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FIXED POINTS OF RATIONAL F-CONTRACTIONS IN S-METRIC SPACES

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Abstract. The concept of F-contraction generalizes Banach contraction theorem. In this paper, we introduce a generalized F-contraction and used it to obtain fixed points in S-metric spaces.

Keywords: rational F-contraction; fixed points; S-metric space.

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1. Introduction

The concept of F-contraction was introduced by Wardowski [1]. By introducing F-contraction, Wardowski [1] generalized the famous Banach Contraction Theorem. His result was extended or generalized by various researchers. For our study, we will use the notation $\mathbb{R}$,$\mathbb{R}^+$,$\mathbb{N}$ as the set of real numbers, set of positive real numbers, set of natural numbers respectively.

Wardowski [1] defined the following:

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2. Preliminaries

Definition 1. [1] *A self mapping P in a metric space (X,d) is said to be an F-contraction if for all $x,y \in X$ and $d(Px,Py) > 0$ implies*

$$\tau + F(d(Px,Py)) \leq F(d(x,y)) \tag{1}$$

where $\tau > 0$ and $F \in \mathscr{F}$.

Here $\mathscr{F}$ is the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying

$(F1)$: *F is strictly increasing;*

$(F2)$: *$\lim_{n\to+\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$ for each sequence $\{\alpha_n\} \subset \mathbb{R}^+$;*

$(F3)$: *for $0 < k < 1$, $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.*

Wardowski also pointed out that by considering different types of mappings in (1) variety of contractions can be obtained. He also remarked that from $(F1)$ and (1), it can be concluded that F-contraction mappings are contractive and hence continuous. Further, if F_1, F_2 be such that the properties $(F1)$-$(F3)$ in Definition 1 are satisfied. If $F_1(\alpha) \leq F_2(\alpha)$ for all $\alpha > 0$ and a mapping $G = F_2 - F_1$ is decreasing then every F_1-contraction P is F_2-contraction.

The following theorem was proved by Wardowski :

Theorem 1. [1] *In a complete metric space (X,d), a self mapping P be an F-contraction. Then for every $x \in X$, the sequence $\{P^n x\}_{n\in\mathbb{N}}$ converges to $x^* \in X$ where x^* is the unique fixed point of P.*

Secelean [2] replaced $(F2)$ of Definition 1 by either of the property given as under:

$(F2')$: $\inf F = -\infty$ or

$(F2'')$: a sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive real numbers exist such that $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

Secelean [2] also proved the following:

Lemma 1. [2] *Consider a sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ and an increasing mapping $F : \mathbb{R}^+ \to \mathbb{R}$. Then the following conditions hold true*

(i): $\lim_{n\to\infty} F(\alpha_n) = -\infty$, *implies* $\lim_{n\to\infty} \alpha_n = 0$;

(ii): $\inf F = -\infty$, *and* $\lim_{n\to\infty} \alpha_n = 0$, *implies* $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

Wardowski also pointed out that Banach contractions are F-contractions but the converse is not true.

F-contraction is introduced by Cosentino and Verto [3].

Definition 2. [3] *In a complete metric space (X,d), a self mapping P is said to be Hardy-Rogers type F-contraction if $F \in \mathscr{F}$ and $\tau > 0$ satisfies*

$$(2) \quad \tau + F(d(Px,Py)) \leq F(a_1.d(x,y) + a_2.d(x,Px) + a_3.d(y,Py) + a_4.d(x,Py) + a_5.d(y,Px))$$

with $d(Px,Py) > 0$ for all $x,y \in X$, where a_1, a_2, a_3, a_4 and a_5 are non-negative numbers, $a_3 \neq 1$ and $a_1 + a_2 + a_3 + 2a_4 = 1$.

Theorem 2. [3] *In a complete metric space (X,d), a self mapping P be a Hardy-Rogers-type contraction and $a_3 \neq 1$. Then P has a fixed point. Further, P has a unique fixed point if $a_1 + a_4 + a_5 \leq 1$.*

In Definition 1, the condition $(F3)$ was replaced by Piri and Kumam [4] as under:

$(F3')$: F is continuous on $(0,+\infty)$.

They defined a family of functions $\mathscr{F}$ satisfying $(F1)$, $(F2')$ and $(F3')$ and proved the following :

Theorem 3. [4] *In a complete metric space (X,d), let P be a self mapping. Let $F \in \mathscr{F}$ satisfy*

$$\forall x,y \in X, [d(Px,Py) > 0 \text{ implies } \tau + F(d(Px,Py)) \leq F(d(x,y))].$$

where $\tau > 0$. Then P has a unique fixed point $x^* \in X$ and the sequence $\{P^n x\}_{n\in\mathbb{N}}$ converges to x^* for each $x \in X$.

Piri and Kumam [4] showed the independence of $(F3)$ and $(F3')$.

The next result was proved by Popescu and Gabrial [5] by generalizing the results in [1, 3].

Theorem 4. [5] *In a complete metric space (X,d), let P be a self mapping. For $\tau > 0$, let $x, y \in X,\ d(Px,Py) > 0$ implies*

$$\begin{aligned} & \tau + F(d(Px,Py)) \\ \leq \ & F(a_1.d(x,y) + a_2.d(x,Px) + a_3.d(y,Py) + a_4.d(x,Py) + a_5.d(y,Px)), \end{aligned}$$

where the mapping $F : \mathbb{R}^+ \to \mathbb{R}$ is increasing, a_1, a_2, a_3, a_4, a_5 are non-negative numbers, $a_4 < 1/2, a_3 < 1, a_1 + a_2 + a_3 + 2a_4 = 1,\ 0 < a_1 + a_4 + a_5 \leq 1$. Then P has a unique fixed point $x^* \in X$, also the sequence $\{P^n x\}_{n\in\mathbb{N}}$ converges to x^* for each $x \in X$.

For more results on F-contraction, readers are suggested to see research papers [6, 7, 8]. For other type of contractions one can see [10, 11, 12, 13, 14, 15, 16]

Definition 3. [9] *Let X be a non-empty set. An S-metric on X is a function $S : X \times X \times X \to [0, +\infty)$ that satisfies the following conditions.*

(1): *$S(x,y,z) \geq 0$ for all $x, y, z \in X$;*

(2): *$S(x,y,z) = 0$ if and only if $x = y = z$ for every $x, y, z \in X$;*

(3): *$S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$ for every $x, y, z, a \in X$.*

The pair (X,S) is called an S-metric.

Definition 4. *Let (X,S) be an S-metric space and $A \subset X$.*

(1): *If for every $x \in A$ there exists $r > 0$ such that $B_S(x,r) \subset A$, then the subset A is called an open subset of X.*

(2): *A subset A of X is said to be S-bounded if there exists $r > 0$ such that $S(x,x,y) < r$ $\forall x, y \in A$.*

(3): *A sequence $\{x_n\}$ in X converges to x if and only if $S(x_n,x_n,x) \to 0$ as $n \to \infty$. That is for each $\varepsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that $\forall\ n \geq n_0, S(x_n,x_n,x) < \varepsilon$ and we denote this by $lim_{n\to\infty} x_n = x$.*

(4): *A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\varepsilon > 0$, there exists $n_0 \in N$ such that $S(x_n,x_n,x_m) < \varepsilon$ for each $n, m \geq n_0$.*

(5): *The S-metric space (X,S) is said to be complete if every Cauchy sequence is convergent.*

(6): *Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exists $r > 0$ such that $B_S(x,r) \subset A$. Then τ is a topology on X (induced by the S-metric S).*

There are various forms of S-metric space and these are used with the generalized forms of Banach Contraction Theorems. These can be found in research papers [17, 18, 19, 20, 21, 22, 23, 24]. In this paper, we use the concept of rational F-contraction in S-metric space to obtain fixed points.

3. Main Results

Theorem 5. *Let P be a self-mapping of a complete S-metric space X into itself. Suppose that there exists $\tau > 0$ such that for all $x, y \in X$, $S(Px,Px,Py) > 0$ implies*

$$\tau + F(S(Px,Px,Py)) \leq F(a_1 S(x,x,y) + a_2 S(x,x,Px) + a_3 S(y,y,Py) + a_4 S(x,x,Py) + a_5 S(y,y,Px))$$

where $F : R^+ \to R$ is an increasing mapping, a_1, a_2, a_3, a_4, a_5 are non negative numbers, $a_1 + a_2 + a_3 + 3a_4 + a_5 \leq 1$. Then P has a unique fixed point $x^* \in X$ and for every $x \in X$, the sequence $\{P^n x\}_{n \in N}$ converges to x^*.

Proof. Let $x_0 \in X$ be an arbitrary point and we construct a sequence $\{x_n\}_{n \in N} \in X$ by

$$\begin{aligned} x_1 &= Px_0, \\ x_2 &= Px_1 = P^2 x_0, \\ &\dots \\ x_n &= Px_{n-1} = P^n x_0, \ \forall n \in N \end{aligned} \tag{3}$$

If there exists $n \in N \cup \{0\}$ such that S(x_n, x_n,Px_n)=0, then x_n is a fixed point of P and the proof is complete. Hence, we assume that

$$0 < S(x_n, x_n, Px_n) = S(Px_{n-1}, Px_{n-1}, Px_n) \ \forall n \in N \tag{4}$$

Now, Let S_n= $S(x_n,x_n,x_{n+1})$. By the hypothesis and the monotony of F, we have for all $n \in N$

$$\begin{aligned}
\tau + F(S_n) &= \tau + F(S(x_n,x_n,x_{n+1})) \\
&= \tau + F(S(Px_{n-1},Px_{n-1},Px_n)) \\
&\leq F(a_1 S(x_{n-1},x_{n-1},x_n) + a_2 S(x_{n-1},x_{n-1},Px_{n-1}) + a_3 S(x_n,x_n,Px_n) \\
&\quad + a_4 S(x_{n-1},x_{n-1},Px_n) + a_5 S(x_n,x_n,Px_{n-1})) \\
&= F(a_1 S(x_{n-1},x_{n-1},x_n) + a_2 S(x_{n-1},x_{n-1},x_n) + a_3 S(x_n,x_n,x_{n+1}) \\
&\quad + a_4 S(x_{n-1},x_{n-1},x_{n+1}) + a_5 S(x_n,x_n,x_n)) \\
&\leq F(a_1 S_{n-1} + a_2 S_{n-1} + a_3 S_n + a_4 2S(x_{n-1},x_{n-1},x_n) + a_4 S(x_n,x_n,x_{n+1}) + a_5.0)) \\
&= F((a_1+a_2+2a_4)S_{n-1} + (a_3+a_4)S_n)
\end{aligned}$$

It follows that

$$\begin{aligned}
F(S_n) &\leq F((a_1+a_2+2a_4)S_{n-1} + (a_3+a_4)S_n) - \tau \\
&< F((a_1+a_2+2a_4)S_{n-1} + (a_3+a_4)S_n) \qquad (5)
\end{aligned}$$

So from the monotony of F, we get

$$S_n \leq (a_1+a_2+2a_4)S_{n-1} + (a_3+a_4)S_n$$

and hence

$$(1-a_3-a_4)S_n \leq (a_1+a_2+2a_4)S_{n-1} \; \forall n \in N$$

Since $a_1+a_2+a_3+3a_4+a_5 \leq 1$

$$\begin{aligned}
S_n &\leq \frac{a_1+a_2+2a_4}{1-a_3-a_4} S_{n-1} \\
&< S_{n-1} \; \forall n \in N.
\end{aligned}$$

Thus, we conclude that the sequence $\{S_n\}_{n\in N}$ is strictly decreasing , so there exists $lim_{n\to\infty} S_n =$ S.

Suppose that $S > 0$. Since F is an increasing mapping, there exists $lim_{x\to S_+} F(x) = F(S+0)$, so taking the limit as $n \to \infty$ in inequality (5), we get $F(S+0) \leq F(S+0) - \tau$, which is a contradiction.

Therefore,

$$lim_{n\to\infty}S_n = 0. \tag{6}$$

Now, we claim that $\{x_n\}_{n\in N}$ is a Cauchy sequence.

Arguing by contradiction , we assume that there exists $\varepsilon > 0$ and sequences $\{p(n)\}_{n\in N}$ and $\{q(n)\}_{n\in N}$ of natural numbers such that $p(n) > q(n) > n$,

$$S(x_{p(n)}, x_{p(n)}, x_{q(n)}) > \varepsilon, S(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)}) \le \varepsilon,\ \forall n \in N \tag{7}$$

Then, we have

$$\begin{aligned} \varepsilon &< S(x_{(p(n)}, x_{p(n)}, x_{q(n)}) \\ &\le 2S(x_{p(n)}, x_{p(n)}, x_{p(n)-1}) + S(S(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)}) \\ &\le 2S(x_{p(n)}, x_{p(n)}, x_{p(n)-1}) + \varepsilon \end{aligned}$$

It follows from relation (6) and above inequality that

$$lim_{n\to\infty}S(x_{p(n)}, x_{p(n)}, x_{q(n)}) = \varepsilon \tag{8}$$

Since $S(x_{p(n)}, x_{p(n)}, x_{q(n)}) > \varepsilon > 0$, by the hypothesis and monotony of F, we have

$$\begin{aligned} \tau + F(S(x_{p(n)}, x_{p(n)}, x_{q(n)})) &= \tau + F(S(Px_{p(n)-1}, Px_{p(n)-1}, Px_{q(n)-1})) \\ &\le F(a_1S(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)-1}) + a_2S(x_{p(n)-1}, x_{p(n)-1}, Px_{p(n)-1}) \\ &\quad + a_3S(x_{q(n)-1}, x_{q(n)-1}, Px_{q(n)-1}) + a_4S(x_{p(n)-1}, x_{p(n)-1}, Px_{q(n)-1}) \\ &\quad + a_5S(x_{q(n)-1}, x_{q(n)-1}, Px_{p(n)-1})) \\ &= F(a_1S(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)-1}) + a_2S(x_{p(n)-1}, x_{p(n)-1}, x_{p(n)}) \\ &\quad + a_3S(x_{q(n)-1}, x_{q(n)-1}, x_{q(n)}) + a_4S(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)}) \\ &\quad + a_5S(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)})) \end{aligned}$$

$$\begin{aligned}
&\leq F(a_1(2S(x_{p(n)-1},x_{p(n)-1},x_{p(n)})+S(x_{p(n)},x_{p(n)},x_{q(n)-1}))+a_2S_{p(n)-1}\\
&\quad +a_3S_{q(n)-1}+a_4(2S(x_{p(n)-1},x_{p(n)-1},x_{p(n)})+S(x_{p(n)},x_{p(n)},x_{q(n)}))\\
&\quad +a_5(2S(x_{q(n)-1},x_{q(n)-1},x_{q(n)})+S(x_{q(n)},x_{q(n)},x_{p(n)})))\\
&\leq F(2a_1S_{p(n)-1}+a_1(2S(x_{p(n)},x_{p(n)},x_{q(n)})+S(x_{q(n)},x_{q(n)},x_{q(n)-1}))\\
&\quad +a_2S_{p(n)-1}+a_3S_{q(n)-1}+2a_4S_{p(n)-1}\\
&\quad +a_4S(x_{p(n)},x_{p(n)},x_{q(n)})+2a_5S_{q(n)-1}+a_5S(x_{p(n)},x_{p(n)},x_{q(n)}))\\
&\quad =F((2a_1+a_4+a_5)S(x_{p(n)},x_{p(n)},x_{q(n)})+(2a_1+a_2+2a_4)S_{p(n)-1}\\
&\quad +(a_1+a_3+2a_5)S_{q(n)-1})
\end{aligned}$$

Taking the limit as $n\to\infty$ in the above inequality, we get

$$\tau+F(\varepsilon+0)\leq F(\varepsilon+0),$$

which is a contradiction and hence, the sequence $\{x_n\}_{n\in N}$ is a Cauchy sequence. Since (X,S) is a complete S-metric space, we have that $\{x_n\}_{n\in N}$ converges to some point x^* in X.

If there exists a sequence $\{p(n)\}_{n\in N}$ of natural numbers such that $x_{p(n)+1}=Px_{p(n)}=Px^*$, then $lim_{n\to\infty}x_{p(n)+1}=x^*$, so $Px^*=x^*$. Otherwise, there exists $n\in N$ such that $x_{n+1}=Px_n\neq Px^*$,$\forall n\geq N$.

Assume that $Px^*\neq x^*$. By the hypothesis, we have

$$\begin{aligned}
\tau+F(S(Px_n,Px_n,Px^*)) &\leq F(a_1S(x_n,x_n,x^*)+a_2S(x_n,x_n,Px_n)+a_3S(x^*,x^*,Px^*)\\
&\quad +a_4S(x_n,x_n,Px^*)+a_5S(x^*,x^*,Px_n))
\end{aligned}$$

so

$$\begin{aligned}
\tau+F(S(x_{n+1},x_{n+1},Px^*)) &\leq F(a_1S(x_n,x_n,x^*)+a_2S(x_n,x_n,x_{n+1})+a_3S(x^*,x^*,Px^*)\\
&\quad +a_4S(x_n,x_n,Px^*)+a_5S(x^*,x^*,x_{n+1}))
\end{aligned}$$

Since F is increasing , we deduce that

$$\begin{aligned}
S(x_{n+1},x_{n+1},Px^*) &\leq a_1S(x_n,x_n,x^*)+a_2S(x_n,x_n,x_{n+1})+a_3S(x^*,x^*,Px^*)\\
&\leq +a_4S(x_n,x_n,Px^*)+a_5S(x^*,x^*,x_{n+1})
\end{aligned}$$

so letting $n \to \infty$, we get

$$\begin{aligned} S(x^*,x^*,Px^*) &\leq a_3S(x^*,x^*,Px^*)+a_4S(x^*,x^*,Px^*) \\ &= (a_3+a_4)S(x^*,x^*,Px^*) \\ &< S(x^*,x^*,Px^*) \end{aligned}$$

This is a contradiction . Therefore, $Px^* = x^*$. Now, we will show that P has a unique fixed point. Let $x,y \in X$ be two distinct fixed points of P. Thus, $Px = x \neq y = Py$. Hence, $S(Px,Px,Py) = S(x,x,y) > 0$. By the hypothesis, since $a_1+a_2+a_3+3a_4+a_5 \leq 1$, we have

$$\begin{aligned} \tau + F(S(x,x,y)) &= \tau + F(S(Px,Px,Py)) \\ &\leq F(a_1S(x,x,y)+a_2S(x,x,Px)+a_3S(y,y,Py) \\ &\quad +a_4S(x,x,Py)+a_5S(y,y,Px)) \\ &= F(a_1S(x,x,y)+a_4S(x,x,y)+a_5S(y,y,x)) \\ &= F((a_1+a_4+a_5)S(x,x,y)) \\ &\leq F(S(x,x,y)) \end{aligned}$$

This is a contradiction. Therefore, P has a unique fixed point. □

Corollary 1. *Let (X,S) be a complete S-metric space and let P be a self mapping on X. Assume that there exists an increasing mapping $F : R^+ \to R$ and $\tau > 0$ such that*

$$\tau + F(S(Px,Px,Py)) \leq F(a_1S(x,x,y)+a_2S(x,x,Px)+a_3S(y,y,Py)) \ \forall x,y \in X, Px \neq Py,$$

where $a_1+a_2+a_3 \leq 1$. Then, P has a unique fixed point in X.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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