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A FUZZY INVENTORY MODEL HAVING LEAKAGE ALONG WITH SHORTAGE

HUIDROM MALEMNGANBI[1,*], M. KUBER SINGH[2]

[1]Department of Mathematics, Manipur University, Canchipur-795003, India

[2]Department of Mathematics, D M College of Science, Imphal-795001, India

Abstract. In this paper, a crisp leakage inventory model is extended to fuzzy environment. Leakage is an unavoidable phenomenon in an inventory system involving liquid or gaseous stock. The rate of leakage and the rate of demand cannot be considered as a fixed quantity through out the functioning of the system. Fuzzy set theory is applied to deal these variabilities and uncertainties of the situation. Triangular fuzzy number is considered for the process of fuzzification. The concept of α-cut and fuzzy intervals is used for the signed distance method of defuzzification. Numerical example is provided to support the result of the proposed fuzzy model.

Keywords: leakage; α-cut; fuzzy interval; signed distance.

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1. Introduction

Inventory model is a mathematically formulated model or replica of the business set-up or inventory system. It helps the management of the business in determining the optimum level of inventories that should be maintained in a production process, managing frequency of ordering, deciding on quantity of goods or raw materials to be stored, tracking flow of supply of raw materials and goods to provide etc. The features of the actual inventory system are represented

*Corresponding author

E-mail address: malem_feb82012@yahoo.in

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in the model. So, a leakage inventory model will be depicting an inventory system dealing with liquid or gaseous stock with the possibility of having leakage. Leakage may be defined as the process where the material (liquid or gas) is lost through a leak in the storage facility or in-transit, which is a common phenomenon. This reduces the quantity of the stock and hence is not a favourable condition thereby becoming an important issue while developing or discussing the economic order quantity(EOQ) or production order quantity(POQ) inventory models. Leakage can also be treated as a form of deterioration as it refer to the loss of the value of the stock quantitatively. So, in this view, Tomba and Geeta [1] developed leakage inventory model with finite and infinite demands having no shortages with instantaneous or finite production . Geeta [2] treated leakage as a separate entity to develop a model with uniform rate of demand and instantaneous production with shortage.

In general, the parameters involved in the inventory model are considered to have exact value. But in reality, there are always fluctuations and variations. So these exactness seem to be unrealistic and hence the idea of fuzziness of Zadeh[3] is applied at this point. Thereby, fuzzy inventory models are discussed as an extension of the traditional or crisp inventory models. Yao and Lee [4] discussed inventory model with shortages in fuzzy sense. Lee and Yao [5] portrayed production inventory problems with fuzziness to suit the real world situations in a more realistic way. They used triangular fuzzy numbers and defuzzify the demand quantity and production quantity. Chang[6] developed a fuzzy production inventory by using triangular fuzzy number. Yao and Su [7] studied the fuzzy inventory model with backorder based on interval-valued fuzzy set. Chen and Ouyang [8], in their paper extended an ordering policy for deteriorating items with allowable shortage and permissible delay in payment to fuzzy model by fuzzifying the carrying cost, interest paid rate and interest earned rate simultaneously based on the interval-valued triangular fuzzy numbers. Chang et al. [9] discussed a fuzzy mixed inventory model involving variable lead-time with backorder and lost sales using probabilistic fuzzy set and triangular fuzzy number. They used two methods of defuzzification. α-cut approach of fuzzy arithmetic operations for fuzzy intervals is a standard way of fuzzification and defuzzification process. Shang et. al [10] explained the α-cut interval method of multiplication operation of fuzzy numbers. Chandrasekaran and Tamilmani [11], in their paper, highlighted the arithmetic operation of fuzzy numbers using α-cut method. Chang [12] discussed an EOQ model with imperfect quality items by applying the fuzzy set theory considering fuzzy defective rate and fuzzy annual demand. Mahata [13] also applied fuzzy theory in an EOQ model for items with imperfect quality using decomposition principle and α-cut theory. Malemnganbi and Kuber[14] extended a crisp leakage inventory model with shortage to fuzzy model taking holding cost and shortage cost as triangular as well as trapezoidal fuzzy numbers.

On the basis of the work of Geeta[2], we propose a fuzzy model incorporating the fuzziness of the demand rate and leakage rate simultaneously. To do so, we use the Yao and Wu's [15] ranking method for fuzzy numbers to find the optimum total profit per unit time in the fuzzy sense and also calculate the optimum lot size for the proposed model.

2. Brief Review of the Crisp Model

Geeta [2] developed a leakage inventory model with instantaneous production and lead time time zero, allowing shortages. The graphical representation of the inventory system is shown in Figure 1 below, where Q is the total demand per production run, z is the order quantity per run and t is the time interval between consecutive runs.

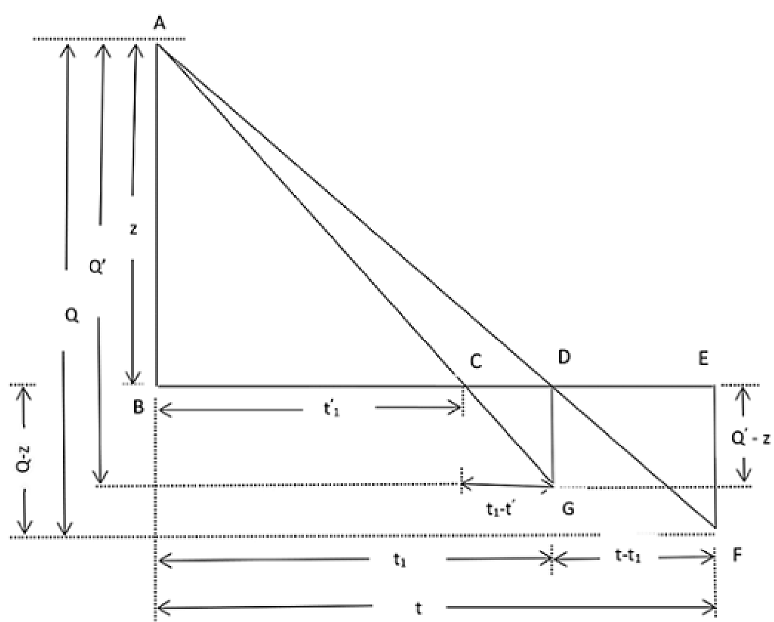

Figure 1: Graphical representation of the inventory system

From the figure,

average total cost, TC(z) $=\frac{1}{t}$(total holding cost + total shortage cost)

$=\frac{1}{t}\{C_h$(area of ΔABD + area of ΔACD) + C_s(area of ΔCDG + area of ΔDEF)$\}$

$=\frac{1}{t}[C_h\{\frac{1}{2}t_1z+\frac{1}{2}z(t_1-t'_1)\}+C_s\{\frac{1}{2}(t_1-t'_1)(Q'-z)+\frac{1}{2}(t-t_1)(Q-z)\}]$

$$=\frac{C_hz^2}{2Q}(\frac{\phi+2\psi}{\phi+\psi})+\frac{C_s}{2Q}[(Q-z)^2+\frac{\psi^2z^2}{\phi(\phi+\psi)}] \tag{1}$$

(by making use of the relations $t_1=\frac{z}{\phi}$, $Q=\phi t$, $t_1{}'=\frac{z}{\phi+\psi}$, $Q'=(\phi+\psi)t_1$ which are self explanatory from the figure)

Here, ϕ, ψ, C_h and C_s denote the demand rate per unit time, leakage rate per unit time, holding cost per unit quantity per unit time and shortage cost per unit quantity per unit time respectively.

3. Definitions and Preliminaries

3.1. Fuzzy set. If X is a collection of objects whose members are denoted by x, then a fuzzy set $\tilde{A}$ in X is defined as $\tilde{A}$=$\{(x,\mu_{\tilde{A}}(x)):x\in X\}$, where $\mu_{\tilde{A}}:X\to[0,1]$ is a function called the membership function of the fuzzy set $\tilde{A}$.

3.2. Fuzzy point. A fuzzy set $\tilde{a}$ defined on $\mathbb{R}$ is called a fuzzy point if its membership function is given by

$$\mu_{\tilde{a}}(x)=\begin{cases}1 & \text{if } x=a\\ 0 & \text{if } x\neq a\end{cases}$$

3.3. α-level fuzzy point. Let $\tilde{a}_\alpha$ $(0\leqslant\alpha\leqslant1)$ be a fuzzy set defined on $\mathbb{R}$. It is called an α-level fuzzy point if its membership function is defined as

$$\mu_{\tilde{a}_\alpha}(x)=\begin{cases}\alpha & \text{if } x=a\\ 0 & \text{if } x\neq a\end{cases}$$

3.4. Triangular Fuzzy Number. A fuzzy set $\tilde{A}$ denoted by $\tilde{A}=(a,b,c)$ defined on $\mathbb{R}$, where $a<b<c$ is called a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leqslant x \leqslant b \\ \frac{c-x}{c-b} & \text{if } b \leqslant x \leqslant c \\ 0 & \text{if otherwise} \end{cases}$$

When a = b = c, $\tilde{A} = (a,a,a) = \tilde{a}$ which becomes a fuzzy point.

3.5. α-level fuzzy interval. For $0 \leqslant \alpha \leqslant 1$, the fuzzy set $[c_\alpha, d_\alpha]$ defined on $\mathbb{R}$ is called an α-level fuzzy interval if the membership function of $[c_\alpha, d_\alpha]$ is given by

$$\mu_{[c_\alpha, d_\alpha]}(y) = \begin{cases} \alpha & \text{if } c \leqslant y \leqslant d \\ 0 & \text{if otherwise} \end{cases}$$

3.6. α-cut of a fuzzy set. Let $\tilde{B}$ be a fuzzy set on $\mathbb{R}$ and $0 \leqslant \alpha \leqslant 1$. Then, the α-cut $B(\alpha)$ of $\tilde{B}$ consists of points y such that $\mu_{\tilde{B}}(y) \geqslant \alpha$, i.e., $B(\alpha) = \{y | \mu_{\tilde{B}}(y) \geqslant \alpha\}$.

3.7. Decomposition Principle [15],[16]**.** Let $\tilde{B}$ be a fuzzy set on $\mathbb{R}$ and $0 \leqslant \alpha \leqslant 1$. Let us suppose the α-cut of $\tilde{B}$ as a closed interval $[B_L(\alpha), B_R(\alpha)]$, i.e., $B(\alpha) = [B_L(\alpha), B_R(\alpha)]$. Then, by decomposition theorem, we have

$$\tilde{B} = \bigcup_{0 \leqslant \alpha \leqslant 1} \alpha B(\alpha) = \bigcup_{0 \leqslant \alpha \leqslant 1} [B_L(\alpha)_\alpha, B_R(\alpha)_\alpha] \tag{2}$$

or

$$\mu_{\tilde{B}}(y) = \bigvee_{0 \leqslant \alpha \leqslant 1} \alpha C_{B(\alpha)}(y) = \bigvee_{0 \leqslant \alpha \leqslant 1} \mu_{[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]}(y) \tag{3}$$

where, $\alpha B(\alpha)$ is a fuzzy set whose membership function is given by

$$\mu_{\alpha B(\alpha)}(y) = \begin{cases} \alpha & \text{if } y \in B(\alpha) \\ 0 & \text{if otherwise} \end{cases}$$

and $C_{B(\alpha)}(y)$ is called characteristic function of $B(\alpha)$ whose membership function is defined as

$$C_{B(\alpha)}(y) = \begin{cases} 1 & \text{if } y \in B(\alpha) \\ 0 & \text{if } y \notin B(\alpha) \end{cases}$$

Properties: For any a, b, c, d, k $\in \mathbb{R}$ and $a < b,\ c < d$, the interval operations are defined as follows:

(i) $[a,b]+[c,d]=[a+c,b+d]$

(ii) $[a,b]-[c,d]=[a-d,b-c]$

(iii) $k(.)[a,b]=\begin{cases}[ka,kb] & \text{if } k>0\\ [kb,ka] & \text{if } k<0\end{cases}$

Furthermore, for $a>0$ and $c>0$,

(iv) $[a,b](.)[c,d]=[ac,bd]$

(v) $[a,b](\div)[c,d]=[\frac{a}{d}.\frac{b}{c}]$

3.8. Signed distance (as in Yao and Wu[15]**).** For any $a\in\mathbb{R}$, the signed distance from a to 0 is defined as $d_0(a,0)=a$. If $a>0$, then the distance from a to 0 is $a=d_0(a,0)$. If $a<0$, then the distance from a to 0 is $-a=-d_0(a,0)$. Hence, $d_0(a,0)=a$ is known as the signed distance from a to 0.

Let Ω denote the family of all fuzzy sets defined on $\mathbb{R}$. For $\tilde{B}\in\Omega$ with the α-cut $B(\alpha)=[B_L(\alpha),B_R(\alpha)],\ \alpha\in[0,1]$ where both $B_L(\alpha)$ and $B_R(\alpha)$ are continuous functions on $\alpha\in[0,1]$, the signed distance of the two end-points $B_L(\alpha)$ and $B_R(\alpha)$ of this α-cut to the origin 0 is $d_0(B_L(\alpha),0)=B_L(\alpha)$ and $d_0(B_R(\alpha),0)=B_R(\alpha)$ respectively (by the definition stated above). Their average $\frac{B_L(\alpha)+B_R(\alpha)}{2}$ is taken as the signed distance of the α-cut $[B_L(\alpha),B_R(\alpha)]$ to 0, i.e., the signed distance of the interval $[B_L(\alpha),B_R(\alpha)]$ to 0 is defined as

$$d_0([B_L(\alpha),B_R(\alpha)],0)=\frac{d_0(B_L(\alpha),0)+d_0(B_R(\alpha),0)}{2}=\frac{B_L(\alpha)+B_R(\alpha)}{2}$$

Since the crisp interval $[B_L(\alpha),B_R(\alpha)]$ has a one-to-one correspondence with the α-level fuzzy interval $[B_L(\alpha)_\alpha,B_R(\alpha)_\alpha]$, for every $\alpha\in[0,1]$, we have

$$[B_L(\alpha),B_R(\alpha)]\leftrightarrow[B_L(\alpha)_\alpha,B_R(\alpha)_\alpha] \tag{4}$$

Also, the real number 0 maps to fuzzy point $\tilde{0}$. Hence, the signed distance of $[B_L(\alpha)_\alpha,B_U(\alpha)_\alpha]$ to $\tilde{0}$ is defined as

$$d([B_L(\alpha)_\alpha,B_U(\alpha)_\alpha],\tilde{0})=d_0([B_L(\alpha),B_R(\alpha)],0)=\frac{B_L(\alpha)+B_R(\alpha)}{2} \tag{5}$$

Using integration, the mean value of the signed distance is obtained as

$$\int_0^1 d([B_L(\alpha)_\alpha, B_R(\alpha)_\alpha], \tilde{0})d\alpha = \frac{1}{2}\int_0^1 \{B_L(\alpha)+B_R(\alpha)\}d\alpha \tag{6}$$

From eqns. (6) and (2), we get

$$d(\tilde{B},\tilde{0}) = \int_0^1 d_0([B_L(\alpha)_\alpha, B_R(\alpha)_\alpha], \tilde{0})d\alpha = \frac{1}{2}\int_0^1 \{B_L(\alpha)+B_R(\alpha)\}d\alpha \tag{7}$$

and accordingly, the following results can also be obtained:

Results:

1. If $\tilde{A} = (a,b,c)$ is a triangular fuzzy number, then the α-cut of $\tilde{A}$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$, where $A_L(\alpha)$ = a+(b-a)α, $A_R(\alpha)$= c-(c-b)α.

2. If $\tilde{A} = (a,b,c)$ is a triangular fuzzy number, then the signed distance of $\tilde{A}$ to $\tilde{0}$ is $d(\tilde{A},\tilde{0}) = \frac{1}{2}\int_0^1 \{A_L(\alpha)+A_R(\alpha)\}d\alpha = \frac{1}{4}(a+2b+c)$

3. For two fuzzy sets $\tilde{B},\tilde{D} \in \Omega$, where $\tilde{B} = \bigcup_{0\leqslant\alpha\leqslant1}[B_L(\alpha)_\alpha, B_R(\alpha)_\alpha]$ and $\tilde{D} = \bigcup_{0\leqslant\alpha\leqslant1}[D_L(\alpha)_\alpha, D_R(\alpha)_\alpha]$, from the above properties and eqn. (4), we have

(i) $\tilde{B}(+)\tilde{D} = \bigcup_{0\leqslant\alpha\leqslant1}[(B_L(\alpha)+D_L(\alpha))_\alpha, (B_R(\alpha)+D_R(\alpha)_\alpha]$

(ii) $\tilde{B}(-)\tilde{D} = \bigcup_{0\leqslant\alpha\leqslant1}[(B_L(\alpha)-D_L(\alpha))_\alpha, (B_R(\alpha)-D_R(\alpha))_\alpha]$

(iii) $$k(.)\tilde{B} = \begin{cases} \bigcup_{0\leqslant\alpha\leqslant1}[(kB_L(\alpha))_\alpha, (kB_R(\alpha))_\alpha] & \text{if } k>0 \\ \bigcup_{0\leqslant\alpha\leqslant1}[(kB_R(\alpha))_\alpha, (kB_L(\alpha))_\alpha] & \text{if } k<0 \\ \tilde{0} & \text{if } k=0 \end{cases}, k \in \mathbb{R}$$

(iv) $\tilde{B}(.)\tilde{D} = \bigcup_{0\leqslant\alpha\leqslant1}[(B_L(\alpha).D_L(\alpha))_\alpha, (B_R(\alpha).D_R(\alpha))_\alpha]$

(v) $\tilde{B}(\div)\tilde{D} = \bigcup_{0\leqslant\alpha\leqslant1}\left[\left(\frac{B_L(\alpha)}{D_R(\alpha)}\right)_\alpha, \left(\frac{B_R(\alpha)}{D_L(\alpha)}\right)_\alpha\right]$

Special case: $\tilde{B}(.)\tilde{B} = \bigcup_{0\leqslant\alpha\leqslant1}[(B_L(\alpha).B_L(\alpha))_\alpha, (B_R(\alpha).B_R(\alpha))_\alpha]$

4. Fuzzy Model

To develop the fuzzy model, we consider the demand rate ϕ and leakage rate ψ of Geeta's[2] model to be imprecise in nature. So, we treat them as triangular fuzzy numbers $\tilde{\phi}$ and $\tilde{\psi}$ respectively, defined as $\tilde{\phi} = (\phi - \Delta_1, \phi, \phi + \Delta_2)$ and $\tilde{\psi} = (\psi - \Delta_3, \psi, \psi + \Delta_4)$, where $0 < \Delta_1, \Delta_2 < \phi$ and $0 < \Delta_3, \Delta_4 < \psi$. $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are the independent variables to be determined by the decision makers.

Incorporating the fuzziness of the demand rate and leakage rate in eqn. (1), we get the fuzzy total cost as

$$\tilde{TC} = \frac{C_h z^2}{2Q}\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}\right) + \frac{C_s}{2Q}\left[(Q-z)^2 + \frac{z^2\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}\right] \tag{8}$$

On defuzzification, we get

$$Z(z) = d(\tilde{TC}, \tilde{0}) = \frac{C_h z^2}{2Q} d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}, \tilde{0}\right) + \frac{C_s}{2Q}\left[(Q-z)^2 + z^2 d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}, \tilde{0}\right)\right] \tag{9}$$

Let $\tilde{\eta} = \tilde{\phi} + 2\tilde{\psi}$

$$\begin{aligned} &= (\phi - \Delta_1, \phi, \phi + \Delta_2) + 2(\psi - \Delta_3, \psi, \psi + \Delta_4) \\ &= ((\phi + 2\psi - \Delta_1 - 2\Delta_3, \phi + 2\psi, \phi + 2\psi + \Delta_2 + 2\Delta_4)) \\ &= (\eta - \Delta_5, \eta, \eta_+ \Delta_6) \end{aligned}$$

where

$$\Delta_5 = \Delta_1 + 2\Delta_3 \tag{10}$$

$$\Delta_6 = \Delta_2 + 2\Delta_4 \tag{11}$$

Let $\tilde{\beta} = \tilde{\phi} + \tilde{\psi}$

$$\begin{aligned} &= (\phi + \psi - \Delta_1 - \Delta_3, \phi + \psi, \phi + \psi + \Delta_2 + \Delta 4) \\ &= (\beta - \Delta_7, \beta, \beta + \Delta_8) \end{aligned}$$

where

$$\Delta_7 = \Delta_1 + \Delta_3 \tag{12}$$

$$\Delta_8 = \Delta_2 + \Delta_4 \tag{13}$$

Let $\tilde{\nu} = \tilde{\phi}(\tilde{\phi} + \tilde{\psi})$

$$= \tilde{\phi}\tilde{\beta}$$

$$= (\phi-\Delta_1,\phi,\phi+\Delta_2)(\beta-\Delta_7,\beta,\beta+\Delta_8)$$
$$= ((\phi-\Delta_1)(\beta-\Delta_7),\phi\beta,(\phi+\Delta_2)(\beta+\Delta_8))$$
$$= (\phi\beta-\phi\Delta_7-\beta\Delta_1+\Delta_1\Delta_7,\phi\beta,\phi\beta+\phi\Delta_8+\beta\Delta_2+\Delta_2\Delta_8)$$
$$= (\nu-\Delta_9,\nu,\nu+\Delta_{10})$$

where

$$\Delta_9 = \phi\Delta_7+\beta\Delta_1-\Delta_1\Delta_7 \tag{14}$$

$$\Delta_{10} = \phi\Delta_8+\beta\Delta_2-\Delta_2\Delta_8 \tag{15}$$

Let $\tilde{\theta} = \tilde{\psi}^2$

$$= (\psi-\Delta_3,\psi,\psi+\Delta_4)(\psi-\Delta_3,\psi,\psi+\Delta_4)$$
$$= (\psi^2+{\Delta_3}^2-2\psi\Delta_3,\psi^2,\psi^2+{\Delta_4}^2+2\Delta_4\psi)$$
$$= (\theta-\Delta_{11},\theta,\theta+\Delta_{12})$$

where

$$\Delta_{11} = 2\psi\Delta_3-{\Delta_3}^2 \tag{16}$$

$$\Delta_{12} = 2\Delta_4\psi+{\Delta_4}^2 \tag{17}$$

For $0 \leqslant \alpha \leqslant 1$,

Left end-point of $\tilde{\alpha}$-cut of $\tilde{\phi}+2\tilde{\psi}$, i.e., $\tilde{\eta}$ is $(\eta-\Delta_5)+\Delta_5\alpha$.

Right end-point of α-cut of $\tilde{\phi}+2\tilde{\psi}$, i.e., $\tilde{\eta}$ is $(\eta+\Delta_6)-\Delta_6\alpha$.

Left end-point of α-cut of $\tilde{\phi}+\tilde{\psi}$, i.e., $\tilde{\beta}$ is $(\beta-\Delta_7)+\Delta_7\alpha$.

Right end-point of α-cut of $\tilde{\phi}+\tilde{\psi}$, i.e., $\tilde{\beta}$ is $(\beta+\Delta_8)-\Delta_8\alpha$.

Left end-point of α-cut of $\tilde{\phi}(\tilde{\phi}+\tilde{\psi})$, i.e., $\tilde{\nu}$ is $(\nu-\Delta_9)+\Delta_9\alpha$.

Right end-point of α-cut of $\tilde{\phi}(\tilde{\phi}+\tilde{\psi})$, i.e., $\tilde{\nu}$ is $(\nu+\Delta_{10})-\Delta_{10}\alpha$.

Left-end point of α-cut of $\tilde{\psi}^2$, i.e., $\tilde{\theta}$ is $(\theta-\Delta_{11})+\Delta_{11}\alpha$.

Right end-point of α-cut of $\tilde{\phi}^2$, i.e., $\tilde{\theta}$ is $(\theta+\Delta_{12})-\Delta_{12}\alpha$.

From the above results, the left and right end-points of α-cut of $\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}$ are

$$\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}\right)_L(\alpha) = \left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_L(\alpha) = \frac{\eta_L(\alpha)}{\beta_R(\alpha)} = \frac{(\eta-\Delta_5)+\Delta_5\alpha}{(\beta+\Delta_8)-\Delta_8\alpha} \tag{18}$$

$$\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}\right)_R(\alpha)=\left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_R(\alpha)=\frac{\eta_R(\alpha)}{\beta_L(\alpha)}=\frac{(\eta+\Delta_6)-\Delta_6\alpha}{(\beta-\Delta_7)+\Delta_7\alpha} \tag{19}$$

Then, the signed distance of $\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}}$ to $\tilde{0}$ is

$$\begin{aligned}
d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}},\tilde{0}\right)&=d\left(\frac{\tilde{\eta}}{\tilde{\beta}},\tilde{0}\right)\\
&=\frac{1}{2}\int_0^1\left[\left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_L(\alpha)+\left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_R(\alpha)\right]d\alpha\\
&=\frac{1}{2}\int_0^1\left[\frac{(\eta-\Delta_5)+\Delta_5\alpha}{(\beta+\Delta_8)-\Delta_8\alpha}+\frac{(\eta+\Delta_6)-\Delta_6\alpha}{(\beta-\Delta_7)+\Delta_7\alpha}\right]d\alpha\\
&=\frac{1}{2}\left\{\left(\frac{\eta\Delta_8+\beta\Delta_5}{\Delta_8^2}\right)\log\left(\frac{\beta+\Delta_8}{\beta}\right)+\left(\frac{\eta\Delta_7+\beta\Delta_6}{\Delta_7^2}\right)\log\left(\frac{\beta}{\beta-\Delta_7}\right)-\frac{\Delta_5}{\Delta_8}-\frac{\Delta_6}{\Delta_7}\right\} \qquad (20)
\end{aligned}$$

which is positive since $\left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_L(\alpha)>0$ and $\left(\frac{\tilde{\eta}}{\tilde{\beta}}\right)_R(\alpha)>0$ are continuous functions on $0\leqslant\alpha\leqslant 1$ and hence the above definite integral is positive.

Similarly, the left and right end-points of α-cut of $\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}$ are

$$\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}\right)_L(\alpha)=\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_L(\alpha)=\frac{\theta_L(\alpha)}{\nu_R(\alpha)}=\frac{(\theta-\Delta_{11})+\Delta_{11}\alpha}{(\nu+\Delta_{10})-\Delta_{10}\alpha} \tag{21}$$

$$\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}\right)_R(\alpha)=\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_R(\alpha)=\frac{\theta_R(\alpha)}{\nu_L(\alpha)}=\frac{(\theta+\Delta_{12})-\Delta_{12}\alpha}{(\nu-\Delta_9)+\Delta_9\alpha} \tag{22}$$

Then, the signed distance of $\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})}$ to $\tilde{0}$ is

$$\begin{aligned}
d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})},\tilde{0}\right)&=d\left(\frac{\tilde{\theta}}{\tilde{\nu}},\tilde{0}\right)\\
&=\frac{1}{2}\int_0^1\left[\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_L(\alpha)+\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_R(\alpha)\right]d\alpha\\
&=\frac{1}{2}\int_0^1\left[\frac{(\theta-\Delta_{11})+\Delta_{11}\alpha}{(\nu+\Delta_{10})-\Delta_{10}\alpha}+\frac{(\theta+\Delta_{12})-\Delta_{12}\alpha}{(\nu-\Delta_9)+\Delta_9\alpha}\right]d\alpha\\
&=\frac{1}{2}\left\{\left(\frac{\theta\Delta_{10}+\nu\Delta_{11}}{\Delta_{10}^2}\right)\log\left(\frac{\nu+\Delta_{10}}{\nu}\right)+\left(\frac{\theta\Delta_9+\nu\Delta_{12}}{\Delta_9^2}\right)\log\left(\frac{\nu}{\nu-\Delta_9}\right)-\frac{\Delta_{11}}{\Delta_{10}}-\frac{\Delta_{12}}{\Delta_9}\right\} \qquad (23)
\end{aligned}$$

which is positive since $\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_L(\alpha)>0$ and $\left(\frac{\tilde{\theta}}{\tilde{\nu}}\right)_R(\alpha)>0$ are continuous functions on $0\leqslant\alpha\leqslant 1$ and hence the above definite integral is positive.

Using the values of $d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}},\tilde{0}\right)$ and $d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})},\tilde{0}\right)$ from eqns. (20) and (23) in eqn. (9), we get

$$Z(z) = d(\tilde{TC},\tilde{0}) = \frac{C_h z^2}{2Q}\omega + \frac{C_s}{2Q}[(Q-z)^2 + z^2\rho)] \tag{24}$$

where

$$\omega = d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}},\tilde{0}\right)$$
$$= \frac{1}{2}\left\{\left(\frac{\eta\Delta_8+\beta\Delta_5}{\Delta_8^2}\right)\log\left(\frac{\beta+\Delta_8}{\beta}\right) + \left(\frac{\eta\Delta_7+\beta\Delta_6}{\Delta_7^2}\right)\log\left(\frac{\beta}{\beta-\Delta_7}\right) - \frac{\Delta_5}{\Delta_8} - \frac{\Delta_6}{\Delta_7}\right\} \tag{25}$$

$$\rho = d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})},\tilde{0}\right)$$
$$= \frac{1}{2}\left\{\left(\frac{\theta\Delta_{10}+\nu\Delta_{11}}{\Delta_{10}^2}\right)\log\left(\frac{\nu+\Delta_{10}}{\nu}\right) + \left(\frac{\theta\Delta_9+\nu\Delta_{12}}{\Delta_9^2}\right)\log\left(\frac{\nu}{\nu-\Delta_9}\right) - \frac{\Delta_{11}}{\Delta_{10}} - \frac{\Delta_{12}}{\Delta_9}\right\} \tag{26}$$

Equating the first order partial derivative of Z(z) of eqn.(24) (with respect to z) to zero, we get

$$z = \frac{C_s Q}{C_h\omega + C_s(\rho+1)}$$

Again, taking second order partial derivative of Z(z) with respect to z, we get

$$\frac{\partial^2 Z}{\partial z^2} = \frac{C_h\omega + C_s(1+\rho)}{Q} > 0.$$

$\therefore$ z obtained above is the optimal value, i.e., optimal order quantity of the model proposed is given by

$$z^* = \frac{C_s Q}{C_h\omega + C_s(\rho+1)} \tag{27}$$

Also, Z(z) is minimum when $z = z^* = \frac{C_s Q}{C_h\omega + C_s(\rho+1)}$. So, from eqn.(24), minimum average total cost

$$Z^*(z) = \frac{C_h z^{*2}}{2Q}\omega + \frac{C_s}{2Q}[(Q-z^*)^2 + z^{*2}\rho)] \tag{28}$$

5. Numerical Example

To illustrate the applicability of the model, the example used by Geeta[2] is taken into account. The crisp inventory system has the data:

demand rate $\phi = 25$ items per day

holding cost C_h= ₹ $\frac{8}{15}$ per item per day

shortage cost C_s= ₹ 10 per item per day

$t = 30$ days

leakage rate ψ_1 is 0.5 items per day to 2.5 items per day

Table 1. Optimal solution when the parameters are in crisp environment

ψ	z^*	C^*_{min}
0.5	711.06	194.73
1.0	709.61	201.97
1.5	707.72	211.42
2.0	705.41	222.94
2.5	702.72	236.37

In our fuzzy model, instead of taking the demand rate and leakage rate as fixed quantity, we assumed them to be imprecise and hence represented by triangular fuzzy numbers $\tilde{\phi} = (25-\Delta_1, 25, 25+\Delta_2)$ and $\tilde{\psi} = (1.5-\Delta_3, 1.5, 1.5+\Delta_4)$ respectively in accordance to our fuzzy model.

Table 2. Optimal solution for the proposed model with fuzzy demand rate and fuzzy leakage rate

Δ_1	Δ_2	Δ_3	Δ_4	$d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}},\tilde{0}\right)$	$d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})},\tilde{0}\right)$	z^*	Z^*
1	1	0.1	0.2	1.022023833	0.000458593	710.9230277	195.3848617
1	1	0.1	0.5	1.025367267	0.000669346	710.6609375	196.6953124
1	1	0.1	1	1.031027925	0.001186558	710.1097856	199.4510719
1	1	0.5	0.2	1.018860496	0.000402093	711.0748257	194.6258714
1	1	0.5	0.5	1.02232738	0.0006156	710.806333	195.9683348
1	1	0.5	1	1.028191698	0.001139196	710.2433566	198.7832172
1	1	1	0.2	1.015047046	0.000498218	711.1471439	194.2642805
1	1	1	0.5	1.018670605	0.000713993	710.8714379	195.6428106
1	1	1	1	1.024793435	0.001243722	710.2949579	198.5252104
1	2	0.1	0.2	1.023226929	0.000453233	710.8834019	195.5829906
1	2	0.1	0.5	1.026699737	0.000664032	710.6166641	196.9166793
1	2	0.1	1	1.03256909	0.00118132	710.0580483	199.7097586
1	2	0.5	0.2	1.020360579	0.000399662	711.022532	194.8873398
1	2	0.5	0.5	1.023953778	0.000613182	710.7495319	196.2523404
1	2	0.5	1	1.030021869	0.001136801	710.1793219	199.1033905
1	2	1	0.2	1.016922299	0.000491211	711.0844344	194.5778282
1	2	1	0.5	1.02066835	0.000707051	710.8043327	195.9783365
1	2	1	1	1.026988798	0.001236884	710.220803	198.8959848
1	3	0.1	0.2	1.024849238	0.000448178	710.8285126	195.857437
1	3	0.1	0.5	1.028441356	0.000659019	710.5575038	197.2124808
1	3	0.1	1	1.034503376	0.001176375	709.9920283	200.0398586
1	3	0.5	0.2	1.022269983	0.000397324	710.9554707	195.2226465
1	3	0.5	0.5	1.025979687	0.000610857	710.6783288	196.6083558
1	3	0.5	1	1.032235926	0.001134497	710.1014721	199.4926397
1	3	1	0.2	1.019194508	0.000484628	711.0071803	194.9640985
1	3	1	0.5	1.023053558	0.000700526	710.7230409	196.3847955
1	3	1	1	1.029556506	0.001230454	710.133036	199.3348202

Δ_1	Δ_2	Δ_3	Δ_4	$d\left(\frac{\tilde{\phi}+2\tilde{\psi}}{\tilde{\phi}+\tilde{\psi}},\tilde{0}\right)$	$d\left(\frac{\tilde{\psi}^2}{\tilde{\phi}(\tilde{\phi}+\tilde{\psi})},\tilde{0}\right)$	z^*	Z^*
2	1	0.1	0.2	1.023810526	0.000472855	710.8492096	195.7539519
2	1	0.1	0.5	1.027394591	0.000695587	710.5704696	197.1476521
2	1	0.1	1	1.033453688	0.001242159	709.9854415	200.0727924
2	1	0.5	0.2	1.020903838	0.000416569	710.9916063	195.0419685
2	1	0.5	0.5	1.024615667	0.000642254	710.7061765	196.4691175
2	1	0.5	1	1.030885616	0.001195697	710.108744	199.4562802
2	1	1	0.2	1.01742534	0.000512972	711.0516776	194.7416121
2	1	1	0.5	1.021299491	0.000741184	710.7586656	196.2066721
2	1	1	1	1.027837363	0.001301396	710.1469851	199.2650746
2	2	0.1	0.2	1.02552872	0.000467496	710.7910857	196.0445717
2	2	0.1	0.5	1.029238335	0.000690273	710.5078531	197.4607347
2	2	0.1	1	1.035499973	0.00123692	709.9156191	200.4219047
2	2	0.5	0.2	1.022926209	0.000414138	710.9205532	195.3972338
2	2	0.5	0.5	1.026760531	0.000639836	710.6307724	196.8461381
2	2	0.5	1	1.033228097	0.001193302	710.0263669	199.8681657
2	2	1	0.2	1.019832293	0.000505965	710.9698724	195.1506381
2	2	1	0.5	1.023825112	0.000734242	710.6726225	196.6368875
2	2	1	1	1.030554447	0.001294558	710.0541556	199.7292222
3	1	0.1	0.2	1.026257785	0.00048824	710.7509207	196.2453965
3	1	0.1	0.5	1.030093607	0.000724	710.4544525	197.7277373
3	1	0.1	1	1.036569686	0.001302506	709.8332193	200.8339034
3	1	0.5	0.2	1.023633843	0.000432192	710.8829569	195.5852157
3	1	0.5	0.5	1.027602212	0.000671129	710.5794807	197.1025966
3	1	0.5	1	1.034297112	0.001257054	709.9451997	200.2740015
3	1	1	0.2	1.020525034	0.000528903	710.9295141	195.3524296
3	1	1	0.5	1.024661994	0.000770659	710.618046	196.9097701
3	1	1	1	1.031635266	0.001364067	709.9686894	200.1565529

From the above Table 2, we observed that

(1) when $\Delta_1,\ \Delta_2$ and Δ_3 are fixed and Δ_4 increases, q^* decreases and Z^* increases.

(2) when $\Delta_1,\ \Delta_2$ and Δ_4 are fixed and Δ_3 increases, q^* increases and Z^* decreases.

(3) when $\Delta_1,\ \Delta_3$ and Δ_4 are fixed and Δ_2 increases, q^* decreases and Z^* increases.

(4) when $\Delta_2,\ \Delta_3$ and Δ_4 are fixed and Δ_1 increases, q^* decreases and Z^* increases.

From the above example and the results from Table 2, we can ascertain that with change in the value of the decision variables, i.e., $\Delta_1,\ \Delta_2,\ \Delta_3$ and Δ_4, there is either increase or decrease in the optimum values of order quantity and the total cost. Also, there is a slight variation in value of the result obtained from the crisp model of the same example. But, in real life situation, there is wide range of variabilities in the parameters concern like in the example considered. So, the results from fuzzy sense will be more reliable as it incorporates the variabilities of reality.

6. Conclusion

In an inventory system, it is quite obvious that there may be leakage of the stock. Also, the leakage rate may also be imprecise in nature. So, in this paper, we have discussed a leakage inventory model with shortage in fuzzy environment by considering the demand rate and the leakage rate of the model as fuzzy numbers. To be precise, they are assumed to be triangular fuzzy numbers. The optimum results are obtained by using signed distance method of defuzzification. The concept of α-cut approach, ranking of fuzzy numbers and decomposition principles are used. It is observed that for different sets of fuzzy demand rate and fuzzy leakage rate, the optimum ordering quantity and minimum total cost are almost more or less with that obtained in crisp environment. Also, uncertainties that are prevalent in real inventory problems are highlighted in this fuzzy model and from the sensitivity analysis by taking different values of $\Delta_1,\ \Delta_2,\ \Delta_3$ and Δ_4 the variations or the effect of uncertainties the optimum ordering quantity and minimum total cost are analysed.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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