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A HYBRID DEEP LEARNING NETWORK FOR NON-LINEAR TIME SERIES PREDICTION

SAMEER POONGADAN*, M.C. LINEESH

Department of Mathematics, National Institute of Technology Calicut, Calicut, India

Abstract. Non-linear time series prediction is highly significant because most of the practical situations deals with time series which are non-linear in nature. This study suggests a new time series prediction CEEMDAN-SVD-LSTM model amalgamating Complete Ensemble EMD with Adaptive Noise, Singular Value Decomposition and Long Short Term Memory network. Non-linear and non-stationary data can be analysed by deploying the above model. CEEMDAN stage, SVD stage and LSTM stage are the main parts of the model. The break down of the data into some IMF components plus a residue is carried out by CEEMDAN in the first stage. Each IMF component and residue so obtained is de-noised by SVD in the second stage. Third stage deployed LSTM to predict all the de-noised IMF components. The foretold values of the actual data is then obtained by adding all the predicted IMF components and residue. We compared the model with other models such as LSTM model, EMD-LSTM model, EEMD-LSTM model, CEEMDAN-LSTM model and EEMD-SVD-LSTM model. The results show that the suggested CEEMDAN-SVD-LSTM model works better than other models in terms of efficiency in predicting future values.

Keywords: IMF; EMD; EEMD; CEEMDAN; SVD; LSTM.

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*Corresponding author

E-mail address: sameerpoongadan@gmail.com

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1. INTRODUCTION

Non-linear time series prediction is highly significant in the modern age. ARIMA model is a good tool that can be applied to model linear and stationary time series as in [1], [2] and [3]. But ARIMA cannot be applied to model non-linear characteristics as it takes in to account only linearly dependent variables and thus has limited applications, [4]. Badr et al. used Holt's linear trend, BATS and TBATS models to forecast internet web traffic, [5]. A vast collection of non-linear prophecy models are developed by various researchers. Many models by making use of Artificial intelligence like Genetic Algorithm, [6], [7], Artificial Neural Network, [8], [9], [10], etc. are proposed. A hybrid prediction model to forecast Carbon monoxide emanations by employing Imperialistic Competitive Algorithm and Artificial Neural Network was constructed by Mahmoudzadeh,[11]. Some models use Adaptive Neuro-Fuzzy Inference System for predictions as in [12] for constructing a divination model of carbon monoxide ejection, in [13] to predict the coefficient of heat transmission in distilled water pool boiling, in [14] to predict daily carbon monoxide concentration in the atmosphere, etc. Rosadi et al. proposed a non-linear autoregressive network with exogenous inputs (NARX) model to compare learning algorithms for seasonal time series forecasting, [15]. Other forecasting approaches to assimilate Empirical mode decomposition (EMD) that helps to examine the non-linear and non-stationary data are available in the literature, [16], [17], [18] etc. EMD is established by Huang et al., [19]. But mode mixing is the major shortcoming of EMD, which can be minimized by utilizing models incorporating Ensemble Empirical mode decomposition (EEMD). EEMD is put forth by Wu and Huang, [20]. A few prediction models incorporating EEMD were proposed by various researchers like Jiang et al. [21], Bao et al. [22], Xie et al. [23] etc. Sameer Poongadan and Lineesh MC proposed a model incorporating EEMD and SVD to predict Carbon monoxide levels in the atmosphere in the Indian region [24]. CEEMDAN is superior to EMD and EEMD in eliminating mode mixing and reducing computational cost. CEEMDAN has fewer number of sifting iterations in comparison with EEMD. The core of EMD, EEMD and CEEMDAN is to fragment the series into some Intrinsic Mode Functions (IMFs). IMF proposed by Huang et al., [19], is a function having the conditions: (a) in the graph of the series, either there are equal numbers of zero crossings and extrema or their difference is one, (b) the envelopes defined by the local extrema are at an equal distance from the axis.

Herein we propose a new time series prophecy technique encompassing CEEMDAN, SVD and LSTM. The series is broken down in to some IMF components plus a residue using CEEMDAN followed by denoising using SVD. The components are converted into Hankel matrices prior to denoising. A matrix with equal elements on the diagonals perpendicular to the main diagonal is called Hankel matrix[25].

A $c \times d$ Hankel matrix is of the form

$$P = \begin{pmatrix} q_1 & q_2 & q_3 & \cdots & q_d \\ q_2 & q_3 & q_4 & \cdots & q_{d+1} \\ q_3 & q_4 & q_5 & \cdots & q_{d+2} \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ q_c & q_{c+1} & q_{c+2} & \cdots & q_n \end{pmatrix}$$

where $c+d-1=n$. Prediction is carried out by using LSTM network.

2. Methodology

2.1. Singular Value Decomposition (SVD). The SVD of a $c \times d$ real matrix P can be expressed as:

$$P = L\,D\,R^T$$

where $L_{c\times r}$ and $R_{d\times r}$ are orthogonal matrices. The columns of L and R are called left and right singular vectors respectively, [25]. The singular values are the diagonal entries of the r-diagonal matrix D given by:

$$D = \begin{pmatrix} S & 0 \\ 0 & 0 \end{pmatrix}$$

where $S = diag(\sigma_1,\ \sigma_2,\ \ldots)$ with components such that $\sigma_1 \geq \sigma_2 \geq \ldots > 0$.

The orthonormal eigenvectors of PP^* and P^*P are respectively referred as left and right singular vectors of P.

2.2. Empirical Mode Decomposition (EMD). In 1998, Huang et al. developed EMD to interpret a time series [19]. EMD is applicable to non-linear and non-stationary data. The EMD aims to fragment the data into a finite number of IMF components.

The following steps gives the design of EMD:

Step 1: Detect every local maximum and local minimum in the data x_n.

Step 2: By a cubic spline, build the envelope l_M of local maxima and the envelope l_m of local minima.

Step 3: Calculate the mean value a_{1_n} of l_M and l_m,
i.e;

$$a_{1_n} = (l_M + l_m)/2$$

Step 4: Calculate $i_{1_n} = x_n - a_{1_n}$

Step 5: If i_{1_n} meet all the characteristics of an IMF, fix i_{1_n} as the first IMF c_{1_n}, and replace the original data by the residue $r_{1_n} = x_n - i_{1_n}$. Otherwise, the original data x_n can be replaced by i_{1_n}.

Step 6: Replicate steps 1 to 5.

The process terminates if one of the ensuing situations met:

(i) the residue is not so much as a fixed value or turns a monotone function there by making extraction of more IMF components impossible.

(ii) the current and previous sifting steps are same in number of zero crossings and extrema.

The process enables us to express the actual series x_t as summation of n modes and a residue:

$$x_t = \sum_{j=1}^{n} c_{j_t} + r_{n_t}$$

where c_{j_t} denotes the IMF components and r_{n_t} is the last residue which is a constant or a trend.

2.3. Ensemble Empirical Mode Decomposition (EEMD). The signal analysis method EEMD designed by Wu and Huang in 2009 [20] is accompanied with some noise. The method intents to minimize mode mixing, which is accounted as the major short coming of EMD. Mode mixing is the appearance of vibrations of very unmatching magnitude in a mode, or the appearance of very matching vibrations in disparate modes.

The following steps gives the design of EEMD:

Step 1: With the actual data x_n, add separate realizations of white noise w_n^j $(j = 1,\ 2,\ \ldots, N)$ to produce new data $x_n^j = x_n + w_n^j$.

Step 2: Break down each x_n^j $(j = 1,\ 2,\ \ldots, N)$ into their modes IMF_k^j, where $k = 1,\ 2,\ \ldots, K$, using EMD.

Step 3: Obtain the k-th mode $\overline{IMF}_k$ of x_n by averaging IMF_k^j (for $j = 1,\ 2,\ \ldots, N$):

$$\overline{IMF}_k = \frac{1}{N}\sum_{j=1}^{N} IMF_k^j$$

2.4. Complete Ensemble Empirical Mode Decomposition With Adaptive Noise (CEEMDAN).

CEEMDAN, developed by Torres et al., in 2011, can be deployed to interpret non-linear and non-stationary data [26]. CEEMDAN is superior to EMD and EEMD in eliminating mode mixing and reducing computational cost. The number of sifting iterations of CEEMDAN is less than that of EEMD. The design of CEEMDAN is to add adaptive white noise in every level of fragmentation and to calculate only residual signal to obtain each modal component with a minimal reformation error.

The following steps gives the design of CEEMDAN:

If x_n is the data, define $E_r(x_n)$ as the r-th mode of x_n procured by EMD. Let w^j be white noise.

Step 1. Fragment N realizations $x_n + \varepsilon_0 w_n^j,\ j = 1, \ldots, N$, by EMD to find their first modes and calculate

$$\widetilde{IMF}_1 = \frac{1}{N}\sum_{j=1}^{N} IMF_1^i = \overline{IMF}_1$$

Step 2. At stage 1 $(m = 1)$ compute the first residue as

$$r_1 = x_n - \widetilde{IMF}_1$$

Step 3. Fragment realizations $r_1 + \varepsilon_1 E_1(w_n^j),\ j = 1, \ldots, N$, to obtain their first EMD modes and compute the second mode:

$$\widetilde{IMF}_2 = \frac{1}{N}\sum_{j=1}^{N} E_1(r_1 + \varepsilon_1 E_1(w_n^j))$$

Step 4. For $m = 2,...,M$ compute the m-th residue

$$r_m = r_{m-1} - \widetilde{IMF}_m$$

Step 5. Fragment realizations $r_m + \varepsilon_m E_m(w_n^j),\ j = 1,...,N$, up to their first EMD mode and compute the $(m+1)$-th mode:

$$\widetilde{IMF}_{m+1} = \frac{1}{N}\sum_{i=1}^{N} E_1(r_m + \varepsilon_m E_m(w_n^j))$$

Step 6. Go to step 4 for next m

Steps 4 to 6 are carried out till the acquired residue can't be further fragmented (the residue does not possess at least two extrema). Then the terminal residue is:

$$R = x_n - \sum_{m=1}^{M} \widetilde{IMF}_m$$

Therefore, the actual signal x_n can be written as:

$$x_n = \sum_{m=1}^{M} \widetilde{IMF}_m + R$$

2.5. SVD based Time series De-noising. SVD can be used to split a time series into clean part and noise part [27]. Consider a signal, which is a mixture of clean signal and a white noise:

$$i.e.,\ x_n = x_{n_s} + w_{n_w}$$

where $x_n,\ x_{n_s}$ and w_{n_w} respectively denote the given signal, clean signal and white noise. We can represent the series $x_n,\ n = 1,2,...,j$, in Hankel matrix as:

$$Q = \begin{pmatrix} x_1 & x_2 & \dots & x_k \\ x_2 & x_3 & \dots & x_{k+1} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ x_l & x_{l+1} & \dots & x_j \end{pmatrix}$$

Since $x_n = x_{n_s} + w_{n_w}$, the Hankel representation will be of the form

$$Q = Q_s + Q_w$$

where Q, Q_s and Q_w are respectively the Hankel matrix representations of given signal, clean signal and the white noise. The SVD decomposes Hankel matrix Q as:

$$Q = L\, D\, R^T$$

where L and R are orthogonal matrices and D is a diagonal matrix with singular values as its diagonal elements.

The uncoupling of the data matrix into signal and noise parts is effected by applying SVD. Since the matrix is spanned by the set of singular vectors, the singular values close to zero correspond to the noise part of the data.

The uncoupling of basis into clean and noisy subspaces can be viewed as:

$$Q = L\, D\, R^T = \begin{pmatrix} L_1 & L_2 \end{pmatrix} \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \begin{pmatrix} R_1^T \\ R_2^T \end{pmatrix}$$

then

$$Q = L_1\, D_1\, R_1^T + L_2\, D_2\, R_2^T$$

where D_1 and D_2 respectively offer the singular values representing clean and noise subspaces. Thus we have

$$Q_s = L_1\, D_1\, R_1^T$$

and

$$Q_w = L_2\, D_2\, R_2^T$$

A threshold value has to be determined in D so that the singular values lower than the threshold value is regarded as the noise subspace singular values and can so be served as zero. This value is obtained by delineating the singular values against their index, marked by the juncture point at which slope of the curve deviates strongly.

2.6. Long Short Term Memory (LSTM) network. The LSTM network is a deep learning design possessing time-depending targets and inputs. It can efficiently analyse and predict time series as it has the proficiency to solve long-term dependent problems. The memory cell is the key component of the LSTM network. The basic ideas of the LSTM network are depicted in [28].

2.7. CEEMDAN-SVD-LSTM Prediction Model. Herein we propose a new time series forecasting approach encompassing CEEMDAN, SVD and LSTM network. The model is composed of three parts which are CEEMDAN stage, SVD stage and LSTM stage. The first stage employs CEEMDAN to decompose the data into a limited number of IMF components and a residue. To de-noise the components SVD is applied in the second stage. The components are converted into Hankel matrices prior to denoising. In the third stage, LSTM forecasts every de-noised IMF component. All the forecast series are added to produce the prediction of the actual series. The step-by-step diagram of the CEEMDAN-SVD-LSTM Prediction Model is illustrated in the Figure 1. For programming and curve plotting we used the software *Matlab 9.10.0*.

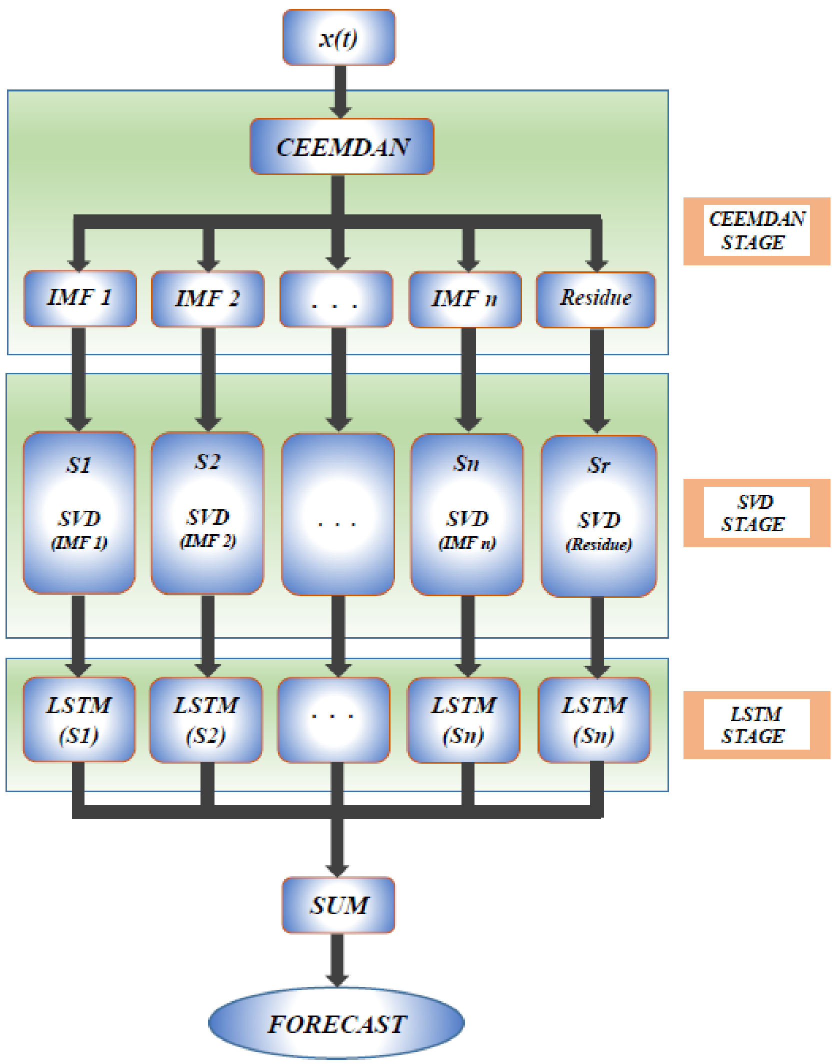


FIGURE 1. Step-by-step diagram of CEEMDAN-SVD-LSTM Prediction Model

3. EXPERIMENT SETUP

3.1. Data. The popular non-linear data Wolf's Sunspot Numbers from the year 1700 to 1988 is used for analysis of the approach. There are 289 data points. The time series data plot is given in Figure 2.

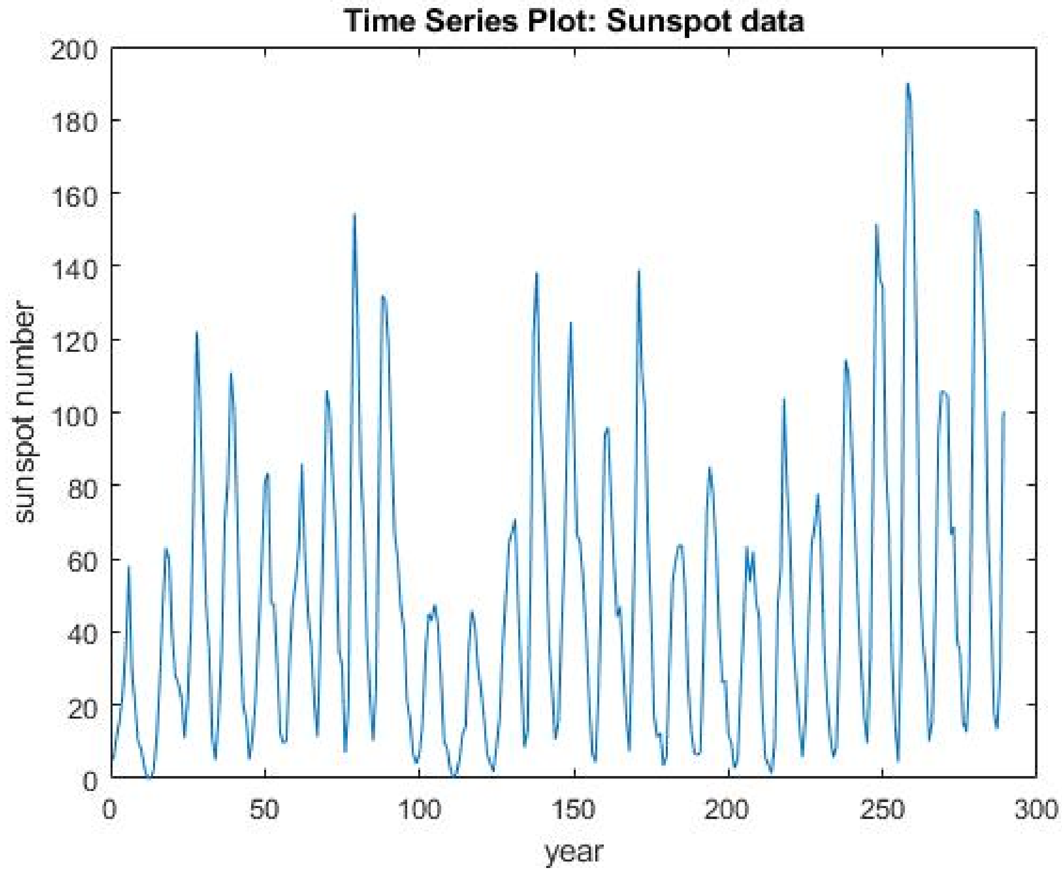


FIGURE 2. Sunspot data

3.2. Segregation of IMF Components. In the CEEMDAN stage, the series is broken down into eight IMF components (IMF1, IMF2, . . . , IMF8) plus a residue using CEEMDAN. Figure 3 shows the delineation of all the IMF components drawn out by CEEMDAN. They are given in the order of their separation in terms of the frequency from the highest to the lowest. The last component indicates the trend of the data.

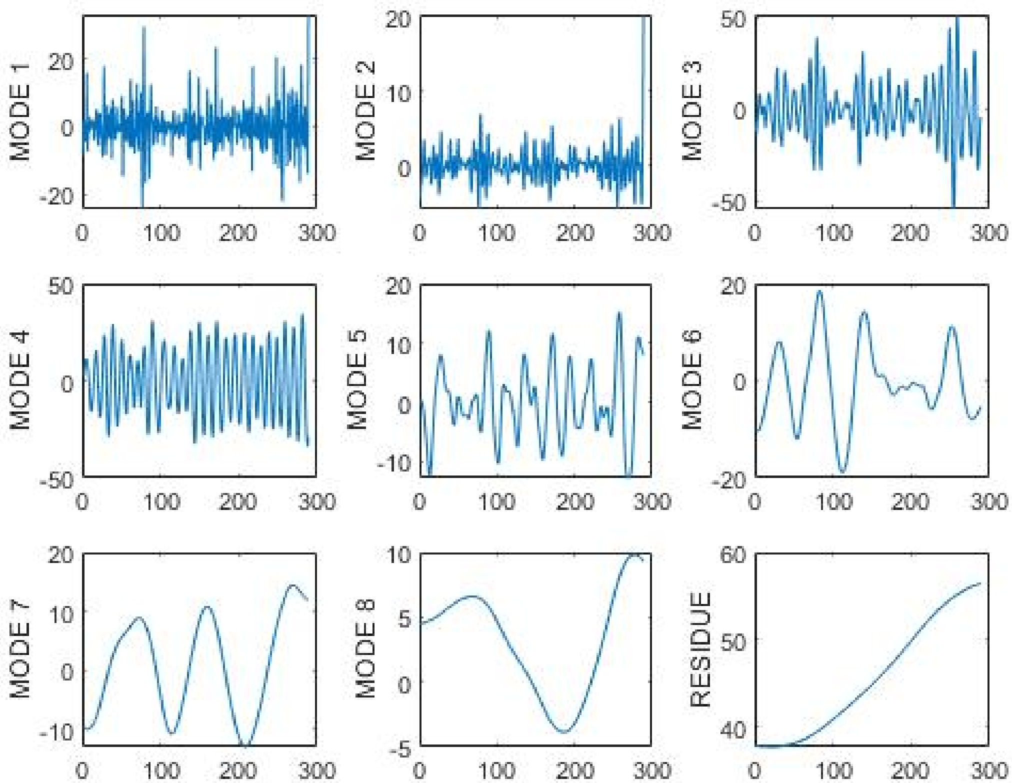


FIGURE 3. The Extricated IMF components of Sunspot data by CEEMDAN

3.3. De-noising the IMF Components by SVD. All the IMF components (IMF1, IMF2, . . . , IMF8) and residue are transformed into Hankel matrices in the second stage, SVD is then applied separately to each component to de-noise the components. The series corresponding to that portion of SVD with non-zero singular values will be the noise reduced portion of the data (sum of the products of the non-zero singular values, left and right singular vectors).

3.4. Forecasting by LSTM. In the LSTM stage each component that is produced as a consequence of SVD, is foretold by LSTM network. As training data we have taken the first 90 % of the data points of the corresponding series and considered the last 10% as testing data in LSTM network. The foretold values of the original data are created by adding all the foretold series of the IMF components and residue series. The Figure 4 gives CEEMDAN-SVD modes and their LSTM-based forecasts.

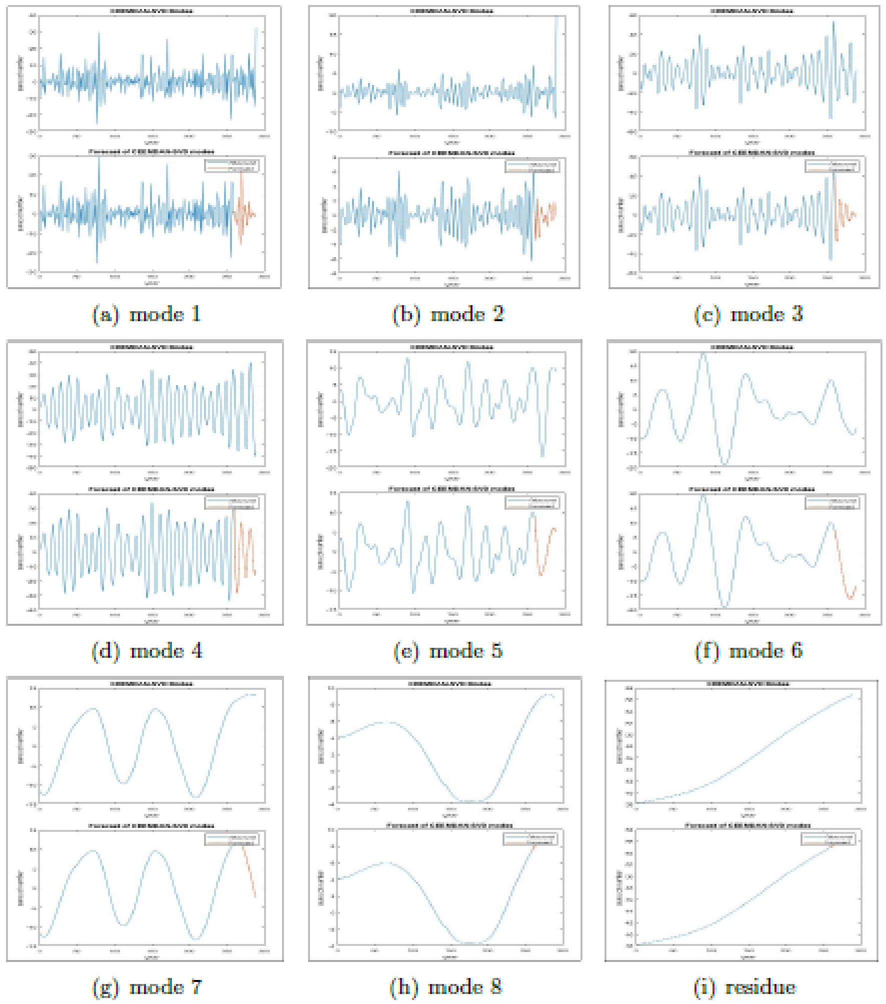
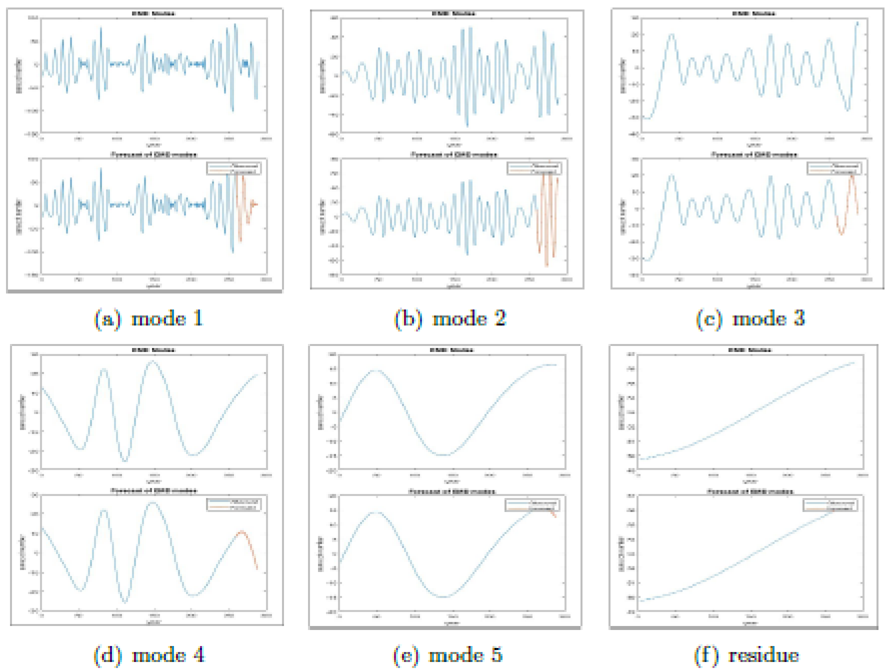


FIGURE 4. Actual and predicted CEEMDAN-SVD modes by LSTM

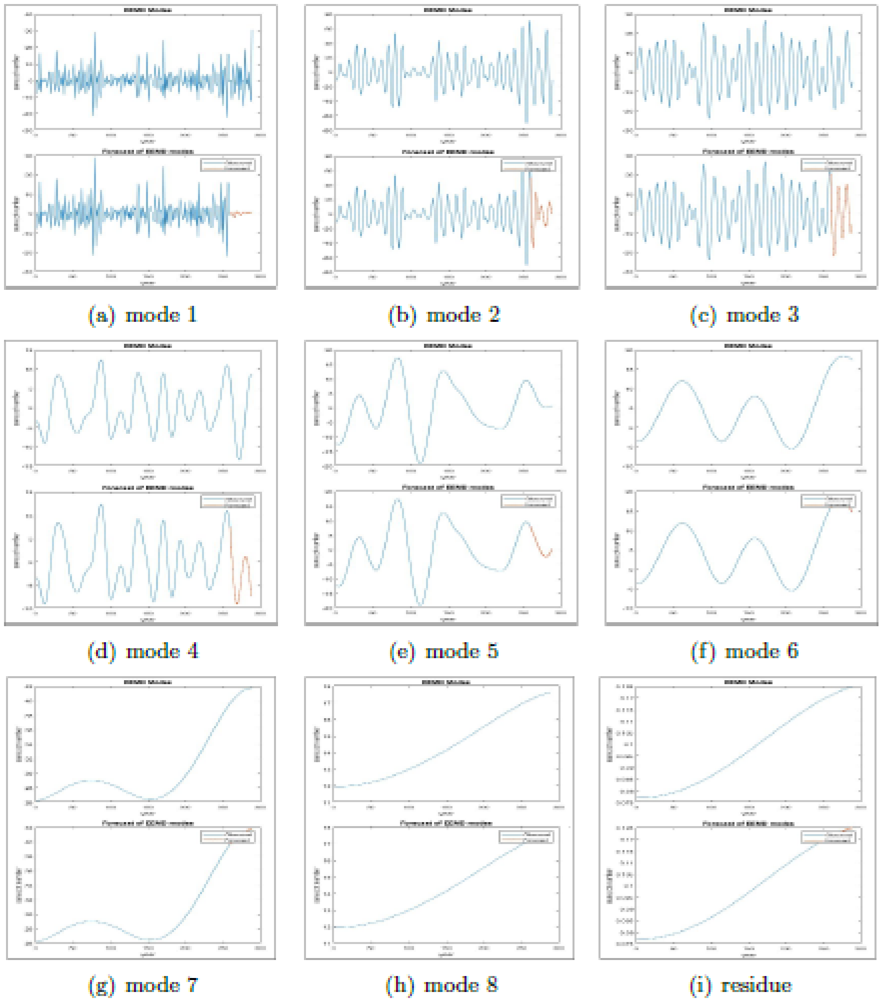
3.5. Comparison. Five other models namely LSTM model, EMD-LSTM model, EEMD-LSTM model, CEEMDAN-LSTM model and EEMD-SVD-LSTM model are used for comparison. In the LSTM model, LSTM is used directly to the data to foretell the series. In the EMD-LSTM model, EMD is carried out first to the data and divided the same into five IMFs plus a residue. LSTM is then used to foretell each IMF and residue and obtained the predicted values of the original data by adding all the foretold series. In the EEMD-LSTM model, the series is fragmented into eight IMF components plus a residue by EEMD, all components including residue are foretold by LSTM and added them to procure the predicted values of the original data. CEEMDAN-LSTM model produced eight IMF components and a residue by applying CEEMDAN to the series, foretold each of the components and residue by LSTM and then added them to produce the foretold series of the actual series. EEMD-SVD-LSTM model generated eight IMF components and residue by EEMD followed by denoising of each component and residue by SVD. Then LSTM foretold each of the components and residue. Adding them we obtained the foretold series of the original series. Figures 5 through 8 respectively illustrates EMD, EEMD, CEEMDAN and EEMD-SVD modes with forecasts by LSTM.



(a) mode 1 (b) mode 2 (c) mode 3

(d) mode 4 (e) mode 5 (f) residue

FIGURE 5. Actual and predicted EMD modes by LSTM

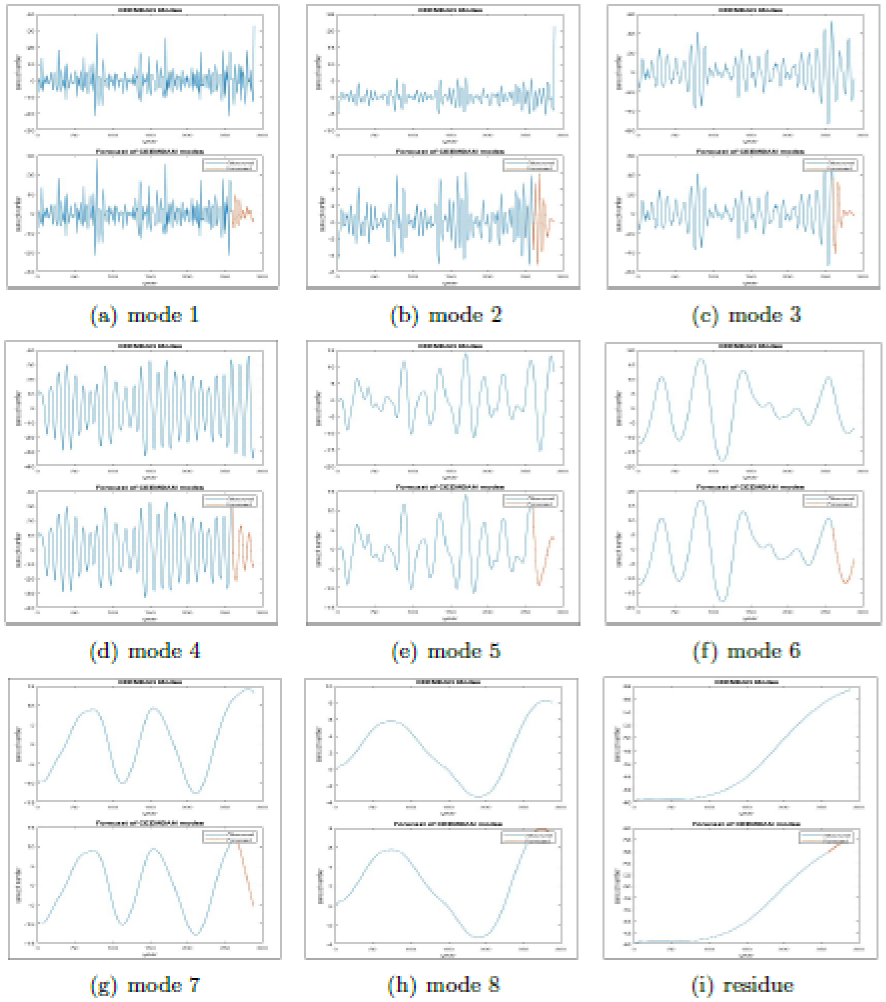


(a) mode 1 (b) mode 2 (c) mode 3

(d) mode 4 (e) mode 5 (f) mode 6

(g) mode 7 (h) mode 8 (i) residue

FIGURE 6. Actual and predicted EEMD modes by LSTM

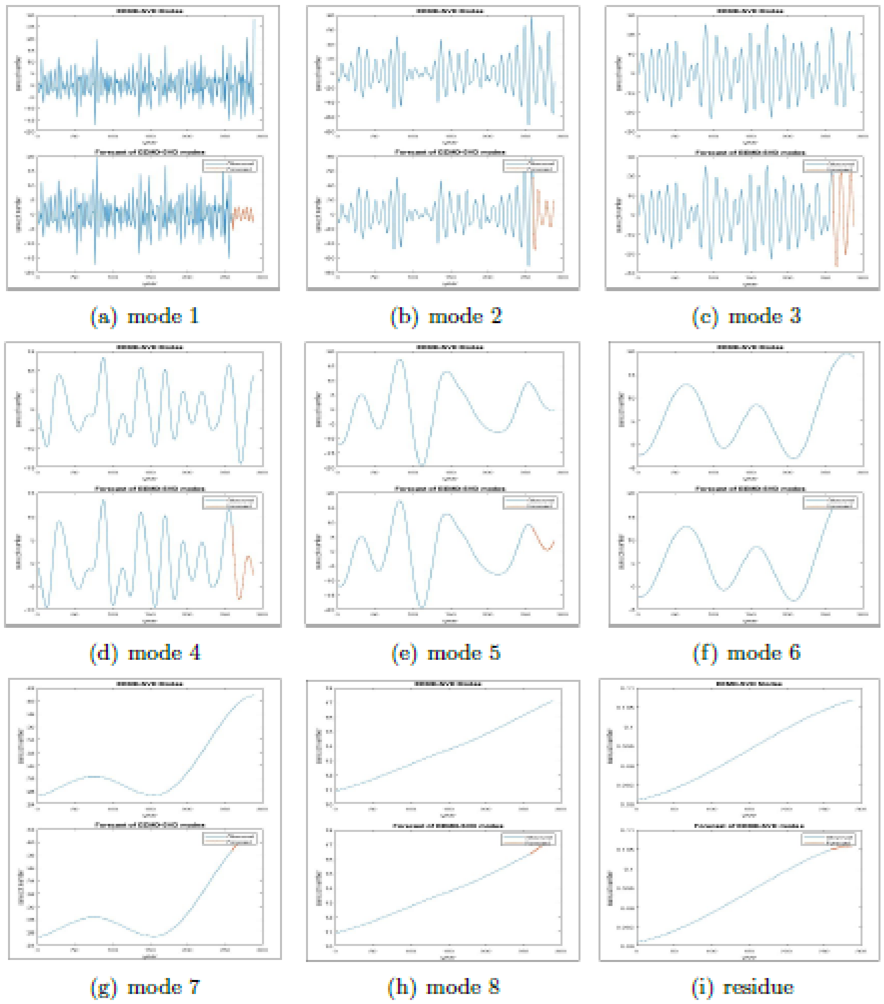


(a) mode 1 (b) mode 2 (c) mode 3

(d) mode 4 (e) mode 5 (f) mode 6

(g) mode 7 (h) mode 8 (i) residue

FIGURE 7. Actual and predicted CEEMDAN modes by LSTM



(a) mode 1 (b) mode 2 (c) mode 3

(d) mode 4 (e) mode 5 (f) mode 6

(g) mode 7 (h) mode 8 (i) residue

FIGURE 8. Actual and predicted EEMD-SVD modes by LSTM

3.6. Performance measures. The validity of prediction models are quantified using various performance measures. Herein Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) are deployed to ascertain the prediction validity. RMSE and MAE are stated as:

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i - \widehat{y_i})^2}$$

$$MAE = \frac{1}{m}\sum_{i=1}^{m}|y_i - \widehat{y_i}|$$

4. Results and discussion

A hybrid CEEMDAN-SVD-LSTM model is proposed in this study to forecast the non-linear sunspot data. The result obtained is compared with five other techniques like LSTM, EMD-LSTM, EEMD-LSTM, CEEMDAN-LSTM and EEMD-SVD-LSTM. The comparison of the suggested model with other models in terms of performance measures is depicted in Table 1. The plots of the values of Observed and Predicted Sunspot data and the errors by LSTM, EMD-LSTM, EEMD-LSTM, CEEMDAN-LSTM, EEMD-SVD-LSTM and CEEMDAN-SVD-LSTM models are displayed in the Figure 9.

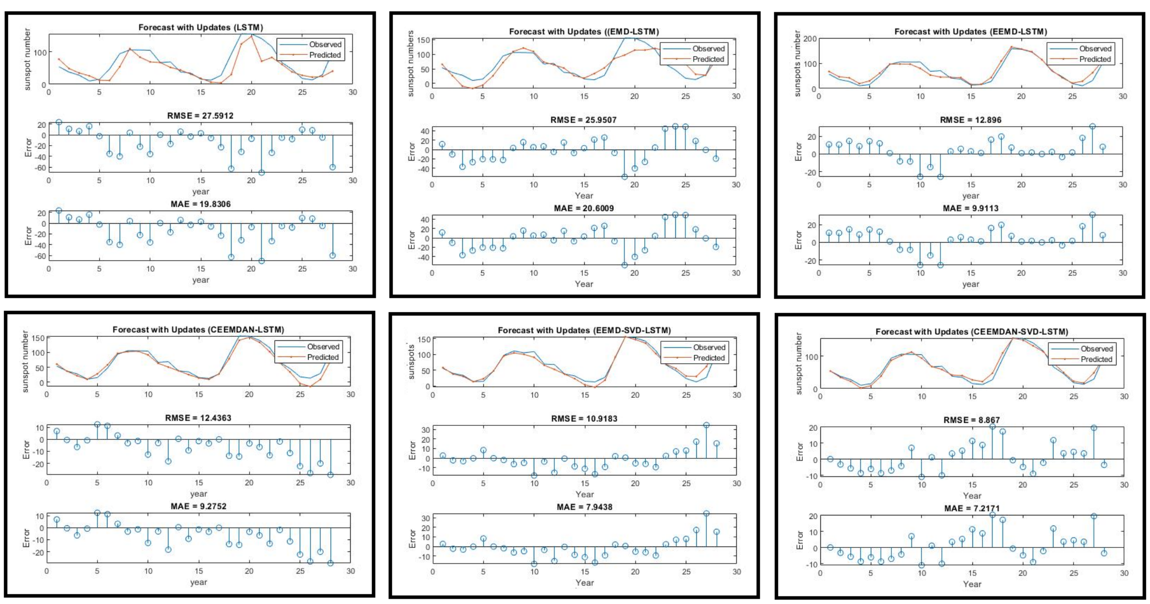


FIGURE 9. Observed versus Predicted Sunspot data and the Errors by LSTM, EMD-LSTM, EEMD-LSTM, CEEMDAN-LSTM, EEMD-SVD-LSTM and CEEMDAN-SVD-LSTM models

TABLE 1. Comparison of the proposed model with other models

Sl. No.	Model	RMSE	MAE
1	LSTM	27.5912	19.8306
2	EMD-LSTM	25.9507	20.6009
3	EEMD-LSTM	12.8960	9.9113
4	CEEMDAN-LSTM	12.4363	9.2752
5	EEMD-SVD-LSTM	10.9183	7.9438
6	CEEMDAN-SVD-LSTM	8.867	7.2171

5. CONCLUSION

Time series forecasting finds wide applications in different fields. The non-linear time series prediction is of utmost significance in the current era as it can be utilized for planning and designing future in various sectors. This study introduces a hybrid non-linear time series prediction CEEMDAN-SVD-LSTM model which encompasses CEEMDAN, SVD and LSTM Network to prophesy non-linear time series data. The comparison of results obtained using proposed CEEMDAN-SVD-LSTM model with prophecy models like LSTM, EMD-LSTM, EEMD-LSTM, CEEMDAN-LSTM and EEMD-SVD-LSTM models show that it surpasses the others. The drawbacks of the above model includes the appearance of residue noise in CEEMDAN modes and the presence of factitious modes.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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