraph equations using the Sumudu Iterative Trasform Method (SITM)[4, 18, 5].

Example 1: We take into account the following: Fractional telegraph equation in homogeneous space-time:

$$\begin{aligned} & D_x^\alpha y(\xi,\mu) = D_t^{k\delta} y(\xi,\mu) + D_t^{n\delta} y(\xi,\mu) + y(\xi,\mu), \;\; 0 < \xi \leq 1, \mu > 0 \\ & where \;\; \delta = \frac{1}{m}, k,m,n \in N, 1 < \alpha \leq 2, 1 < k\delta \leq 2, 0 < n\delta \leq 1, \\ & D_\mu^k \delta \equiv D_\mu^\delta D_\mu^\delta ... D_\mu^\delta (k - times) \\ & D_\mu^n \delta \equiv D_\mu^\delta D_\mu^\delta ... D_\mu^\delta (n - times) \end{aligned} \tag{26}$$

$D_\xi^\delta, D_\mu^\delta$ are caputo fractional derivatives defined by equation (2)k and r is odd and initial conditions are given by with initial conditions

$$y(0,\mu) = E_\delta(-\mu^\delta) \text{and } y_x(\xi,\mu) = E_\delta(-\mu^\delta) \tag{27}$$

applying the sumudu trasform on the both sides of equation (27) and subject to the initial conditions (28),we get,

$$S\left[D_\xi^\alpha y(\xi,\mu)\right] = S\left[(D_\mu^{k\delta} + D_\mu^{n\delta} + 1) y(\xi,\mu)\right], \;\; 0 < \xi \leq 1, \mu > 0 \tag{28}$$

using the properties of the Sumudu transform, we get

$$S[y(\xi,\mu)] = E_\delta(-\mu^\delta) + uE_\delta(-\mu^\delta) + \frac{1}{u^{-\alpha}} S\left[(D_\mu^{k\delta} + D_\mu^{n\delta} + 1) y(\xi,\mu)\right] \tag{29}$$

Applying insverse Sumudu transform to the equation eq.(30) we get

$$y(\xi,\mu) = (1+\xi) E_\delta(-\mu^\delta) + s^{-1}\left[\frac{1}{u^{-\alpha}} S\left[(D_\mu^{k\delta} + D_\mu^{n\delta} + 1) y(\xi,\mu)\right]\right] \tag{30}$$

$$y_0(\xi,\mu) = (1+\xi)E_\delta(-\mu^\delta)$$

$$y_1(\xi,\mu) = s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_\mu^{k\delta}+D_\mu^{n\delta}+1)y_0(\xi,\mu)\right]\right]$$

$$= \left(\frac{\xi^\alpha}{\Gamma(1+\alpha)}+\frac{\xi^\alpha}{\Gamma(2+\alpha)}\right)E_\delta(-\mu^\delta)$$

$$y_2(\xi,\mu) = s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_\mu^{k\delta}+D_\mu^{n\delta}+1)y(\xi,\mu)+y_0(\xi,\mu)\right]\right]$$

(31)
$$= \left(\frac{\xi^2\alpha}{\Gamma(1+2\alpha)}+\frac{\xi^\alpha}{\Gamma(2+\alpha)}+\frac{\xi^2\alpha+1}{\Gamma(2+2\alpha)}+\frac{\xi^\alpha+1}{\Gamma(2+\alpha)}\right)E_\delta(-\mu^\delta)$$

$$-\left(\frac{\xi^\alpha}{\Gamma(1+\alpha)}+\frac{\xi^\alpha+1}{\Gamma(2+\alpha)}\right)E_\delta(-\mu^\delta)$$

$$= \left(\frac{\xi^2\alpha}{\Gamma(1+2\alpha)}+\frac{\xi^2\alpha+1}{\Gamma(2+2\alpha)}\right)E_\delta(-\mu^\delta)$$

(32)

The series form of the solution is then provided by

$$y(\xi,\mu) = y_0(\xi,\mu)+y_1(\xi,\mu)+y_2(\xi,\mu)+\dots$$

$$= E_\delta(-\mu^\delta)\left[1+\xi+\frac{\xi^\alpha}{\Gamma(\alpha+1)}+\frac{\xi^{\alpha+1}}{\Gamma(\alpha+2)}+\frac{\xi^{2\alpha}}{\Gamma(2\alpha+1)}+\dots\right]$$

(33)
$$= [E_\alpha(\xi^\alpha)+\xi E_{\alpha,2}(\xi^\alpha)]E_\delta(-\mu^\delta)$$

Remark 1: setting α=2,equation (27) reduces to time fractional telegraph equation ,with the meaning of various symbols and parameters asgiven equation (27) ,as followws.

(34)
$$D_\xi^2 y(\xi,\mu) = D_\mu^{k\delta}y(\xi,\mu)+D_\mu^{n\delta}y(\xi,\mu)+y(\xi,\mu), \quad 0<\xi\leq 1, \mu>0$$

with solution

(35)
$$y(\xi,\mu) = e^\xi E_\delta(-\mu^\delta)$$

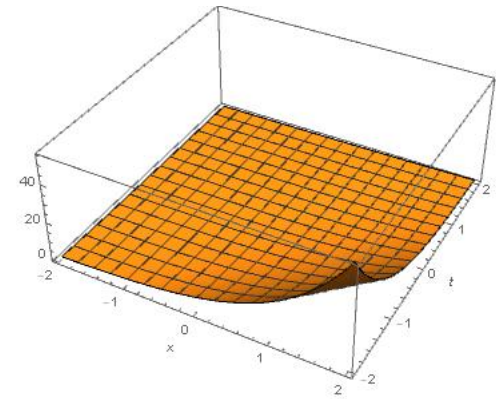

FIGURE 1. for $\delta = 1$

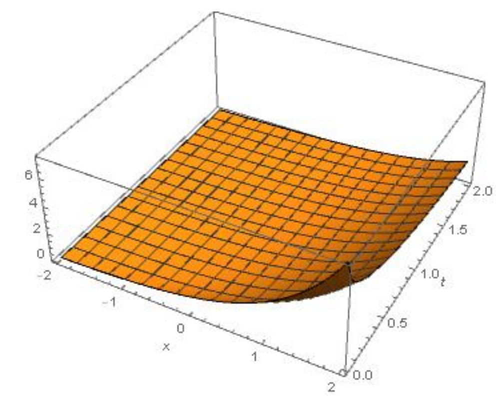

FIGURE 2. for $\delta = \frac{1}{2}$

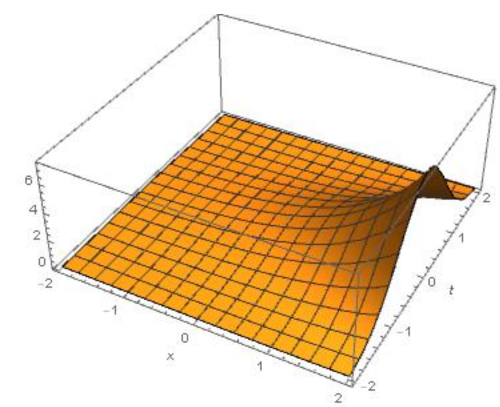

FIGURE 3. for $\delta = 2$

Remark 2: setting α=2,k=2,m=n=1, equation eq(27) reduces to classical telegraph equation.

Remark 3: setting k=2,m=n=1, the space-time fractional telegraph equation eq(27) reduces space fractional telegraph equation.

Example 2: We take into account the following: fractional telegraph equation for non-homogeneous space-time:

(36)
$$D_{\xi}^{\alpha}y(\xi,\mu)=D_{\mu}^{k\delta}y(\xi,\mu)+D_{\mu}^{n\delta}y(\xi,\mu)+y(\xi,\mu)-2E_{\alpha}(\xi^{\alpha})E_{\delta}(-\mu^{\delta})\ \ 0<\xi\leq 1,\mu>0$$
$$where\ \ \delta=\frac{1}{m},k,m,n\in N,1<\alpha\leq 2,1<k\delta\leq 2,0<n\delta\leq 1,$$
$$D_{\mu}^{k}\delta\equiv D_{\mu}^{\delta}D_{\mu}^{\delta}...D_{\mu}^{\delta}(k-times)$$
$$D_{\mu}^{n}\delta\equiv D_{\mu}^{\delta}D_{\mu}^{\delta}...D_{\mu}^{\delta}(n-times)$$

The caputo fractional derivatives $D_{\xi}^{\delta},D_{\mu}^{\delta}$ are defined by equation (ref2). Initial conditions are given by with initial conditions, and k and n are odd.

(37)
$$y(0,\mu)=E_{\delta}(-\mu^{\delta})\text{and } y_{\xi}(\xi,\mu)=E_{\delta}(-\mu^{\delta})$$

aplying the sumudu trasform on the both sides of equation (26) and subject to the initial conditions (27),we get,

(38)
$$S\left[D_{\xi}^{\alpha}y(\xi,\mu)\right]=S\left[(D_{\mu}^{k\delta}+D_{\mu}^{n\delta}+1)y(\xi,\mu)-2S[E_{\alpha}(x^{\alpha})E_{\delta}(-\mu^{\delta})]\right],\ \ 0<\xi\leq 1,\mu>0$$

using the properties of the Sumudu transform, we get

(39)
$$S[y(\xi,\mu)]=uE_{\delta}(-\mu^{\delta})+\frac{1}{u^{-\alpha}}S[(D_{\mu}^{k\delta}+D_{\mu}^{n\delta}+1)y(\xi,\mu)]-\frac{2}{u^{-}\alpha}S[E_{\alpha}(\xi^{\alpha})E_{\delta}(-\mu^{\delta})]$$

operating with the sumudu inverse on both side of equation 38

(40)
$$y(\xi,\mu)=E_{\delta}(-\mu^{\delta})-S^{-1}[\frac{2}{u^{-}\alpha}S[E_{\alpha}(\xi^{\alpha})E_{\delta}(-\mu^{\delta})]]+s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_{\mu}^{k\delta}+D_{\mu}^{n\delta}+1)y(\xi,\mu)\right]\right]$$

$$y_0(\xi,\mu) = E_\delta(-\mu^\delta) - S^-1[\frac{2}{u^-\alpha}S[E_\alpha(\xi^\alpha)E_\delta(-\mu^\delta)]]$$
$$= E_\alpha(\xi^\alpha)E_\delta(-\mu^\delta) - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+1)}}{\Gamma(\alpha(z+1)+1)}$$
$$y_1(\xi,\mu) = s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_\mu^{k\delta}+D_\mu^{n\delta}+1)y_0(\xi,\mu)\right]\right]$$
$$= 3\left[E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+1)}}{\Gamma(\alpha(z+1)+1)} - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+2)}}{\Gamma(\alpha(z+2)+1)}\right]$$
$$y_2(\xi,\mu) = s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_\mu^{k\delta}+D_\mu^{n\delta}+1)y_1(\xi,\mu)+y_0(\xi,\mu)\right]\right] - s^{-1}\left[\frac{1}{u^{-\alpha}}S\left[(D_\mu^{k\delta}+D_\mu^{n\delta}+1)y_0(\xi,\mu)\right]\right]$$
$$= 3^2\left[E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+2)}}{\Gamma(\alpha(z+2)+1)} - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+3)}}{\Gamma(\alpha(z+3)+1)}\right]$$

the solution in series form is then given by

$$\begin{aligned}
y(\xi,\mu) &= y_0(\xi,\mu)+y_1(\xi,\mu)+y_2(\xi,\mu)+\dots \\
&= [E_\alpha(\xi^\alpha)E_\delta(-\mu^\delta) - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+1)}}{\Gamma(\alpha(z+1)+1)}] \\
&+ 3[E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+1)}}{\Gamma(\alpha(z+1)+1)} - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+2)}}{\Gamma(\alpha(z+2)+1)}] \\
&+ 3^2[E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+2)}}{\Gamma(\alpha(z+2)+1)} - 3E_\delta(-\mu^\delta)\sum_{z=0}^{\infty}\frac{\xi^{\alpha(z+3)}}{\Gamma(\alpha(z+3)+1)}] \\
&= E_\alpha(\xi^\alpha)E_\delta(-\mu^\delta)
\end{aligned}$$

Remark:1.setting $\alpha = 2$ equation(36) reduces to a non-homogeneous time fractional telegraph equation, with the following symbols and parameters as provided in equation(36).

$$D_\xi^2 y(\xi,\mu) = D_\mu^{k\delta}y(\xi,\mu) + D_\mu^{n\delta}y(\xi,\mu) + y(\xi,\mu) - 2e^x E_\delta(-\mu^\delta), \;\; 0<\xi\le 1, \mu>0 \tag{41}$$

with solution

$$y(\xi,\mu) = e^\xi E_\delta(-\mu^\delta) \tag{42}$$

Remark 2:setting α=2, m=2,k=4,n=2, equation(36) reduces to a non-homogeneous time fractional telegraph equation, with the following symbols and parameters as provided in equation(36).

$$D_x^2 y(\xi,\mu) = D_t^2 y(\xi,\mu) + D_t y(\xi,\mu) + y(\xi,\mu) - 2e^x E_{1/2}(-t^{1/2}), \;\; 0 < \xi \leq 1, \mu > 0 \tag{43}$$

with solution

$$y(\xi,\mu) = e^{\xi} E_{1/2}(-\mu^{1/2}) \tag{44}$$

Remark 3: setting m=2,k=4,n=2 ,equation 30 reduces to non-homo the space-time fractional telegraph equation with the meaning of various symbols and parameters asgiven equation(36) ,as followws.

$$D_{\xi}^{\alpha} y(\xi,\mu) = D_{\mu}^2 y(\xi,\mu) + D_{\mu} y(\xi,\mu) + y(\xi,\mu) - 2E_{\alpha} e^{(-\mu)}, \;\; 0 < \xi \leq 1, \mu > 0 \tag{45}$$

with solution

$$y(\xi,\mu) = E_{\alpha}(\xi^{\alpha}) e^{(-\mu)} \tag{46}$$

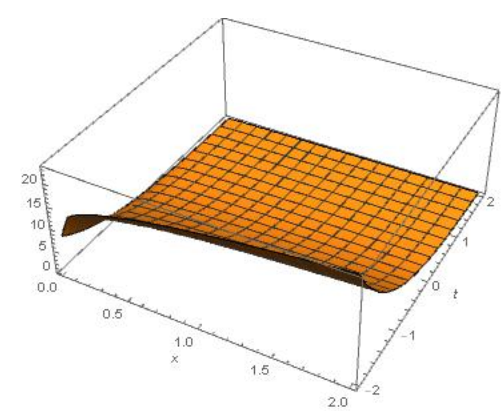

FIGURE 4. for $\alpha = 0.2$

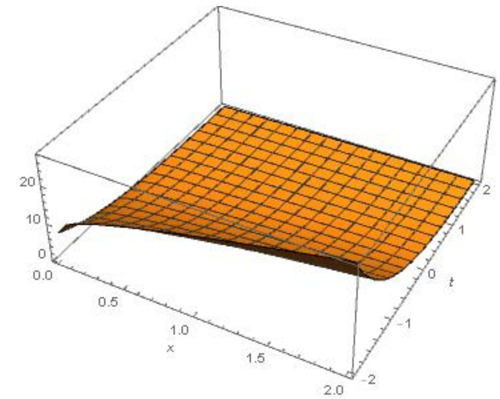

FIGURE 5. for $\alpha = 0.4$

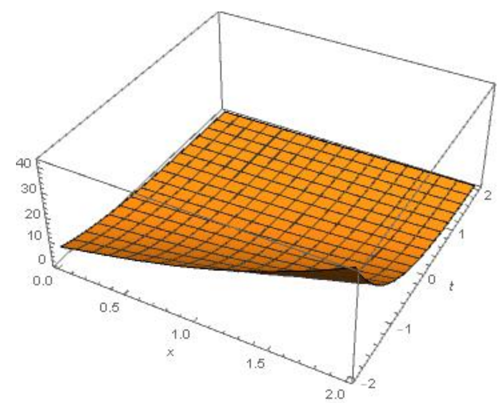

FIGURE 6. for $\alpha = 0.8$

CONCLUSION

Applying Sumudu Iterative method on the space-time Fractional Telegraph Equations we get the exact solutions.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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