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FRW COSMOLOGICAL MODELS IN PRESENCE OF A PERFECT FLUID WITHIN THE FRAMEWORK OF SAEZ-BALLESTER THEORY IN FIVE DIMENSIONAL SPACE TIME

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Abstract: In the framework of Saez-Ballester scalar-tensor theory of gravity, five-dimensional FRW space-time is considered in the presence of perfect fluid. We employed a power law between the scalar field and the universe's scale factor to get a definite solution to the field equations. Models that radiate flat, closed and open universe are shown. The model's physical features are also discussed.

Keywords: five dimension; FRW cosmological models; perfect fluid; Saez-Ballester theory; radiating models.

2010 AMS Subject Classification: 83C10, 83C15.

1. INTRODUCTION

The study of five-dimensional space-time is crucial because the cosmos may have had a higher-dimensional phase earlier in its existence. The experimental observation of time change of fundamental constants, according to Marciano [1], provides significant evidence for the presence of extra dimension. The extra dimension in space-time contracted or remained invariant to a very

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small size of Planck length. Furthermore, extra dimensions produce a substantial quantity of entropy during the contraction process, providing a different solution to the flatness and horizon problem [2]. Higher-dimensional cosmology drew the attention of Witten [3], Appelquist et al. [4], and Chodos and Detweiler [5] because it has physical significance to the early cosmos before compactification changes. A number of authors have recently investigated higher-dimensional space time using various cosmological models [6-10].

Perfect fluid and viscous fluid are the two most commonly encountered fluids in simulating our cosmos. Perfect fluid scientists refer to fluids whose rest frame pressure and density can be completely described. The pressure of a perfect fluid is isotropic, meaning that it has the same pressure in all directions. There is no shear stress, no conduction, and no viscosity in a perfect fluid. A number of authors [11-15] have investigated the perfect fluid using different cosmological models and different scalar-tensor.

Different cosmological models for the universe have been discovered using Einstein's general theory of relativity. However, in recent years, various modifications to Einstein's theory of gravitation have been made to accommodate certain desirable aspects that the original theory lacked. The field equations, for example, do not properly include Mach's principle into Einstein's theory. Furthermore, the current hypothesis of early inflation and late-time accelerated expansions of the cosmos is unaffected by general theory of relativity. As a result, various modifications to general relativity have been made to integrate the above desirable qualities. Scalar-tensor theories of gravitation presented by Brans and Dicke [16] and Saez and Ballester [17] are noteworthy, as are modified theories of gravity such as Nojiri and Odinstov’s [18] f(R) theory of gravity and Harko et al’s [19] f(R, T) theory of gravity. The construction of cosmological models of the universe to understand the universe's origin, mechanics, and ultimate fate has sparked a lot of interest in recent years. Scientists are increasingly interested in cosmological models based on the Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation.

The Brans-Dicke theory is a well-known challenger to Einstein's gravitational theory. It is the most basic scalar-tensor theory, in which the gravitational interaction is mediated by a scalar field ϕ as well as Einstein's tensor field g_{ij}. The scalar field ϕ has the same dimension as the global gravitational constant in this theory. Saez-Ballester proposed a scalar-tensor theory in which the metric is simply coupled with a dimensionless scalar field ϕ. Despite the scalar field's dimensionlessness, an anti-gravity regime develops in this theory. In addition, this theory adequately describes weak fields and proposes a solution to the ‘missing matter' problem in non-flat FRW cosmologies. Saez-Ballester's field equations for combined scalar and tensor fields are as follows:

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega \phi^n \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \tag{1}$$

where the equation is satisfied by the scalar field ϕ.

$$2\phi^n \phi^{;i}_{;i} + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \tag{2}$$

We also have

$$T^{ij}_{;j} = 0 \tag{3}$$

as a result of field equations (1) and (2). The conservation of matter source is physically represented by this equation. In this theory, we can also find equations of motion. The gravitational theory is strengthened by this equation. The constants are $\boldsymbol{\omega}$ and n, and partial and covariant differentiation are denoted by a comma and a semicolon, respectively. In general relativity, the other symbols have their usual meaning.

Some of the authors who have explored various parts of the Saez-Ballester theory in four and five dimensional space include Singh and Agrawal [20], Reddy and Rao [21], Reddy et al. [22], Mohanty and Sahoo [23], Adhav et al. [24], and Tripathy et al. [25]. Recently many researchers [26-32] have investigated Saez-Balester scalar-tensor theory using different cosmological models. We examine five-dimensional FRW models in Saez-Ballester theory of gravity as a result of the preceding investigations. The second section contains metric and field equations. We consider cosmological models in Saez-Ballester theory with an equation of state akin to disordered radiation in general relativity in Sect. 3. The physical explanation of the models is covered in Section 4, and the final section offers some conclusions.

2. METRIC AND FIELD EQUATIONS

In this paper, we look at the FRW five-dimensional space-time metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-mr^2} + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2) + (1-mr^2)d\psi^2\right] \tag{4}$$

The Einstein tensor's non-vanishing components for the metric (4) are

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{m}{a^2} \tag{5}$$

$$G_5^5 = 6\frac{\dot{a}^2}{a^2} + 6\frac{m}{a^2} \tag{6}$$

where an overhead dot denotes ordinary differentiation with respect to t and $m = 1, -1, 0$ for closed, open, and flat models, respectively.

For a perfect fluid distribution, the energy momentum tensor is given by

$$T_j^i = (\rho + p)u^i u_j - g_{ij}p \tag{7}$$

where,

$$u^i u_i = 1, \qquad u^i u_j = 0 \tag{8}$$

The field equations (1)–(3) can be expressed as with the help of equations (5) – (8) for the metric (4) using co moving coordinates

$$6\frac{\dot{a}^2}{a^2} + 6\frac{m}{a^2} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = -8\pi\rho \tag{9}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{m}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = 8\pi p \tag{10}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \tag{11}$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{12}$$

Hubble parameter H and deceleration parameter q are the physical parameters of importance for the observation

$$H = \frac{\dot{a}}{a} \tag{13}$$

$$q = \frac{-\left(\dot{H} + H^2\right)}{H^2} \tag{14}$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations (9)–(11) are three independent equations with four unknowns: a, p, ρ and ϕ [Eq. (12) is the result of Eqs. (9) - (11)]. As a result, an additional condition is required to obtain a definite answer. So, we employ the well-known relationship between a scalar field and the universe's scale factor *a(t)* given by [33]

$$\phi = \phi_0 a^n \tag{15}$$

Here $n > 0$ and ϕ_0 are constants.

In this particular case, we obtain the following physically significant models

3.1. CASE (I): For m=1 (i.e., closed model)

In this situation, using Eq. (15) and the field equations (9)–(11), the scale factor solutions are as follows

$$a(t) = \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}}\right)(a_0 t + t_0)\right]^{\frac{1}{\left(\frac{n^2}{2}+n+4\right)}} \tag{16}$$

We can now write the metric (4) with the help of (16) as (i.e. we choose $a_0 = 1, t_0 = 0$) with the right choice of coordinates and constants.

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \left[\frac{dr^2}{1 - r^2} + r^2(d\Theta^2 + sin^2\,\Theta\, d\phi^2) + (1 - r^2)d\psi^2\right] \tag{17}$$

In the model, together with the scalar field, given by

$$\phi = \phi_0 \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} \right) t \right]^{\frac{n}{\left(\frac{n^2}{2}+n+4\right)}} \tag{18}$$

The model (17) is a five-dimensional FRW radiating model with the physical parameters Volume *V*, Hubble parameter *H*, Energy Density *ρ*, and Isotropic Pressure *p*, all of which are relevant in cosmology discussions.

$$V = a^4 = \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} \right) t \right]^{\frac{4}{\left(\frac{n^2}{2}+n+4\right)}} \tag{19}$$

$$H = \left(\frac{1}{\frac{n^2}{2} + n + 4} \right) \frac{1}{t} \tag{20}$$

$$\rho = \frac{1}{8\pi} \left[\frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right] - 6 \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} t \right) \right]^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \tag{21}$$

$$p = \frac{1}{8\pi} \left[\frac{(\omega n^2 + 6)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} + 3 \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} t \right)^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} - 3 \left(\frac{\frac{n^2}{2} + n + 3}{\frac{n^2}{2} + n + 4} \right) \right] \tag{22}$$

3.2. CASE (II): For m=-1 (i.e., open model)

In this situation, the model is

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \left[\frac{dr^2}{1+r^2} + r^2(d\Theta^2 + sin^2\Theta\, d\phi^2) + (1+r^2)d\psi^2\right] \quad (23)$$

Here the energy density is given by

$$\rho = \frac{1}{8\pi}\left[\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right] + 6\left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}t\right)\right]^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \quad (24)$$

and the pressure is given by

$$p = \frac{1}{8\pi}\left[\frac{(\omega n^2+6)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2} - 3\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}t\right)^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} - 3\left(\frac{\frac{n^2}{2}+n+3}{\frac{n^2}{2}+n+4}\right)\right] \quad (25)$$

3.3. CASE (III): For m=0 (i.e., flat model)

In this situation, the model is transformed

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} [dr^2 + r^2(d\Theta^2 + sin^2\Theta\, d\phi^2) + d\psi^2] \quad (26)$$

In this model, the energy density is

$$\rho = \frac{1}{8\pi}\left[\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right] \quad (27)$$

The model's pressure is

$$p = \frac{1}{8\pi}\left[\frac{(\omega n^2 + 6)}{2t^2\left(\frac{n^2}{2} + n + 4\right)^2} - 3\left(\frac{\frac{n^2}{2} + n + 3}{\frac{n^2}{2} + n + 4}\right)\right] \qquad (28)$$

For all the Cases (i.e. closed, open, flat), the scalar field ϕ, volume V and the Hubble parameter H are same which are given by equation (18), (19) and (20) respectively.

Also the deceleration parameter q is same for all the cases, which is define by as follows

$$q = \left(\frac{n^2}{2} + n + 3\right) \qquad (29)$$

When $q > 0$, the cosmos decelerates in the usual fashion, and when $q < 0$, the universe accelerates. The models decelerate in the usual way here.

Here, $\pi = 3.14,\ n = 1,\ \omega = 500,\ \phi_0 = .001$ are used to draw all graphs.

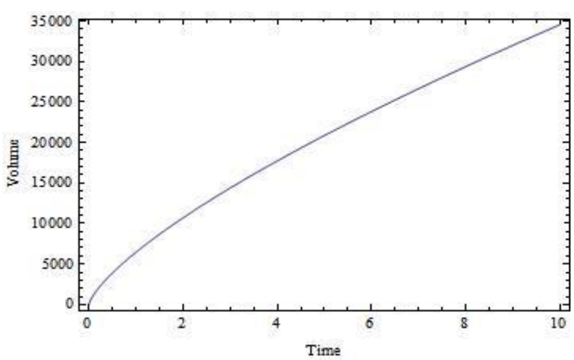

Fig-1. Volume V vs. time

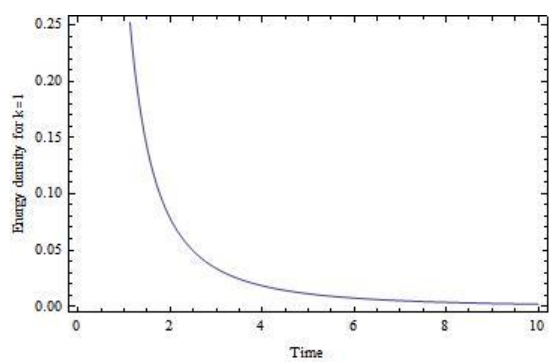

Fig-2. Density ρ vs. time for k=1

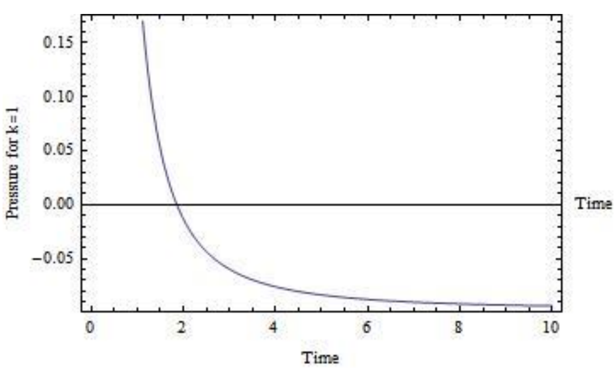

Fig-3. Pressure p vs. time for k=1

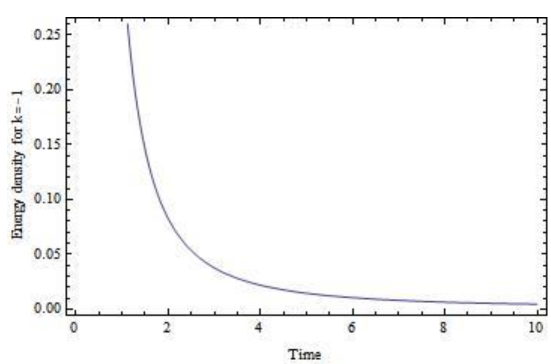

Fig-4. Density ρ vs. time for k= -1

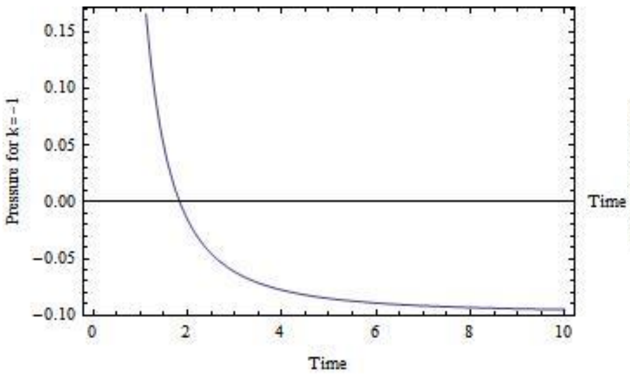

Fig-5. Pressure p vs. time for k= -1

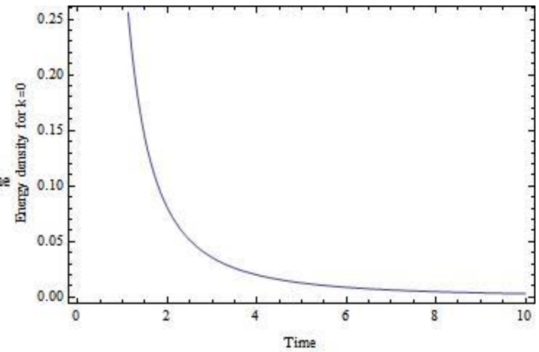

Fig-6. Density ρ vs. time for k= 0

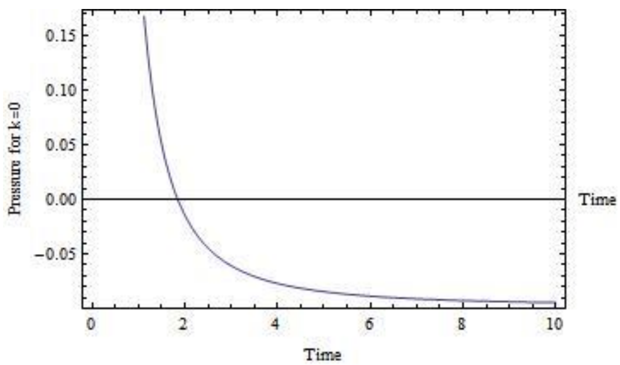

Fig-7, Pressure p vs. time for k= 0

4. PHYSICAL AND GEOMETRICAL INTERPRETATION

In Saez-Ballester theory, Equations (17), (23) and (26) represent FRW five-dimensional radiating closed, open, and flat models. It is worth noting that none of the models have an initial singularity. The pressure and energy density of closed and open models diverge at t = 0 and decrease over time [As shown in fig-2, fig-3, fig-4 and fig-5]. The energy density and pressure in the flat model diminish with time and eventually vanish for infinitely large t [As shown in fig-6 and fig-7]. They also diverge in the first epoch. All of the models have the same spatial volume, which grows with time and goes to infinity after an infinitely long period of time [As shown in fig-1]. For all models, the average Hubble's parameter, provided by Eq. (20), will diverge at the start epoch, i.e. at t = 0, and will approach infinity as time t gets indefinitely big. The models will aid in our understanding of spatially homogeneous isotropic universes in five dimensions immediately prior to compactification. In all of the models, the scalar field grows with time. In each Case, the deceleration parameter is $q = \left(\frac{n^2}{2} + n + 3\right)$, indicating that the models in five dimensions decelerate in a typical manner. The FRW five-dimensional models obtained here are significantly distinct from the Kaluza–Klein five-dimensional models obtained by various authors previously stated and are quite interesting.

5. Conclusion

In this research, cosmological models corresponding to perfect fluid dispersion with trace free matter source were constructed using the Saez-Ballester theory of gravitation. In five dimensions, the models obtained depict closed, open, and flat FRW radiating perfect fluid types. All physical quantities diverge at the beginning of the epoch and vanish for infinitely large values of cosmic time. The models will aid in our understanding of spatially homogeneous and isotropic universes in five dimensions just prior to compactification.

Conflict of Interests

The authors (s) declare that there is no conflict of interests.

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