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OPTIMAL STRATEGIES TO CONTROL THE TRANSMISSION DYNAMICS OF COVID'19

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Abstract. In this paper, we propose a model of transmission of a novel coronavirus (COVID'19) given by a system of non-linear differential equations. We apply optimal control theory to obtain optimal control strategies by minimizing the number of susceptible, exposed, and infected individuals. The existence and characterization of optimal controls are given using Pontryagin's Maximum Principle. Numerical simulations are carried out to illustrate the different effects and to show the efficiency of the proposed approach.

Keywords: COVID'19; optimal control; mathematical model.

2010 AMS Subject Classification: 93A30, 49J15.

1. Introduction

A mathematical model is a type of scientific model which aims to describe as precisely as possible an object, a phenomenon, a mechanism using equations to verify, understand, predict certain properties or behaviors. These models are used in many fields including economics [10],

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agriculture [5] , social sciences [20], medicine [17], etc. In epidemiology, mathematical models play a very important role in understanding the temporal dynamics of an epidemic and then in applying therapeutic strategy or the fight against infectious diseases, the latter has caused many epidemics such as Ebola and others, which lead to human losses and injury to millions of individuals around the world, as well as negatively affect economic and social development. From this point of view, the researchers were interested in mathematical modeling, which they contributed to developing in order to study the dynamics of the spread of epidemics and their progress and to evaluate the strategies used to combat them. For example, the following articles and books can be considered [9, 11, 12, 19, 14, 8, 3, 1].

For the COVID'19 pandemic, a disease caused by the novel coronavirus called SARS-CoV-2. This novel virus was first detected by the World Health Organization on December 31, 2019, after a cluster of cases of viral pneumonia were reported in Wuhan, People's Republic of China. Symptoms of COVID'19 are fever, tiredness, and dry cough. Some patients may suffer from aches and pains, nasal congestion, cold, or sore throat. These symptoms are usually mild and then gradually worsen so that the patient may lose his or her senses of taste and smell, feel pressure in the chest and, have shortness of breath. Also, some people become infected without showing any symptoms and without feeling sick. When the Covid'19 pandemic first appeared, researchers quickly used, formulated, and developed mathematical models to understand the dynamics of its spread and propose optimal control strategies to tackle the sources of pollution and significantly reduce the number of deaths and injured.

Before the vaccine's emergence and its approval by the World Health Organization, some researchers focused on non-pharmacological measures to control the outbreak such as quarantine, isolation, and public health education, they studied the effect of these different control strategies as time-dependent interventions using the optimal control approach to determine their contributions in the dynamic transmission of COVID'19 [15]. While B. Seidu [18] suggested the optimal control problem with four controls, namely, the use of face masks and social distancing, avoiding touching contact surfaces, preventing surface contamination, and disinfecting the environment. Through it, he proved that the best strategy to control the spread of COVID'19 is social distancing and the use of nose masks as the most important strategy to help curb

COVID'19. From another perspective, Moore and Okyere [16] saw that delayed diagnosis, limited hospital resources, and other treatment resources are causing COVID'19 to spread rapidly. From where, they considered an optimal control COVID'19 transmission model and assessed the impact of certain control measures that may lead to the reduction of exposed, and infected individuals in the population. They studied three control strategies for this deadly infectious disease using personal protection, treatment with early diagnosis, treatment with late diagnosis and spraying of viruses into the environment as time-dependent control functions in the model dynamic to curb the spread of disease. Many other results related to optimal strategies for controlling the COVID'19 pandemic have been established and can be found in numerous articles (see, for example, [4, 13, 6]).

The main objective of this article is to apply optimal control theory to a novel model of coronavirus (COVID'19) transmission provided by a system of non-linear ordinary differential equations to minimize susceptible, exposed and infected individuals. We are studying four strategies for controlling this deadly infectious disease using public health education, vaccination, preventive measures such as quarantine and isolation, and intensive medical care for all confirmed cases to increase the number of recovered individuals.

The model inspired by the COVID'19 pandemic associated with control measures is presented in the following section. In section 3, we provide the necessary condition for the existence of optimal control and its characterization using Pontryagin's Maximum Principle. We present in section 4 the numerical results of the optimal control model.

2. COVID'19 MODEL WITH CONTROLS

In this paper, we divide the total human population into six compartments: susceptible (S) exposed (E), quarantined (Q), asymptomatic infected (I_A), symptomatic infected (I_S), and recovered (R). We introduce four control variables u_1, u_2, u_3, and u_4. The first control u_1 represents vaccination to minimize the infection of susceptible individuals. The control u_2 represents public health education effort to educate people about the importance and necessity of social distancing, mask-wearing, and sanitizing hands and surfaces to curb the spread of COVID'19. The control u_3 represents preventive measures efforts such as quarantine, and isolation that help to reduce contact rate, and the control effort u_4 represents intensive care for confirmed cases,

whether asymptomatic or symptomatic to increase the number of people cured of the disease. Natural rates of human birth and death are denoted by Ξ and μ, respectively. Susceptible individuals (S) become infected following sufficient contact with exposed (E), asymptomatic infected (I_A) and, symptomatic infected (I_S) individuals at the rates of α, ξ and ψ, respectively. Individuals u_3S and u_3E are removed from the susceptible and exposed classes and added to the quarantined class (Q). People exit the exposed compartment and move either to the compartment of infected individuals with symptoms (symptomatic) (I_S) or to the compartment of infected individuals without symptoms (asymptomatic) (I_A) at the rates of, γ and β, respectively. By performing the test on individuals of the (Q) compartment, symptomatic individuals are transferred to the (I_S) compartment at the rate of δ, while asymptomatic individuals are transferred to the (I_A) compartment at the rate of σ. It should be noted that there are people in quarantine who can leave compartment (Q) and move to the compartment (R) at the rate of r_1 if they are found to be in good health after a negative test result. r_2 and r_3 represent respectively the rates of recovery from infection for the individuals of compartments (I_A) and (I_S). Whereas, η represents the disease mortality rate of individuals infected with symptoms.
The total population is given by $N(t) = S(t) + E(t) + Q(t) + I_A(t) + I_S(t) + R(t)$.

Through the schematic diagram in figure (1), the system of non-linear differential equations is expressed as follows:

$$
(1)\quad
\begin{aligned}
\frac{dS(t)}{dt} &= \Xi - (1-u_2(t))\,[\alpha E(t) + \xi I_A(t) + \psi I_S(t)]\,S(t) - (\mu + u_3(t) + u_1(t))\,S(t),\\
\frac{dE(t)}{dt} &= (1-u_2(t))\,[\alpha E(t) + \xi I_A(t) + \psi I_S(t)]\,S(t) - (\mu + \beta + \gamma + u_3(t))\,E(t),\\
\frac{dQ(t)}{dt} &= u_3(t)\,(S(t) + E(t)) - (\mu + \sigma + \delta + r_1)\,Q(t),\\
\frac{dI_A(t)}{dt} &= \beta E(t) + \sigma Q(t) - (\mu + r_2 + u_4(t))\,I_A(t),\\
\frac{dI_S(t)}{dt} &= \gamma E(t) + \delta Q(t) - (\mu + \eta + r_3 + u_4(t))\,I_S(t),\\
\frac{dR(t)}{dt} &= r_1 Q(t) + (r_2 + u_4(t))\,I_A(t) + (r_3 + u_4(t))\,I_S(t) - \mu R(t),
\end{aligned}
$$

with $S(0) \geq 0$, $E(0) \geq 0$, $Q(0) \geq 0$, $I_A(0) \geq 0$, $I_S(0) \geq 0$ and $R(0) \geq 0$ as the initial conditions.

Invariant region.

Consider the state variables $(S,E,Q,I_A,I_S,R) \in \mathbb{R}^6_+$. Differentiating $N(t)$ with respect to time t, we get

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dQ(t)}{dt} + \frac{dI_A(t)}{dt} + \frac{dI_S(t)}{dt} + \frac{dR(t)}{dt},$$

that is

$$\frac{dN(t)}{dt} = \Xi - \mu N(t) - u_1 S(t) - \eta I_S < \Xi - \mu N(t).$$

Using the theory of differential inequality (Birkhoff and Rota, 1982) [2] , we obtain the following result:

$$0 \leq N(t) < N(0)e^{-\mu t} + \frac{\Xi}{\mu}[1 - e^{-\mu t}].$$

This means $N(t) < \frac{\Xi}{\mu}$ as $t \to +\infty$. We conclude that all feasible solutions of the system (1) are bounded in a positive invariant region

$$\Phi = \{(S,E,Q,I_A,I_S,R) \in \mathbb{R}^6_+ : 0 \leq N < \frac{\Xi}{\mu}\}.$$

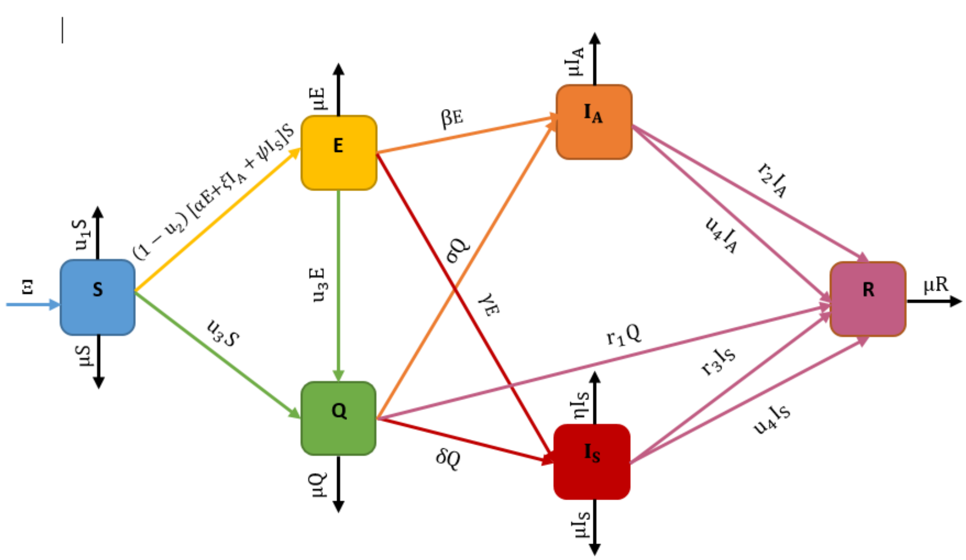


FIGURE 1. Compartmental diagram for the transmission dynamics of COVID'19

3. Optimal Control Problem

We define the objective functional J as follows:

$$J(u_1,u_2,u_3,u_4) = \int_0^{t_f} \left[C_1 S(t) + C_2 E(t) + C_3 I_A(t) + C_4 I_S(t) + \frac{1}{2}\sum_{i=1}^{i=4} \rho_i u_i^2(t) \right] dt, \tag{2}$$

where $C_i \geq 0$, for $i = 1,2,3,4$ denote weights that balance the size of the $S(t), E(t), I_A(t)$ and $I_S(t)$, respectively. The parameters ρ_i, for $i = 1,2,3,4$ represent the balancing factors associated with vaccination, public health education, preventive measures and intensive care, respectively. Furthermore, let $\mathscr{X} = (S,E,Q,I_A,I_S,R)$ and $u = (u_1,u_2,u_3,u_4) \in \Omega$, the integrand of the objective functional is given by

$$\mathscr{L}(\mathscr{X},u) = C_1 S(t) + C_2 E(t) + C_3 I_A(t) + C_4 I_S(t) + \frac{1}{2}\sum_{i=1}^{i=4} \rho_i u_i^2(t).$$

More precisely, the optimal control problem can be defined as follows:

$$J(u_1^*,u_2^*,u_3^*,u_4^*) = \min_{\Omega}\ J(u_1,u_2,u_3,u_4), \tag{3}$$

where

$$\Omega = \left\{ u_1,u_2,u_3,u_4 : u_i(t) \text{ is lebesgue measurable}, \ 0 \leq u_i(t) < 1, \ t \in [0,t_f], \text{for } \ i = 1,2,3,4 \right\}$$

is the set of admissible controls.

3.1. Existence of optimal controls.

In order to solve the optimal control problem, it is first necessary to show the existence of the solution of the system (1). Consider the state variables $S(t)$, $E(t)$, $Q(t)$, $I_A(t)$, $I_S(t)$, $R(t)$ and the control variables $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ with non-negative initial conditions, then the system (1) can be written as

$$\mathscr{X}_t = \mathscr{A}\mathscr{X} + \mathscr{B}(\mathscr{X}), \tag{4}$$

where

$$\mathscr{X} = \begin{bmatrix} S(t) \\ E(t) \\ Q(t) \\ I_A(t) \\ I_S(t) \\ R(t) \end{bmatrix}, \quad \mathscr{B}(\mathscr{X}) = \begin{bmatrix} \Xi - (1-u_2(t))\,[\alpha E(t) + \xi I_A(t) + \psi I_S(t)]S(t) \\ (1-u_2(t))\,[\alpha E(t) + \xi I_A(t) + \psi I_S(t)]S(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathscr{A} = \begin{bmatrix} -\mu - u_3(t) - u_1(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu-\beta-\gamma-u_3(t) & 0 & 0 & 0 & 0 \\ u_3(t) & u_3(t) & -\mu-\sigma-\delta-r_1 & 0 & 0 & 0 \\ 0 & \beta & \sigma & -\mu - r_2 - u_4(t) & 0 & 0 \\ 0 & \gamma & \delta & 0 & -\mu-\eta-r_3-u_4(t) & 0 \\ 0 & 0 & r_1 & r_2+u_4(t) & r_3+u_4(t) & -\mu \end{bmatrix}.$$

$\mathscr{X}_t$ is the derivative of $\mathscr{X}$ with respect to time t. System (4) is a non-linear system with a bounded coefficient.

We pose

$$\mathscr{H}(\mathscr{X}) = \mathscr{A}\mathscr{X} + \mathscr{B}(\mathscr{X}). \tag{5}$$

Then,

$$\mathscr{B}(\mathscr{X}_1) - \mathscr{B}(\mathscr{X}_2)$$

$$= \begin{bmatrix} (1-u_2(t))\,[\alpha[-E_1(t)S_1(t) + E_2(t)S_2(t)] + \xi[-I_{A_1}(t)S_1(t) + I_{A_2}(t)S_2(t)] + \psi[-I_{S_1}(t)S_1(t) + I_{S_2}(t)S_2(t)]] \\ (1-u_2(t))\,[\alpha[E_1(t)S_1(t) - E_2(t)S_2(t)] + \xi[I_{A_1}(t)S_1(t) - I_{A_2}(t)S_2(t)] + \psi[I_{S_1}(t)S_1(t) - I_{S_2}(t)S_2(t)]] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The second right-hand term in equation (5) satisfies

$$\begin{aligned}
||\mathscr{B}(\mathscr{X}_1)-\mathscr{B}(\mathscr{X}_2)|| &= 2\,(1-u_2(t))\,|\alpha\,[S_1(t)E_1(t)-S_2(t)E_2(t)]+\xi\,[S_1(t)I_{A_1}(t)-S_2(t)I_{A_2}(t)] \\
&\quad +\psi\,[S_1(t)I_{S_1}(t)-S_2(t)I_{S_2}(t)]| \\
&\leq \frac{2\,(1-u_2(t))\,\Xi}{\mu}\,[(\alpha+\xi+\psi)\,|S_1(t)-S_2(t)|+\alpha|E_1(t)-E_2(t)| \\
&\quad +\xi\,|I_{A_1}(t)-I_{A_2}(t)|+\psi|I_{S_1}(t)-I_{S_2}(t)|] \\
&\leq M\,[|S_1(t)-S_2(t)|+|E_1(t)-E_2(t)|+|I_{A_1}(t)-I_{A_2}(t)|+|I_{S_1}(t)-I_{S_2}(t)|]
\end{aligned}$$

,

where $M>0$ is independent of the variables $S(t)$, $E(t)$, $I_A(t)$ and $I_S(t)$. Therefore,

$$||\mathscr{H}(\mathscr{X}_1)-\mathscr{H}(\mathscr{X}_2)|| \leq K||\mathscr{X}_1-\mathscr{X}_2||, \text{ with } K=||\mathscr{A}||+M<+\infty.$$

Thus, it follows that the function $\mathscr{H}$ satisfies the Lipschitz condition uniformly with respect to non-negative state variables. Therefore, there exists a solution of the system (1).

Now, we present a result that will demonstrate the existence of optimal controls that minimize the objective functional J in a finite interval, subject to the system (1).

Theorem 1. *Consider the optimal control problem (3) associated with the system (1), then there exists an optimal control quadruple* $u^*=\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$ *in* Ω *such that*

$$J(u_1^*,u_2^*,u_3^*,u_4^*)=\min_{\Omega}\; J(u_1,u_2,u_3,u_4).$$

Proof. To prove the existence of an optimal control, we use Fleming's results [theorem (III.4.1) and its corresponding corollary][7].

The set of solutions for the system (1) with control variables in Ω is non-empty. We consider

$$\Omega=\left\{u\in\mathbb{R}^4 : ||u||<1\right\},$$

let u, $v\in\Omega$ such that $0\leq||u||\ <1$ and $0\leq||v||<1$. Then, for any $\varepsilon\in[0,1]$, one has

$$0\leq||\varepsilon u+(1-\varepsilon)v||\leq\varepsilon||u||+(1-\varepsilon)||v||<1,$$

which implies that Ω is convex and close. Also, the state system can be written as linear function of control variables with coefficients depending on time and state variables.

Since the integrand of the objective function is written as the sum of the convex functions with respect to control variables, then it is also convex on Ω.

Let $\mathscr{X} = (S, E, Q, I_A, I_S, R)$, $u = \{u_1, u_2, u_3, u_4\}$ and $\mathscr{L}(\mathscr{X}, u) \geq \frac{1}{2}\sum_{i=1}^{i=4} \rho_i u_i^2(t)$. We pose $\varphi = min\left(\frac{\rho_1}{2}, \frac{\rho_2}{2}, \frac{\rho_3}{2}, \frac{\rho_4}{2}\right)$ and g is a continuous function defined by $g(\varphi) = \varphi ||u||^2$.
Then,

$$\mathscr{L}(\mathscr{X}, u) \geq g(\varphi) \text{ and } ||u||^{-1} g(u) \to +\infty \text{ as } ||u|| \to +\infty,\ u \in \Omega.$$

Thus, all conditions are achieved. Therefore, we deduce the existence of an optimal control $u^* = \left(u_1^*, u_2^*, u_3^*, u_4^*\right)$ which minimizes the objective functional J. □

3.2. Characterization of optimal controls.

After establishing the existence of the optimal control that minimizes the objective functional J given by (2), we will characterize this optimal control by applying the Pontryagin's Maximum principle to the Hamiltonian.
Let $\mathscr{X} = (S, E, Q, I_A, I_S, R)$, $u = (u_1, u_2, u_3, u_4) \in \Omega$ and $\Lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$ the adjoint variable. The Hamiltonian function is defined as

$$\begin{aligned} \mathrm{H}(\mathscr{X}, u, \Lambda, t) = \quad & C_1 S(t) + C_2 E(t) + C_3 I_A(t) + C_4 I_S(t) + \frac{1}{2}\sum_{i=1}^{i=4} \rho_i u_i^2(t) + \lambda_1(t)\frac{dS(t)}{dt} + \lambda_2(t)\frac{dE(t)}{dt} \\ & + \lambda_3(t)\frac{dQ(t)}{dt} + \lambda_4(t)\frac{dI_A(t)}{dt} + \lambda_5(t)\frac{dI_S(t)}{dt} + \lambda_6(t)\frac{dR(t)}{dt}. \end{aligned} \tag{6}$$

If $(\mathscr{X}^*(t), u^*(t))$ is an optimal solution for the optimal control problem, then there exists a non-trivial vector function $\Lambda(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t))$ such that

$$\begin{aligned} \frac{d\mathscr{X}}{dt} &= \frac{\partial \mathrm{H}(\mathscr{X}^*(t), u^*(t), \Lambda(t), t)}{\partial \Lambda} \\ 0 &= \frac{\partial \mathrm{H}(\mathscr{X}^*(t), u^*(t), \Lambda(t), t)}{\partial u} \\ \frac{d\Lambda(t)}{dt} &= -\frac{\partial \mathrm{H}(\mathscr{X}^*(t), u^*(t), \Lambda(t), t)}{\partial \mathscr{X}}. \end{aligned} \tag{7}$$

Theorem 2. *Given an optimal control* $u^* = \left(u_1^*, u_2^*, u_3^*, u_4^*\right)$ *and a corresponding solutions* $\mathscr{X}^* = \left(S^*, E^*, Q^*, I_A^*, I_S^*, R^*\right)$ *that minimize* $J(u)$ *over* Ω. *Then, there exist adjoint variables* λ_i, $i =$

$1,2,\cdots,6$ *satisfying*

$$
\begin{aligned}
\frac{d\lambda_1(t)}{dt} &= -C_1+(1-u_2(t))\,[\lambda_1(t)-\lambda_2(t)]\,[\alpha E(t)+\xi I_A(t)+\psi I_S(t)]+\\
&\quad (\mu+u_3(t)+u_1(t))\,\lambda_1(t)-u_3(t)\lambda_3(t),\\
\frac{d\lambda_2(t)}{dt} &= -C_2+\alpha\,(1-u_2(t))\,[\lambda_1(t)-\lambda_2(t)]\,S(t)+(\mu+\beta+\gamma+u_3(t))\,\lambda_2(t)-u_3(t)\lambda_3(t)\\
&\quad -\beta\lambda_4(t)-\gamma\lambda_5(t),\\
\frac{d\lambda_3(t)}{dt} &= (\mu+\sigma+\delta+r_1)\,\lambda_3(t)-\sigma\lambda_4(t)-\delta\lambda_5(t)-r_1\lambda_6(t),\\
\frac{d\lambda_4(t)}{dt} &= -C_3+\xi\,(1-u_2(t))\,[\lambda_1(t)-\lambda_2(t)]S(t)+(r_2+u_4)\,[\lambda_4(t)-\lambda_6[t]+\mu\lambda_4(t),\\
\frac{d\lambda_5(t)}{dt} &= -C_4+\psi\,(1-u_2(t))\,[\lambda_1(t)-\lambda_2(t)]S(t)+(r_3+u_4)\,[\lambda_5(t)-\lambda_6[t]+(\mu+\eta)\,\lambda_5(t),\\
\frac{d\lambda_6(t)}{dt} &= \mu\lambda_6(t),
\end{aligned} \tag{8}
$$

where the transversality conditions $\lambda_i(t_f)=0$, $i=1,2,\cdots,6$. *Moreover, the following characterization holds:*

$$
\left\{
\begin{aligned}
u_1^*(t) &= \max\{\,\min\{\,1,\ \frac{\lambda_1(t)S^*(t)}{\rho_1}\,\},\ 0\,\},\\
u_2^*(t) &= \max\{\,\min\{\,1,\ \frac{[-\lambda_1(t)+\lambda_2(t)][\alpha E^*(t)+\xi I_A^*(t)+\psi I_S^*(t)]S^*(t)}{\rho_2}\,\},\ 0\,\},\\
u_3^*(t) &= \max\{\,\min\{\,1,\ \frac{[\lambda_1(t)-\lambda_3(t)]S^*(t)+[\lambda_2(t)-\lambda_3(t)]E^*(t)}{\rho_3}\,\},\ 0\,\},\\
u_4^*(t) &= \max\{\,\min\{\,1,\ \frac{[\lambda_4(t)-\lambda_6(t)]I_A^*(t)+[\lambda_5(t)-\lambda_6(t)]I_S^*(t)}{\rho_4}\,\},\ 0\,\},
\end{aligned}
\right. \tag{9}
$$

Proof. The form of the adjoint system (8) endowed with terminal conditions results from Pontryagin's Maximum Principle by differentiating the Hamiltonian function (6), at the respective solutions of the state system (1). Also, to get the characterization of the optimal control given by (9) we use the optimality conditions. After solving the equation

$$\frac{\partial \mathrm{H}(\mathscr{X}^*(t),u^*(t),\Lambda(t),t)}{\partial u_i}=0, \text{ for } i=1,2,3,4,$$

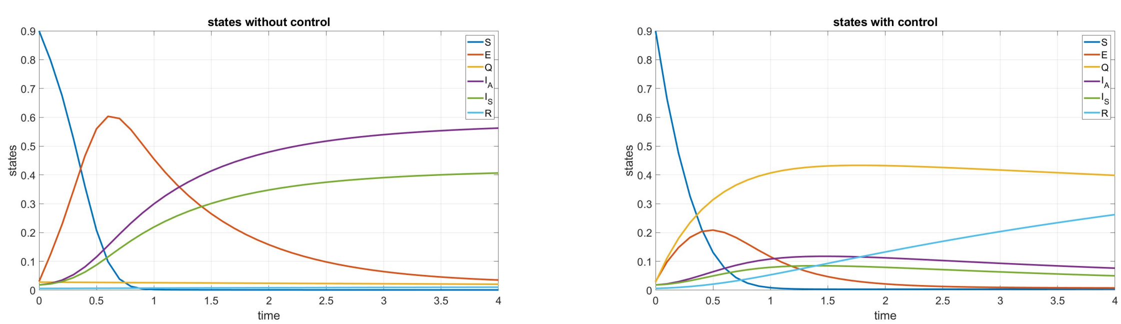
we obtain directly (9) by taking into consideration the boundedness condition. □

4. SIMULATION

In this section, we give numerical results of the optimal control of the COVID'19 pandemic model. The simulations are carried out using MATLAB R2021b. The results are simulated for one year divided into 1.5-month sections. The data are collected based heavily on the model parameters and coefficients values in [21].

The evolution of the six states without and with controls are represented in figures (2a) and (2b), respectively. These figures show that the number of susceptible individuals (S) decreases more rapidly in the case of the system without control, that because this number is transformed to exposed, or infected individuals due to an efficient contact. However, it is reasonable that this number decreases less rapidly in the case of the system with control, that because the transformation of this number to not for quarantined (Q) but for recovered individuals (R) takes more time. The controls bring the number of exposed (E) and infected individuals (I_a) and (I_s) to a small level and the number of recovered (R) to a high level.

To find out the best control strategy, we employ and simulate combinations of one, two, and three controls in the optimization system and examine the evolution of each state with the combination of controls applied. Our goal is to determine precisely the optimal controls for each state.



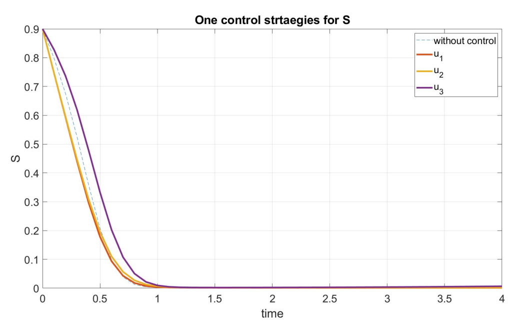
(a) States without controls

(b) States with controls

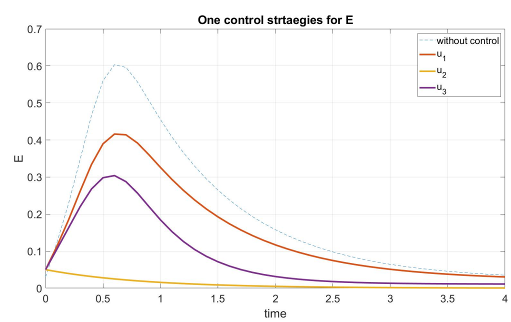
FIGURE 2. Comparison of states with and without controls

Scenario 1

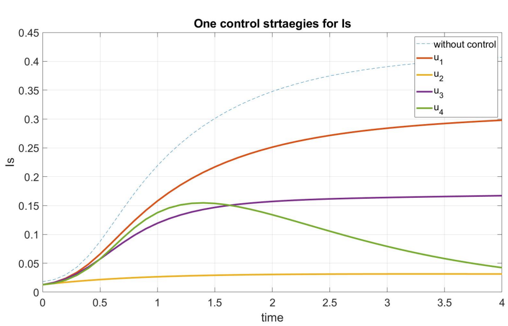
We first apply the four controls separately, in the optimal system and then simulate results in order to represent dynamics for each state when one control is applied. We apply u_1, u_2 and, u_3, separately, to minimize the number of susceptible and exposed individuals. Note that the intensive care control effect is not important for the number of (S), and (E). All three controls are efficient to reach the goal, as shown in figures (3a) and (3b). Furthermore, the vaccination and public health education controls decrease the number of (S) more rapidly. In another hand, the preventive measures efforts and public health education controls are more important for (E), considering vaccination control has an indirect effect on the number of (E). The results in figures (3c) and (3d) show that even if the public health education effort control is not applied directly to infected individuals, it has a more efficient effect on the decrease of the number of (I_a) and (I_s). Besides, intensive care control has also an efficient role for these states.



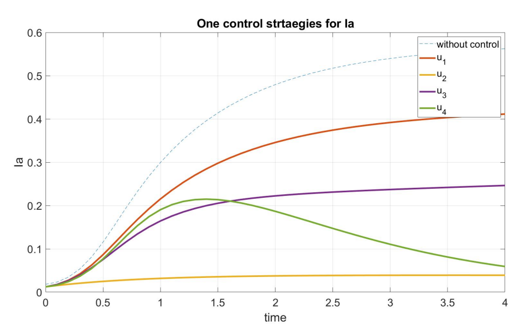
(a) Susceptible individuals (S)



(b) Exposed individuals (E)



(c) Symptomatic infected individuals (I_s)

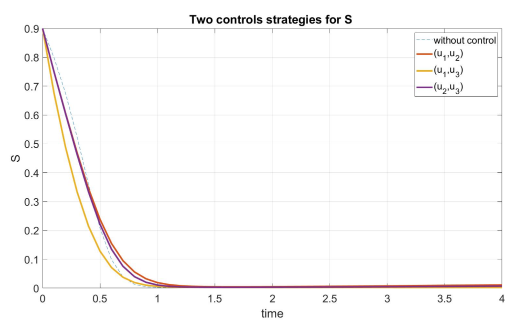


(d) Asymptomatic infected individuals (I_a)

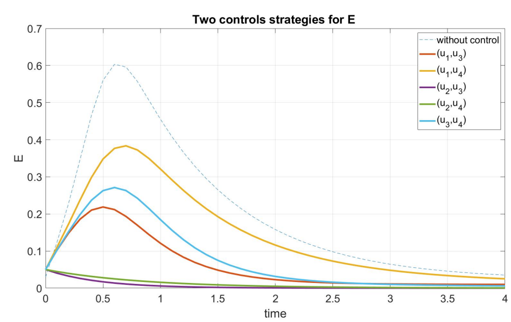
FIGURE 3. One control strategies

Scenario 2

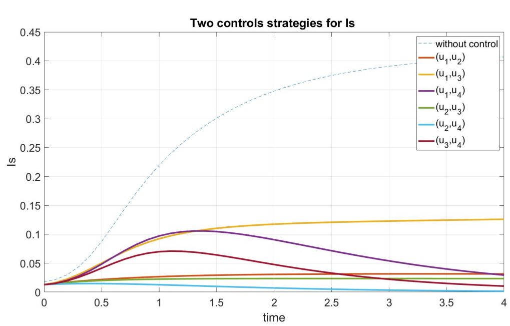
The second scenario is achieved by applying the combination of two controls each time. Applying combination of vaccination and public education health has efficient result on the number of (S) more than applying controls separately, comparing figures (3a) and (4a). However, two control strategy has no real impact on the numbers of exposed individuals regarded to one control strategy, as long as, combinations that have an efficient role contains consistently the public education effort control as shown in figure (4b). Moreover, the control u_2 is efficient also for the number of Infected. The reason is that decreasing exposed individuals means implicitly ing the number of infected ones. Although adding intensive care or preventive measures efforts controls besides control u_2 provides much more powerful control strategies as presented in (4c) and (4d).



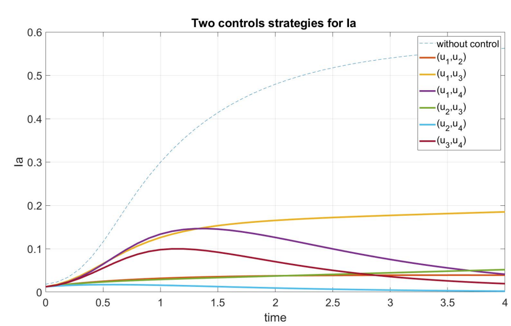
(a) Susceptible individuals S



(b) Exposed individuals E



(c) Symptomatic infected individuals Is

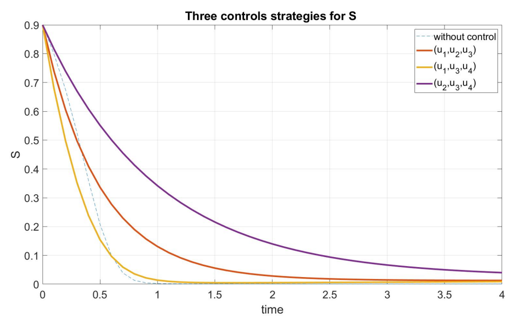


(d) Asymptomatic infected individuals Ia

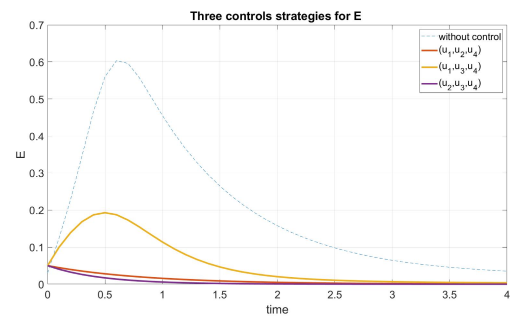
FIGURE 4. Two controls strategies

Scenario 3

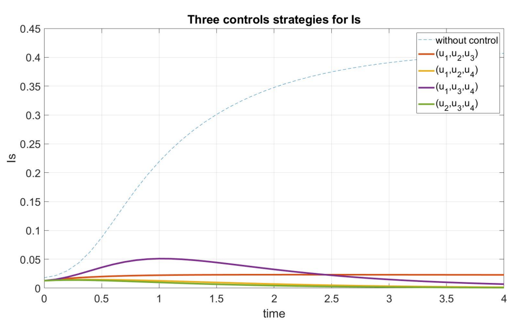
In this scenario, we examine the results of combinations of three controls each time. Either vaccination, public education health and, intensive care or preventive measures, public education health, and intensive care are strongly sufficient to minimize the infected individuals in a short time as presented in (5c) and (5d). However, this strategy has no real effect on the number of (S) and (E) in comparison with one and two strategies, figures (5a) and (5b).



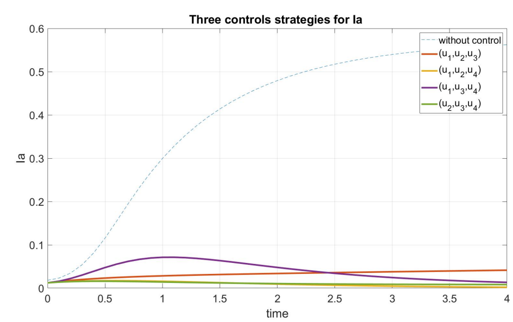
(a) Susceptible individuals (S)



(b) Exposed individuals (E)



(c) Symptomatic infected individuals (I_s)



(d) Asymptomatic infected individuals (I_a)

FIGURE 5. Three controls strategies

5. CONCLUSION

In this article, a mathematical model is proposed to study the dynamics of COVID'19 in an entire population. We linked our model to four control measures: vaccination, public health education, preventive measures, and, intensive care. The optimal control problem is formulated and analysed, and thus optimal control strategies are found by minimizing the number of susceptible individuals, exposed individuals, asymptomatic infected individuals, and symptomatic individuals, using the Pontryagin's Maximum Principle. A comparison between optimal controls and without controls is presented. Also, we tried to study all the possible combinations between the controls and analyse all the scenarios. All strategies are effective in minimizing susceptible individuals, exposed individuals, asymptomatic infected individuals, and symptomatic individuals and each of these strategies is an option depending on the desired purpose of the model control.

DATA AVAILABILITY

The data used to support the findings of this study are available from the corresponding author upon request.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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