Available online at http://scik.org
J. Math. Comput. Sci. 2022, 12:176
https://doi.org/10.28919/jmcs/7462
ISSN: 1927-5307

LOCAL AND GLOBAL STABILITY ANALYSIS OF A COVID-19 MODEL DYNAMICS WITH HEALTHY DIET AS CONTROL

UCHENNA E. MICHAEL*, LOUIS O. OMENYI, KAFAYAT O. ELEBUTE, AKACHUKWU A. OFFIA

Department of Mathematics and Statistics, Alex Ekwueme, Federal University Ndufu-Alike, Nigeria

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Abstract. We construct and analyse a nonlinear deterministic mathematical model for the transmission dynamics of COVID-19 with healthy diet as a control. The effective reproduction number, R_e, of the model is computed and its sensitivity analysis done. Our results are the proof of existence of forward bifurcation using center manifold theory and stability of the equilibrium points. Model fitting was done with data published by Nigeria Centre for Disease Control (NCDC) on COVID-19 and we estimated the model parameters by least square. Our simulation and analysis results show asymptotic stability. Thus, a consistent intake of healthy diet boost the immune system which help in wading-off COVID-19 when an individual is exposed.

Keywords: COVID-19; equilibria; reproduction number; sensitivity analysis; asymptotic stability; bifurcation.
2010 AMS Subject Classification: 53D15, 53C40, 53C22, 53C12.

1. Introduction

On 11th march 2020, the World Health Organisation (WHO) declared that a strain of human coronaviruses, 2019 – nCoV nicknamed COVID – 19 is a Pandemic [23]. Globally, as of 6 : 15pm WAT, 25th March 2022, there have been 476,374,234 confirmed cases of COVID – 19, including 6,108,976 deaths reported to WHO [4], and as of 4 : 41pm WAT, 26 March 2022, there have been 255,296 confirmed cases of COVID – 19, including 3,142 deaths reported to

*Corresponding author
E-mail address: michael.uchenna@funai.edu.ng
Received April 29, 2022

Nigeria center for Disease Control, NCDC. Often, people who are infected by COVID – 19 virus experience mild to moderate respiratory illness and recover without receiving any special treatment [6]. Patients with cardiovascular diseases, diabetes, chronic respiratory diseases, cancer etc and older people with compromised immune system are likely to have a progressive illness. The clinical symptoms of COVID – 19 include fever, dry cough, fatigue, sore throat, diarrhea, loss of taste or sense of smell and headache. Difficulty in breathing or shortness of breath, loss of speech or mobility or confusion, chest pain or pressure are symptoms for acute cases [7]. The most common manifestation of severe COVID – 19 is pneumonia with fever, cough, dyspnoea and pulmonary infiltrates [12]; circulatory shock, myocardial damage, arrhythmias, and encephalopathy [15]. The incubation period is estimated to 2 – 14 days after contact, but in some cases can elongate up to 27 days [17]. Infected individual can either be symptomatic, asymptomatic or presymptomatic; the two later classes have no visible symptoms and are not cognizant of their infection, yet they are capable of transmitting the disease. COVID-19 is a highly contagious disease, which is transmitted from person to person through respiratory droplets, either by being inhaled of deposited on mucosal surfaces, in addition to aerosols produced during coughing and speaking. Children and adolescents have lower susceptibility to SARS – CoV – 2 than adults, and children younger than 10 – 14 years appear to be less susceptible to SARS – CoV – 2 than older children and adults (20 years and older) [20]. The transmission through respiratory droplets is pronounced within a social distance less than 6 feet [21].

Since the first case of COVID-19 in Nigeria on 27th February, 2020, and the declaration of the disease as a global pandemic on 14th March, 2020; there has been daily situation report on states affected, number of new cases confirmed, people on admission, people discharged from the hospital and number of people that have died of the disease by NCDC. Nigerian government imposed its first round of lock-down on 27 April 2020 to take effect from May 4 to 17 spanning two weeks in Lagos, Ogun and Abuja; imposed quarantine for all travelers and wearing of face-mask and proper personally hygiene was encouraged, restricted public gatherings, churches and schools, inter – state travels and all businesses and offices within these locations was closed during this period so as to contain the pandemic. Regardless of all these measures;

people neglected some of the impositions, maybe because the economy bite so hard on them and consequently the disease spread rapidly throughout the country. The poor health system, corrupt national politics, overcrowding nature of Nigerians [26] facilitated in the spread of the pandemic, likewise the shallow knowledge of the disease, daunted awareness and the populace lack of adherence to proper wearing of face - mask , maintaining social distancing and use of alcohol based sanitizer also contributed greatly in the upsurge.

Over the year, mathematical modeling has played an appreciable role in the studying, predicting and controlling the transmission dynamics of different diseases. Different researchers have worked on variant diseases like meningitis [31, 28, 33], conjunctivitis [34, 35], HIV [37], malaria [8, 18, 9], tuberculosis [10, 38]etc. Since the breakout of COVID-19, there have been numerous research on its dynamics; multiple transmission pathways including human – to – human and environment – to – human was studied by [39] which [40] later used optimal control theory to minimize the risk of transmission. A mechanistic model was studied by [42] which they explored the role of asymptomatic class, quarantine and isolation in the transmission dynamics of the disease; their result shows that control of the disease is basically dependent on management of population in quarantine and isolation. The effect of social distancing, lock-down, quarantine and isolation in the dynamic transmission of COVID-19 was studied by [43] using a deterministic mathematical model. The result shows that lock-down and quarantine of asymptomatic individuals reduces the contact rate in a given population and hence help in controlling COVID-19. Mathematical models for COVID-19 were proposed by considering pharmaceutical and non - pharmaceutical as intervention techniques to combat COVID-19 [45, 22, 24]; on which optimal control was carried out to determine the best intervention strategies, their results show that community awareness is crucial to eradication of COVID-19 in any given population. A model for COVID-19 was proposed and analyzed by classifying the compartments into symptomatic, infectious, asymptomatic infectious and hospitalized individuals [25] on which cumulative case report in Nigeria was used to parametrize the model and optimal control model to minimize the number of infectious humans was explored.

Various researchers used fractional calculus which makes use of non - integer or fractional order. A new fractional derivative operator with a nonsingular and Mittag - Leffler kernel was introduced my [46] of which the main merit is that it has a nonlocal and nonsingular kernel of which [46, 48, 50, 52, 53] have used in mathematical modeling. In [3], a model of coronavirus disease containing asymptomatic and symptomatic classes was studied using the Atangana - Toufik numerical scheme; their results shows that the people in authority should ensure that the exposed individual and susceptible individuals contact should be minimized. A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation, and environtal viral load was studied [51], which they used Caputo operator to formulate the fractional derivatives. They concluded that the use of fractional epidemic model provides a better understanding and biologically more insights about the disease dynamics. A novel fractional mathematical model of COVID-19 epidemic considering quarantine and latent time [55], [44] worked on the existence and semi – analytical results to fractional order mathematical model of COVID-19 and crop of other researchers used fractional order model with interesting and fascinating results.

Most of the studies obtained that public health education, personal protective, and treatment of hospitalized individuals are significant to reducing the spread of the disease, [20, 25, 9, 6, 14]. However, authors have not considered the role of healthy diet (HD) to survival from the pandemic in the context that it strengthen the immune system. This study aims to study the transmission dynamics of COVID-19 when the population is conscious of nutritional balance of their meal before and during disease outburst. A mathematical model of the transmission dynamics is formulated, nutritional status of individuals have been deployed as resilience towards destabilization during COVID-19 pandemic [49], and asymptotic analysis of the model is performed to determine the potency of healthy diet in curtailing the contagions. The model parameters are fitted to the reported real data of COVID-19 from February 27, 2020 to March 25, 2022 in Nigeria.

The paper is organized as follows: The proposed mathematical model is described in section 2. Asymptotic analysis which comprises of existence and uniqueness, non - negativity and boundedness, equilibria, stability and bifurcation analysis of the model in section 3. The sensitivity analysis of the effective reproduction number to model parameters are discussed in section 4. The demonstration of the numerical simulations is shown in section 5 including parameter estimation and finally, the discussion and conclusions are presented in section 6.

2. Model Formulation

In this work, we propose a mathematical model to analyze the dynamics of COVID-19 pandemic when the population adhere to some extent to the non - pharmaceutical safety precautions and pharmaceutical when an individual has been exposed to the disease when clinical evaluation has been carried out. The total population is divided into six compartment :S denotes susceptible population, E denotes the exposed population, individuals who have been in an environment or has come in contact with a person who clinically tested positive to COVID-19, such a person may or may not have been infected are grouped as quarantine. Some of these exposed individuals are quarantined so as to take them away from healthy individuals, Q represents these individuals that are quarantined. The population that have the disease but not showing symptoms (asymptomatic) is denoted by A and we assume that they contribute to the distribution of COVID-19. The symptomatic population who are capable of spreading the disease who are either hospitalized or not are denoted by H and R represent the recovered population. The total population at time t is

$$N(t) = S(t) + E(t) + Q(t) + A(t) + H(t) + R(t).$$

The recruitment rate of susceptible individuals is denoted by Λ; the susceptible population are assumed to be additionally increased at the rate τ from the quarantined who test negative after incubation period and at the rate ω from recovered population whose immunity has waned down. We assumed that the susceptible population acquired the disease when they come in contact with either the asymptomatic or symptomatic. regardless, the tendency of asymptomatic spreading the disease is minimal compared to symptomatic because often they do not disperse respiratory droplets,the reduction parameter is η. Therefore the force of infection, $\lambda = \frac{\beta(\eta A+H)}{N-Q}$ excluding the quarantined from the total population [6]. The number of new infection rate is denoted by β which is the number of new contagions per unit time with respect to contact with infectious individuals. The natural mortality rate is assumed uniform at all the compartments. The fraction of the exposed population joining Q, A, and H are r_1, α, r_2 respectively. The rate at which the quarantine individuals test positive before or by the incubation period with symptoms is σ and without symptoms is ε.

The rate of disease induced death are d_1 and d_2 for asymptomatic and symptomatic population respectively; it is assumed that $d_1 < d_2$. The infective individuals in classes$A$ and H are assumed to recovered at rates m and k respectively which is dependent on the health diet taken, to help boost the immune system and for more effective pharmaceutical processes. The parameter that determines how nutritious the diet intake of both asymptomatic and symptomatic population when taking other pharmacological prescription are $u_1 \in (0,1]$ and $u_2 \in (0,1]$ respectively. $\nu \in (0,1)$ is the parameter that represents the nutrition and dietary nutrient intake that impact the immune system of the susceptible population through gene expression, cell activation, and signaling molecules modification.

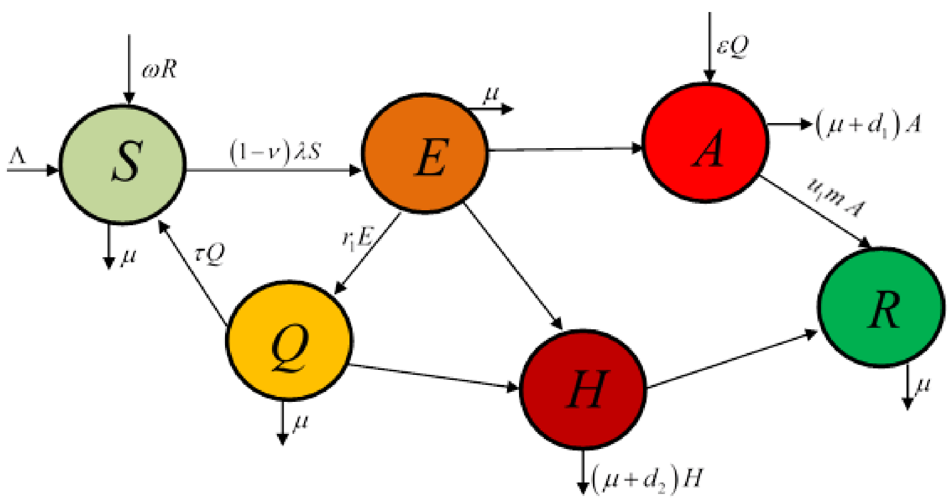

FIGURE 1. Schematic diagram of the proposed model.

Our model assumed that the quarantine exposed individuals move either to the susceptible, asymptomatic or symptomatic class depending on the outcome of the result by the end of the incubation period. We considered HD with the assumption that other non -pharmaceutical interventions are negligible because the population under study is greatly aware of there proper usage and adhere to some extent. So, the system of equations for the model is (2.1):

$$
\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda+\tau Q+\omega R-(1-\nu)\lambda S-\mu S\\
\frac{dE}{dt} &= (1-\nu)\lambda S-(\alpha+r_1+r_2+\mu)E\\
\frac{dQ}{dt} &= r_1E-(\tau+\sigma+\varepsilon+\mu)Q\\
\frac{dA}{dt} &= \alpha E+\varepsilon Q-(u_1m+d_1+\mu)A\\
\frac{dH}{dt} &= r_2E+\sigma Q-(u_2k+d_2+\mu)H\\
\frac{dR}{dt} &= u_1mA+u_2kH-(\omega+\mu)R
\end{aligned}\right\} \tag{2.1}
$$

where $\lambda=\dfrac{\beta(\eta A+H)}{N-Q}$, with initial conditions $S(0)>0, E(0)\geq 0, Q(0)\geq 0, A(0)\geq 0, H(0)\geq 0$, and $R(0)\geq 0$. The biological meaning of all the parameters in (2.1) are given in Table 1, they all assume non-negative numerical values.

3. Analysis of the Model

We have the following results.

Theorem 3.1. *Solutions to (2.1) exists and are unique for $t\in[0,\infty)$.*

Proof. We write (2.1) compactly as $y'=f(t,y(t))$, where $y(t)=(S(t),E(t),Q(t),A(t),H(t),R(t))$, and $f(t,y(t))=(f_1(t,y(t)),f_2(t,y(t)),\cdots,f_6(t,y(t)))$. Applying existence and uniqueness theorem in [56], the proof of existence and uniqueness of solution of (2.1) in $D=\left\{(S,E,Q,A,H,R)\in\mathbb{R}^6_+ : N\leq\frac{\Lambda}{\mu}\right\}$ is easy. To verify this claim we check that each of the partial derivatives of $f_i(t,y(t))$ of the model with respect to S,E,Q,A,H,R are bounded in D. □

Theorem 3.2. *All solutions of model (2.1) with non-negative initial value remain non-negative for all $t\in[0,\infty)$. Moreover,*

$$\limsup_{t\to\infty}\leq\frac{\Lambda}{\mu}.$$

Proof. Let $t_1=\sup\{t\geq 0 : S(t)>0, E(t)\geq 0, Q(t)\geq 0, A(t)\geq 0, H(t)\geq 0, R(t)\geq 0\}\geq 0$. From (2.1)

$$\frac{dS}{dt}+[(1-\nu)\lambda+\mu]S=\Lambda+\tau Q+\omega R.$$

TABLE 1. Biological Interpretation of parameters involved in the model system 2.1.

Parameters	Description
Λ	Natural recruitment rate
τ	Influx from quarantined class after incubation period and testing COVID-19 negative
ω	Rate at which recovered individual immunity wane out
ν	Healthy diet level of an individual
μ	The natural death rate
r_1	Transfer rate of exposed individual to quarantine center
r_2	Rate of transfer of individuals from exposed class to symptomatic class
α	Transfer rate of exposed class to asymptomatic class
σ	Rate at which quarantined show symptom of COVID-19
ε	Rate at which quarantined test COVID-19 positive without symptom
u_1	Value of nutritional value of diet given to asymptomatic individual to boast immunity
u_2	Value of nutritional value of diet given to symptomatic individual
d_1	Disease related death of asymptomatic individual
d_2	Disease related death of symptomatic individual
m	Recovery rate of asymptomatic class
k	Recovery rate of symptomatic class
η	Modification factor for asymptomatic individual
β	Coefficient of transmission

The integrating factor is $\exp\left[\mu t + \int_0^t (1-\nu)\lambda(\xi)d\xi\right]$:

$$\begin{aligned}
\frac{d}{dt}\left[S(t)\exp\left(\mu t + \int_0^t (1-\nu)\lambda(\xi)d\xi\right)\right] &= (\Lambda + \tau Q + \omega R)\exp\left[\mu t + \int_0^t (1-\nu)\lambda(\xi)d\xi\right] \\
\Rightarrow S(t_1)\exp\left(\mu t + \in_0^t (1-\nu)\lambda(\xi)d\xi\right) - S(0) &= \int_0^t (\Lambda + \tau Q(\psi) + \omega R(\psi)) \\
&\quad \exp\left[\mu \psi + \int_0^t (1-\nu)\lambda(\xi)d\xi\right] d\psi.
\end{aligned}$$

Thus,

$$\begin{aligned} S(t_1) &= S(0)\exp\left[-\left(\mu t_1 + \in_0^{t_1} (1-\nu)\lambda(\xi)d\xi\right)\right] + \exp\left[-\left(\mu t_1 + \in_0^{t_1} (1-\nu)\lambda(\xi)d\xi\right)\right] \\ &\quad \times \int_0^{t_1} (\Lambda + \tau Q(\psi) + \omega R(\psi)) \exp\left[\mu\psi + \int_0^t (1-\nu)\lambda(\xi)d\xi\right] d\psi. \end{aligned}$$

Choosing appropriately the value of t_1 such that the initial condition is satisfied, we have that $S(t_1) \geq 0$. In the other-hand, using the same approach , we can also show that

$$E(t) \geq -(\alpha + r_1 + r_2 + \mu) \Rightarrow E \geq 0$$

$$Q(t) \geq -(\tau + \sigma + \mu) \Rightarrow Q(t) \geq 0,$$

$$A(t) \geq -(u_1 m + d_1 + \mu) \Rightarrow A(t) \geq 0,$$

$$H(t) \geq -(u_2 + d_2 + \mu) \Rightarrow H(t) \geq 0,$$

$$R(t) \geq -(\omega + \mu)R \Rightarrow R(t) \geq 0.$$

Thus, any solution of model (2.1) is non-negative with non-negative initial values for $t \in [0, \infty)$. For boundedness, observe that

$$\frac{dN}{dt} = \Lambda - \mu N - d_1 A - d_2 H \leq \Lambda - \mu N.$$

It follows that

$$N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right) e^{-\mu t}.$$

If $t \to \infty$, we obtain

$$\limsup_{t\to\infty} N(t) \leq \frac{\Lambda}{\mu}.$$

Hence the model (2.1) is bounded for all $t \in [0, \infty)$. □

Theorem 3.3. *The closed region* $\omega = \left\{(S, E, Q, A, H, R) \in \mathbb{R}_+^6 : 0 \leq N \leq \frac{\Lambda}{\mu}\right\}$ *is positively invariant set for the model (2.1).*

Proof. To examine the positive invariant of Ω, we check the direction of the vector field $(f_1, f_2, f_3, f_4, f_5, f_6)^T$ of the model (2.1) if it points inward on the boundary $\mathbb{R}_+^6$. From model (2.1), on the plane that $S = 0$, we have

$$\frac{dS}{dt}\Big|_{N=\frac{\Lambda}{\mu}} = \Lambda + \tau Q + \omega R > 0.$$

Therefore, the vector field on the $(E,Q,A,H,R)-$ plane points to the interior of $\mathbb{R}^6_+$. Similarly, one can show for others. If $N = \frac{\Lambda}{\mu}$, then

$$\frac{dN}{dt}|_{N=\frac{\Lambda}{\mu}} = -(d_1A + d_2H) \leq 0.$$

Hence all solutions starting in $\mathbb{R}^6_+$ remain in $\mathbb{R}^6_+$ for $t > 0$. Hence Ω is positively invariant for the model (2.1). □

Generally, for any initial point $S_0, E_0, Q_0, A_0, H_0, R_0 \in \mathbb{R}^6_+$, the trajectory of solutions lies in the manifold Ω. Therefore, the model (2.1) is both epidemiologically and mathematically well - posed.

3.1. Equilibrium Points. The equilibrium point of a mathematical model is a constant that explains the behaviour of a model over a period of time. Solving the dynamic system in model (2.1) we obtain two equilibria

i. Disease Free Equilibrium (DFE) point: This is the equilibrium of model (2.1) when there is no or minimal COVID-19 incidence in the population that is when $E = Q = A = H = R = 0$; plugging these values into model (2.1) we get

$$E^0 = (S_0, E_0, Q_0, A_0, H_0, R_0) = (S_0, 0, 0, 0, 0, 0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0\right).$$

ii. Endemic Equilibrium Point (EEP): This is a point when COVID-19 becomes endemic in the population, at this point we have that

$$\left.\begin{array}{rcl} \Lambda + \tau Q + \omega R - (1-\nu)\lambda S - \mu S & = & 0 \\ (1-\nu)\lambda S - (\alpha + r_1 + r_2 + \mu)ES & = & 0 \\ r_1E - (\tau + \sigma + \varepsilon + \mu)QS & = & 0 \\ \alpha E + \varepsilon Q - (u_1m + d_1 + \mu)AS & = & 0 \\ r_2E + \sigma Q - (u_2k + d_2 + \mu)HS & = & 0 \\ u_1mA + u_2kH - (\omega + \mu)RS & = & 0. \end{array}\right\} \tag{3.1}$$

Solving (3.1) we obtain the endemic equilibrium point as $E^1 = (S^*, E^*, Q^*, A^*, H^*, R^*)$ such that

$$S^* = \frac{(\Lambda g_2g_3g_4 + r_1\tau g_2g_3)(\omega + \mu) + \omega\Big(u_1mg_3(\alpha g_4 + \varepsilon r_1) + u_2kg_2(r_2g_4 + \sigma r_1)\Big)}{g_2g_3g_4(\omega + \mu)((1-\nu)\lambda + \mu)}E^*,$$

$$E^* = \frac{g_2 g_3 g_4 \lambda (1-\nu)(\omega+\mu)}{\left(\left((1-\nu)\lambda+\mu\right)g_1 g_2 g_3 g_4 - r_1 \tau g_2 g_3\right)(\omega+\mu) + \omega\left(u_1 m g_3(\alpha g_4 + \varepsilon r_1) + u_2 k g_2(r_2 g_4 + \sigma r_1)\right)},$$

$$Q^* = \frac{r_1}{g_4}E^*, A^* = \frac{g_4\alpha + \varepsilon r_1}{g_2 g_4}E^*, H^* = \frac{g_4 r_2 + \sigma r_1}{g_3 g_4}E^*,$$

$$R^* = \frac{u_1 m g_3(g_4\alpha + \varepsilon r_1) + u_2 k g_2(g_4 r_2 + \sigma r_1)}{g_2 g_3 g_4(\omega+\mu)}E^*,$$

where $g_1 = \alpha + r_1 + r_2 + \mu, g_2 = u_1 m + d_1 + \mu, g_3 = u_2 k + d_2 + \mu$ and $g_4 = \tau + \sigma + \varepsilon + \mu$.

3.2. Reproduction number. The reproduction number is the expected number of secondary infections produced by an index case in a completely susceptible population by a typical infective individual

The reproductive number, denoted by R_e, is defined as number of secondary infections produced by an index case in a susceptible population by an infected individual. It is a threshold condition for studying the stability of a system at an equilibrium point used as an indicator for disease control. We use next generation matrix to obtain this measure of the potential for COVID-19 spread within a population. It is obtained solving the dominant eigenvalue of the next generation matrix that comprises F and V^{-1} i.e. $\rho\left(FV^{-1}\right)$ [54, 30] where ρ is the spectral radius, F is the influx of the infection into a compartment while V is the reflux. Let f_i be the rate of new appearance of infection in a compartment and v_i be the difference between the transfer rate of population out of a compartment i, then

$$f_i = \begin{pmatrix} (1-\nu)\lambda \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad v_i = \begin{pmatrix} (\alpha + r_1 + r_2 + \mu)E \\ -r_1 E + (\tau + \sigma + \varepsilon + \mu)Q \\ -(\alpha E + \varepsilon Q) + (u_1 m + d_1 + \mu)A \\ -(r_2 E \sigma Q) + (u_2 k + d_2 + \mu)HS \end{pmatrix}.$$

The matrices F and V are obtained by linearizing about DFE the above results, hence the Jacobian matrix are:

$$F = \begin{pmatrix} 0 & (1-\nu)\beta\eta & \beta(1-\nu) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} g_1 & 0 & 0 & 0 \\ -\alpha & g_2 & 0 & \varepsilon \\ -r_2 & 0 & g_3 & -\sigma \\ -r_1 & 0 & 0 & g_4 \end{pmatrix}.$$

Hence, the next generation matrix is given as

$$FV^{-1} = \begin{pmatrix} \frac{\beta(1-\nu)\left(\eta\alpha g_3g_4+g_2(r_2g_4+\sigma r_1)\right)}{g_1g_2g_3g_4} & -\frac{\beta\eta(1-\nu)}{g_2} & \frac{\beta(1-\nu)}{g_3} & -\frac{\sigma\beta(1-\nu)}{g_3g_4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The spectral radius of the next generation matrix is

$$R_e = \frac{\beta(1-\nu)\left(\eta\alpha g_3g_4+g_2(r_2g_4+\sigma r_1)\right)}{g_1g_2g_3g_4}$$

$$= \frac{(1-\nu)\beta\eta\alpha}{g_1g_2} + \frac{\beta(1-\nu)(r_2g_4+\sigma r_1}{g_1g_3g_4} = R_a + R_h.$$

R_a is the part of reproduction number associated with asymptomatic individuals and R_h is related to the symptomatic class, it is believed that $R_a > R_h$ because often R_a substantially drive community transmission even when they individually transmit at a low per capital rate, [57].

3.3. Local and global stability of DFE and EEP. There is also a probability that a population ravaged with an epidemic will be disease - free at a given time. The theorems below discuss the local and global stability of such DFE.

Theorem 3.4. *E^0 of model (2.1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. The Jacobian matrix of model (2.1) evaluated at E^0 is given by

$$J|_{E^0} = \begin{pmatrix} -\mu & 0 & -(1-\nu)\beta\eta & -(1-\nu)\beta & \tau & \omega \\ 0 & -g_1 & (1-\nu)\beta\eta & (1-\nu)\beta & 0 & 0 \\ 0 & \alpha & -g_2 & 0 & \varepsilon & 0 \\ 0 & r_2 & 0 & -g_3 & \sigma & 0 \\ 0 & r_1 & 0 & 0 & -g_4 & 0 \\ 0 & 0 & u_1m & u_2k & 0 & -(\omega+\mu) \end{pmatrix}.$$

The characteristic polynomial of $J|_{E^0}$ is written as $|J|_{E^0} - \lambda I| = 0$, where λ is the eigenvalue and I is a 6×6 identity matrix. Obviously $\lambda_1 = -\mu, \lambda_6 = -(\omega+\mu)$. To obtain the remaining eigenvalues of $J|_{E^0}$, we solve the characteristic polynomial of the reduced matrix

$$\begin{vmatrix} -g_1-\lambda & (1-\nu)\beta\eta & (1-\nu)\beta & 0 \\ \alpha & -g_2-\lambda & 0 & \varepsilon \\ r_2 & 0 & -g_3-\lambda & \sigma \\ r_1 & 0 & 0 & -g_4-\lambda \end{vmatrix} = 0.$$

$$\therefore \lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0$$

where

$$\begin{aligned} p_1 &= g_1+g_2+g_3+g_4 \\ p_2 &= g_4(g_1+g_2+g_3)+g_1g_3+g_2(g_1+g_3)-(1+\eta\alpha)(1-\nu)\beta \\ p_3 &= g_1g_2g_3+g_4\Big(g_1g_3+g_2(g_1+g_3)\Big)-\Big(g_2+g_4+\sigma r_1+(g_3+g_4)(\alpha+r_1\varepsilon)\eta\Big)(1-\nu)\beta \\ p_4 &= g_1g_2g_3g_4(1-R_e)-r_1\varepsilon g_3\eta(1-\nu)\beta. \end{aligned}$$

By Routh - Huriwtz stability criterion, it can be shown that there are no sign changes after some little algebraic simplification if $R_e < 1$. If $R_0 > 1, p_1 > 0, p_2 > 0, p_3 > 0$, $p_4 > 0$ and $p_1p_2p_3 > p_3^2 + p_1^2 p_4$, hence we guarantee that all roots of the equation are real and has negative. Generally, E^0 of model (2.1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. □

Theorem 3.5. *If $R_e < 1$, the DFE of model (2.1) is globally asymptotically stable in its feasible region.*

Proof. Rewrite model (2.1) as

$$\frac{dX}{dt} = F(X,Y), \frac{dY}{dt} = G(X,Y), \text{with } G(X,0) = 0$$

where $X = (S,R) \in \mathbb{R}^2$ is the non - disease compartments and $Y = (E,Q,A,H) \in \mathbb{R}^4$ is the disease compartments. For E^0 to be globally asymptotically stable, the following conditions must satisfy

C_1 : For $\frac{dX}{dt} = F(X,0)$, if $F(X^*,0) = 0$, then X^* is globally asymptotically stable.

$C_2:$ $G(X,Y)=RY-\hat{G}(X,Y), \hat{G}(X,Y)>0$ for $(X,Y)\in\Omega$ where $R=D_yG(X^*,0)$ is a M – matrix. This means that elements of R are non - negative except the elements in the principal diagonal and is the region where the system is biologically and epidemiologically meaningful in Ω.

At DFE, we have

$$\frac{dX}{dt}=\begin{pmatrix}\Lambda-\mu S\\ 0\end{pmatrix}. \tag{3.2}$$

The solution of $S(t)=\frac{\Lambda}{\mu}+\Big(S(0)-\frac{\Lambda}{\mu}\Big)e^{\mu t}$ at DFE such that $\lim_{t\to\infty}=\dfrac{\Lambda}{\mu}$ which shows global convergence of (3.2) in Ω. From model (2.1) we obtain that

$$R=\begin{pmatrix}-g_1 & (1-\nu)\beta\eta & (1-\nu)\beta & 0\\ \alpha & -g_2 & 0 & \varepsilon\\ r_2 & 0 & -g_3 & \sigma\\ r_1 & 0 & 0 & -g_4\end{pmatrix} \quad \text{and} \quad \hat{G}(X,Y)=\begin{pmatrix}\frac{(1-\nu)\beta(\eta A+H)}{N-Q}S\\ 0\\ 0\\ 0\end{pmatrix}.$$

Obviously, $\frac{S}{N-Q}\geq 0$ inside Ω and hence $\hat{G}(X,Y)\geq 0$. This shows that conditions C_1 and C_2 are satisfied and hence DFE of (2.1) is globally asymptotically stable when $R_0<1$. □

Theorem 3.6. *Model (2.1) has an endemic equilibrium, E^1 that is globally asymptotically stable whenever $R_e>1$.*

Proof. We need to show that model (2.1) does not have limit cycles. Let the vector $X=(S,E,Q,A,H,R)$, define a Dulac's function $F=\frac{1}{SR}$. Then

$$\begin{aligned}
F\frac{dS}{dt} &= \frac{\Lambda}{SR}+\frac{\tau Q}{SR}+\frac{\omega}{S}-\frac{(1-\nu)\beta(\eta A+H)}{R(N-Q)}-\frac{\mu}{R},\\
F\frac{dE}{dt} &= \frac{(1-\nu)\beta(\eta A+H)}{R(N-Q)}-\frac{g_1E}{SR}, \quad F\frac{dQ}{dt}=\frac{r_1E}{SR}-\frac{g_2Q}{SR},\\
F\frac{dA}{dt} &= \frac{\alpha E}{SR}+\frac{\varepsilon Q}{SR}-\frac{g_3A}{SR}, \quad F\frac{dH}{dt}=\frac{r_2E}{SR}+\frac{\sigma Q}{SR}-\frac{g_4H}{SR}\\
F\frac{dR}{dt} &= \frac{u_1mA}{SR}+\frac{u_2kH}{SR}-\frac{\omega+\mu}{S}.
\end{aligned}$$

Furthermore,

$$\frac{dFX}{dt}=\frac{\partial}{\partial S}\Big(F\frac{dS}{dt}\Big)+\frac{\partial}{\partial E}\Big(F\frac{dE}{dt}\Big)+\frac{\partial}{\partial Q}\Big(F\frac{dQ}{dt}\Big)+\frac{\partial}{\partial A}\Big(F\frac{dA}{dt}\Big)+\frac{\partial}{\partial H}\Big(F\frac{dH}{dt}\Big)+\frac{\partial}{\partial R}\Big(F\frac{dR}{dt}\Big).$$

$$= \frac{\partial}{\partial S}\Big(\frac{\Lambda}{SR} + \frac{\tau Q}{SR} + \frac{\omega}{S} - \frac{(1-\nu)\beta(\eta A + H)}{R(N-Q} - \frac{\mu}{R}\Big) + \frac{\partial}{\partial E}\Big(\frac{(1-\nu)\beta(\eta A + H)}{R(N-Q} - \frac{g_1 E}{SR}\Big)$$
$$+ \frac{\partial}{\partial Q}\Big(\frac{r_1 E}{SR} - \frac{g_2 Q}{SR}\Big) + \frac{\partial}{\partial A}\Big(\frac{\alpha E}{SR} + \frac{\varepsilon Q}{SR} - \frac{g_3 A}{SR}\Big) + \frac{\partial}{\partial H}\Big(\frac{r_2 E}{SR} + \frac{\sigma Q}{SR} - \frac{g_4 H}{SR}\Big)$$
$$+ \frac{\partial}{\partial R}\Big(\frac{u_1 m A}{SR} + \frac{u_2 k H}{SR} - \frac{\omega + \mu}{S}\Big)$$
$$= -\bigg(\frac{\Lambda}{RS^2} + \frac{\tau Q}{RS^2} + \frac{\omega}{S^2} + \frac{u_1 m A}{SR^2} + \frac{u_2 k H}{SR^2} + \frac{g_1 + g_2 + g_3 + g_4}{SR}\bigg) < 0.$$

Hence by Dulac's criterion having Poincare Bendixson theorem in mind, there is no limit cycle in Ω for model (2.1) since the set is invariant. Therefore the endemic equilibrium is globally asymptotically stable i.e. there are variations in the number of infectious persons in a population, hence the tendency of allocation of resources for COVID-19 control is arduous, making the pandemic to persist in Nigeria □

3.4. Bifurcation analysis.

We study whether a stable DFE co - exists with a stable EEP and an unstable EEP when $R_e = 1$. To obtain the cause(s) of the backward bifurcation in moedl (2.1), we resort to the center manifold theory.

Theorem 3.7. [11] *Assume that*

(i) 0 *is an eigenvalue of* J_{E^0} *and the other eigenvalues of* J_{E^0} *have negative real part;*

(ii) J_{E^0} *has a (nonnegative) right eigenvector* $(w = w_1, w_2, \cdots, w_n)^T$ *and a left eigenvector* $v = (v_1, v_2, \cdots, v_n)$ *with respect to the zero eigenvalue. Let* $f_k(x,\phi)$ *denote the kth component of* $f(x,\phi)$*,* ϕ *is the bifurcation parameter and*

$$a = \sum_{k,i,j=1}^{n} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0,0), b = \sum_{k,i=1}^{n} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}(0,0).$$

Then, the local dynamics of system (2.1) around $x = 0$ *are greatly obtained by the sign of a and b.*

(a) *If* $a > 0$ *and* $b > 0$*, then when* $\phi < 0$ *with* $|\phi| \ll 1, x = 0$ *is locally asymptotically stable and there exists a positive unstable equilibrium. When* $0 < Q \leq 1, x = 0$ *is unstable and there exists a negative asymptotic stable equilibrium;*

(b) *if* $a > 0$ *and* $b > 0$*, then when* $\phi < 0$ *moves from negative to positive,* $x = 0$ *swaps its stability from stable to unstable. Correspondingly, a negative unstable equilibrium becomes positive and local asymptotic stable.*

Particularly, if $a < 0$ and $b > 0$, then a forward bifurcation occurs at $\phi = 0$ and $a > 0$ and $b > 0$, a backward bifurcation occurs at ϕ.

Proof. Let $x_1 = S, x_2 = E, x_3 = Q, x_4 = A, x_5 = H, x_6 = R$ and $\lambda = \frac{\beta(\eta x_4 + x_5)}{N-Q}$. We can rewrite (2.1) in the form of $\frac{dX}{dt} = F(X)$, with $F = \Big(f_1, f_2, f_3, f_4, f_5, f_6\Big)^T$ and $X = (x_1, x_2, x_3, x_4, x_5, x_6)^T$, hence we obtain

$$f_1 \equiv x_1' = \Lambda + \tau x_3 + \omega x_6 - (1-\nu)\lambda x_1 - \mu x_1, \tag{3.3}$$

$$f_2 \equiv x_2' = (1-\nu)\lambda x_1 - (\alpha + r_1 + r_2 + \mu)x_2, \tag{3.4}$$

$$f_3 \equiv x_3' = r_1 x_2 - (\tau + \sigma + \varepsilon + \mu)x_3, \tag{3.5}$$

$$f_4 \equiv x_4' = \alpha x_2 + \varepsilon x_3 - (u_1 m + \mu + d_1)x_4, \tag{3.6}$$

$$f_5 \equiv x_5' = r_2 x_2 + \sigma x_3 - (u_2 k + \mu + d_2), \tag{3.7}$$

$$f_6 \equiv x_6' = u_1 m x_4 + u_2 k x_5 - (\omega + \mu)x_6. \tag{3.8}$$

We choose transmission rate, β as the bifurcation parameter. By simplification at $R_e = 1$ we obtain

$$\beta = \beta^* = \frac{g_1 g_2 g_3 g_4}{(1-\nu)\Big(\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)\Big)}.$$

The matrix J_{E^0} has right eigenvectors given by $w = \Big(w_1, w_2, w_3, w_4, w_5, w_6\Big)^T$ where

$$w_1 = \frac{r_1\tau(\omega+\mu)g_2 g_3 + \omega\Bigg(g_3 u_1 m(\alpha g_4 + \varepsilon r_1) + g_2 u_2 k(r_2 g_4 + \sigma r_1) - (\beta^*(1-\nu)(\omega+\mu)\Big(\tau g_3(\alpha g_4 + \varepsilon r_1) + g_2(r_2 g_4 + \sigma r_1)\Big)\Bigg)}{\mu(\omega+\mu)g_2 g_3 g_4} w_2$$

$$w_2 > 0, w_3 = \frac{\alpha g_4 + \varepsilon r_1}{g_2 g_4} w_2, \quad w_4 = \frac{r_2 g_4 + \sigma r_1}{g_3 g_4} w_2, w_5 = \frac{r_1}{g_4} w_2$$

$$w_6 = \frac{g_3 u_1 m(\alpha g_4 + \varepsilon r_1) + g_2 u_2 k(r_2 g_4 + \sigma r_1)}{(\omega+\mu)g_2 g_3 g_4},$$

and the left eigenvectors given by $v = (v_1, v_2, v_3, v_4, v_5, v_6)$ where $v_1 = 0, v_2 > 0, v_3 = \frac{(1-\nu)\beta^*\eta}{g_2} v_2, v_4 = \frac{(1-\mu)\beta^*}{g_3} v_2, v_5 = \frac{\beta^*(1-\nu)(\eta\varepsilon g_3 + g_2\sigma)}{g_2 g_3 g_4} v_2, v_6 = 0$. We obtain the second - order partial derivative of $f_2, f_3, f_4,$ andf_5 since $v_1 = v_6 = 0$, the second - order partial derivatives of f_1

and f_6 are zero. Hence we have

$$\frac{\partial^2 f_2}{\partial x_2 \partial x_4} = \frac{\partial^2 f_2}{\partial x_4 \partial x_2} = \frac{\partial^2 f_2}{\partial x_4 \partial x_6} = \frac{\partial^2 f_2}{\partial x_6 \partial x_4} = -\beta\eta(1-\nu), \frac{\partial^2 f_2}{\partial x_4^2} = -2(1-\nu)\beta\eta,$$
$$\frac{\partial^2 f_2}{\partial x_2 \partial x_5} = \frac{\partial^2 f_2}{\partial x_5 \partial x_2} = \frac{\partial^2 f_2}{\partial x_5 \partial x_6} = \frac{\partial^2 f_2}{\partial x_6 \partial x_5} = -\beta(1-\nu), \frac{\partial^2 f_2}{\partial x_5 \partial x_4} = \frac{\partial^2 f_2}{\partial x_5 \partial x_4} = -\beta(1-\nu)(1+\eta),$$
$$\frac{\partial^2 f_2}{\partial x_5^2} = -2(1-\nu)\beta, \frac{\partial^2 f_2}{\partial x_2 \partial \beta} = (1-\nu)\eta, \frac{\partial^2 f_2}{\partial x_5 \partial \beta} = 1-\nu,$$

all the other second - order derivatives are zero. Furthermore, we compute for a and b, it follows that

$$\begin{aligned} a &= v_2\Bigg(2\Big(w_2 w_4 \frac{\partial^2 f_2}{\partial x_2 \partial x_4} + w_2 w_5 \frac{\partial^2 f_2}{\partial x_2 \partial x_5} + w_4 w_5 \frac{\partial^2 f_2}{\partial x_4 \partial x_5} + w_4 w_6 \frac{\partial^2 f_2}{\partial x_4 \partial x_6} + w_5 w_6 \frac{\partial^2 f_2}{\partial x_5 \partial x_6}\Big) \\ &\quad + w_4^2 \frac{\partial^2 f_2}{\partial x_4^2} + w_5^2 \frac{\partial^2 f_2}{\partial x_5^2}\Bigg) \\ &= -2v_2(1-\nu)\beta\Big(\eta w_2 w_4 + w_2 w_5 + (1+\eta) w_4 w_5 + \eta w_4 w_6 + w_5 w_6\Big) < 0. \\ b &= v_2\Bigg(w_4 \frac{\partial^2 f_2}{\partial x_4 \partial \beta} + w_5 \frac{\partial^2 f_2}{\partial x_5 \partial \beta} = v_2(1-v)(\eta w_4 + w_5)\Bigg) > 0. \end{aligned}$$

Since $a < 0$ and $b > 0$, by the theorem, model (2.1) undergoes a forward bifurcation at $R_e = 1$. This typically shows that the disease will die out when $R_e < 1$ and persist when $R_e > 1$; in other words, the DFE does not coexist with the EEP when $R_e < 1$. □

4. SENSITIVITY OF THE MODEL PARAMETERS

To study the best control measures of COVID-19 in Nigeria, the sensitivity of model parameters is performed and analyzed considering the reproduction number. The normalized forward sensitivity indices, that tells us how crucial each parameter is to disease transmission and prevalence is applied. The sensitivity index technique measures the most sensitive parameters in the model, those with the positive sign are regarded as highly sensitive to the value of R_0 and others are neutrally sensitive to R_0 which is the quality used in reducing and aborting COVID-19 pandemic. The elasticity indices of R_0 is given by $\gamma_\rho^{R_0} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0}$; where ρ is a parameter present in the reproduction number R_0. Applying sensitivity index, we have

$$\gamma_\beta^{R_0} = 1, \quad \gamma_\nu^{R_0} = -\frac{\nu}{1-\nu}, \quad \gamma_\eta^{R_0} = \frac{\alpha\eta g_3 g_4}{\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)},$$

$$\gamma_\alpha^{R_0} = \frac{\beta(1-\nu)\Big(\eta g_3 g_4(g_1 g_2 - \alpha) - g_2^2(r_2 g_4 + \sigma r_1)\Big)}{g_1^2 g_2^2 g_3^2 g_4^2}, \quad \gamma_{r_1}^{R_0} = \frac{\sigma}{\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)} - \frac{r_1}{g_1},$$

$$\gamma_\eta^{R_0} = r_2\left(\frac{g_2 g_4}{\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)} - \frac{1}{g_1}\right), \quad \gamma_\sigma^{R_0} \frac{r_1\sigma(g_4 - 1)}{\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)},$$

$$\gamma_\tau^{R_0} = -\frac{\sigma\tau r_1}{g_4\Big(\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)\Big)}, \quad \gamma_\varepsilon^{R_0} = -\frac{\varepsilon\sigma r_1}{g_4\Big(\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)\Big)},$$

$$\gamma_m^{R_0} = -\frac{m\eta\alpha g_3 g_4}{g_2\Big(\eta\alpha g_2 g_4 + g_2(r_2 g_4 + \sigma r_1)\Big)}, \quad \gamma_k^{R_0} = -\frac{k g_2(r_2 g_4 + \sigma r_1)}{\Big(\eta\alpha g_3 g_4 + g_2(r_2 g_4 + \sigma r_1)\Big)}.$$

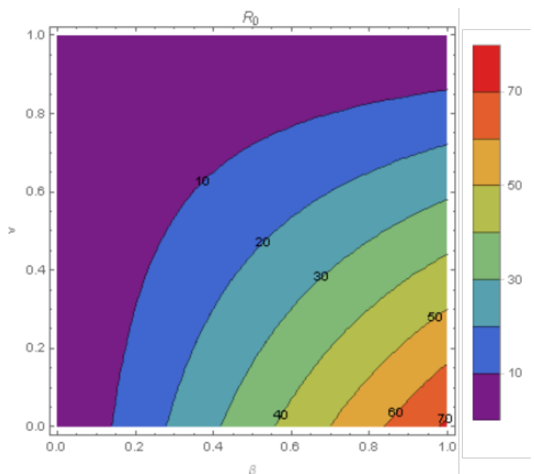

FIGURE 2. Contour plots of R_0 versus transmission rate and healthy diet intake in the population.

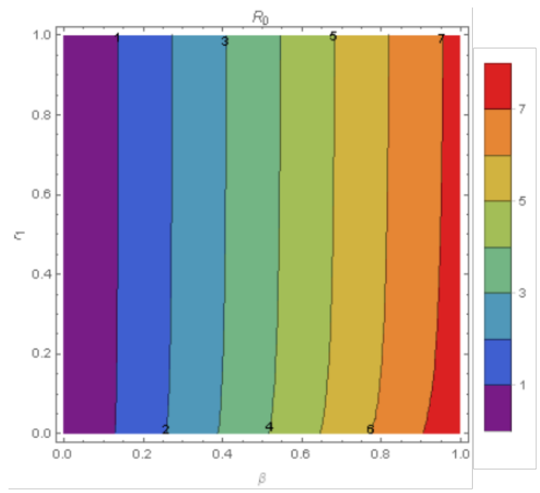

FIGURE 3. Contour plots of R_0 versus transmission rate and rate at which exposed population is quarantine.

The positive relationship for the parameters implies that when the parameter value is increased there will be a remarkable effect on the frequency of the ailment spread. A negative relation shows that positive change of the parameter would help to decrease the prevalence of the disease. In Figure 2, the contours shows that increasing and being conscious of the the dietary and nutritional value of diet consumed by exposed individual lowers the reproduction number R_0, which Figure 7 agrees with in the sense that general practicing of balanced diet fight against any form of diseases and infection. In addition, Figure 3 shows that increase in the rate of number of exposed individuals that are being quarantined reduces the reproduction number slightly, hence case tracing and quarantine is great but should be supported by other means of curtailing the pandemic.

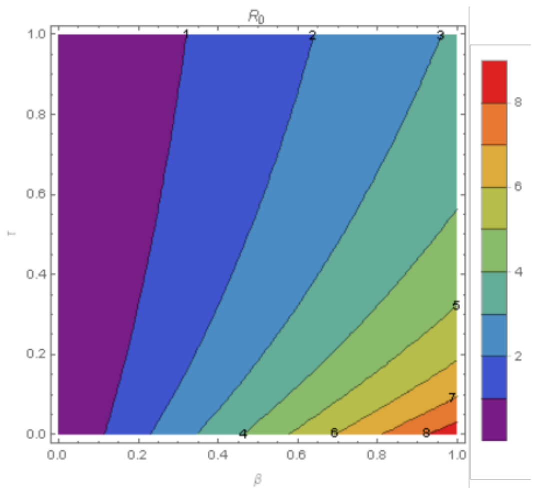

FIGURE 4. Contour plots of R_0 versus transmission rate and rate at which quarantined class become susceptible.

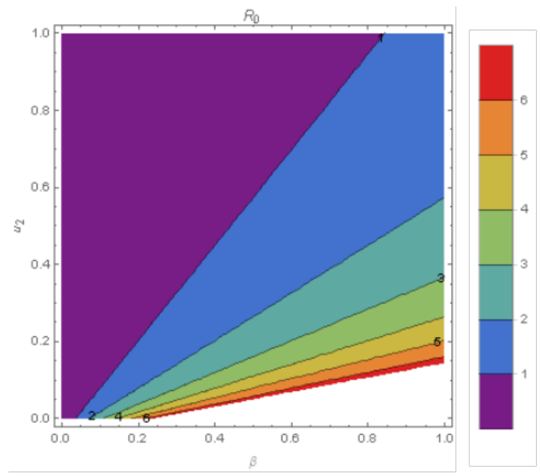

FIGURE 5. Contour plots of R_0 versus value of healthy diet and rate of transmission within the symptomatic class.

When the quarantined population is taken care of properly within the incubation period such that their is an increase in the influx from quarantined class and transmission rate is reduced by proper adherence to the safety measures, Figure 4 shows that the reproduction number is reduced likewise the disease burden. COVID-19 pandemic can be minimized by increasing the nutritional value of foods given to the exposed symptomatic and asymptomatic patients because of its tendency of helping the body in development of antibodies when β is reduced as shown bt the contours in Figure 5. If the body system is not supported either by acquired or latent antibodies, COVID-19 will rampage the population greatly. In Figure 6, we observed that if the rate at which quarantine individual show symptom of COVID-19 is reduced by appropriate care of individual in the center,burden in the population when rate is on the low - ebb.

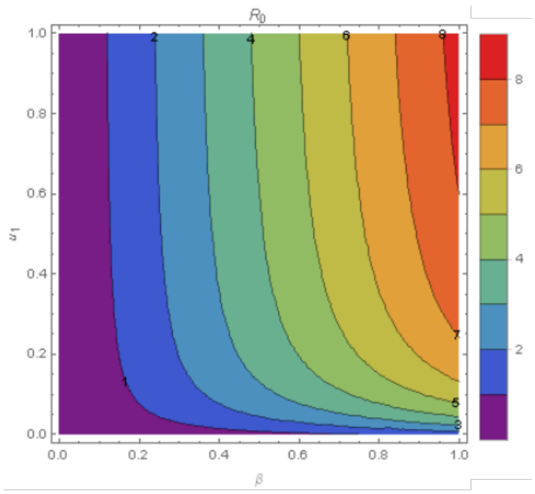

FIGURE 6. Contour plots of R_0 versus transmission rate and rate at which quarantined class develop symptoms.

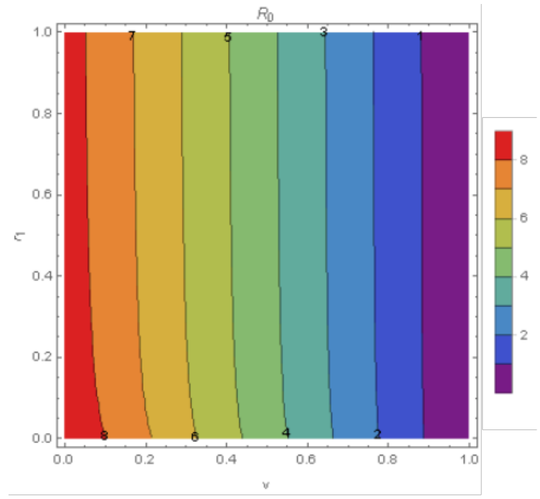

FIGURE 7. Contour plots of R_0 versus value of healthy diet and rate at which Exposed class become Quarantined.

5. NUMERICAL SIMULATION

We estimate the parameters in model (2.1) based on real data of COVID-19 infected cases in Nigeria. We consider the COVID-19 daily data from 27th February 2020 to 16th March, 2022. The data were obtained from Nigeria centre for Disease control (NCDC) site [5]. The data fitting was done using the nonlinear least - squares curve fitting method with the help of "fmin -search" function from MATLAB optimization toolbox. The total population of Nigeria is approximately $N = 215,120,007$, [13, 19] and the life expectancy of Nigerians for the year 2022 is 55.75 years [19]. Then the natural death rate is approximately 0.001495 per month. The recruitment rate of susceptible class $\Lambda = \mu N(0)$ is calculated to be 321,554. According to NCDC report, at March 27th 2020, there have been 83 COVID -19 confirmed cases, hence $H(0) = 83$. At the onset of COVID-19 in Nigeria, the testing kits were readily available, hence there is a great probability that there are handful of asymptomatic individuals [53], consequently, there may be possible greater number of cases reported. Therefore, we can estimate the initial conditions for the exposed, asymptomatic, quarantine and recovered population which will aid in obtaining the initial condition of the susceptible. We therefore assume that

$E(0) = 12,348, Q(0) = 15,600, A(0) = 1200$ and $R(0) = 950$, the $S(0) = 214,781,138$. The values of the calculated and estimated/fitted parameters are summarized in Table 2 and it is to be noted that the values of the parameters used in this work are obtained from estimation, model fitting and from the literature and the unit of parameters(rate constants) is per month. Using the parameter values in Table 2 the basic reproductive value is $R_0 = 2.010$, which is greater than COVID-19 threshold value 1.

TABLE 2. Model 2.1 parameter values.

Parameters	Values	Sources
Λ	321554	Calculated [14, 6]
τ	$\frac{1}{14}$	[43]
ω	0.3274	Fitted
ν	$[0, 0.9]$	Assumed
η	0.45	Fitted
μ	0.001495	Calculated [14]
α	0.02557	Fitted
r_1	0.81692	[6]
r_2	$\frac{1}{7}$	[43]
σ	0.31167	Fitted
ε	0.00872	Fitted
u_1	$[0, 0.9]$	Assumed
m	$\frac{1}{9}$	[2]
d_1	0.16673	[5]
d_2	0.00147	[2]
u_2	$[0, 0.9]$	Assumed
k	0.0667	[16]
β	0.118	[2]

The numerical simulation of model (2.1) is performed in this section using the initial conditions $S(0) = 214,781,138$, $E(0) = 12,348$, $Q(0) = 15,600$, $A(0) = 1200$, $H(0) =$

83 and $R(0) = 950$, with $\beta = 0.118$ and other parameters are as in Table 2. The effective reproduction number is obtained as $R_e = 0.865911$, disease free equilibrium, DFE, $E^0 = (2.15086 \times 10^8, 0, 0, 0, 0, 0)$ which the entire population tends to gradually. At endemic equilibrium the effective reproduction number rises to $R_e = 2.1848$ and $E^1 = (2.16 \times 10^8, 2.56 \times 2.56 \times 10^7, 371913, 449741, 1.36 \times 10^7, 2.151 \times 10^7)$ which signifies that COVID-19 will live with us for some time because all the classes coexist within the population.

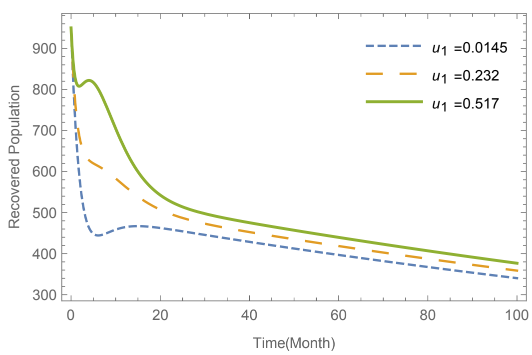

FIGURE 8. Effect of u_1 on the Recovered Population.

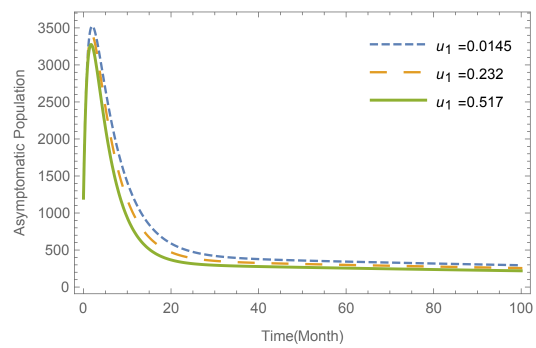

FIGURE 9. Effect of u_1 on the Asymptomatic Population.

An adequate and prescribed intake of zinc, iron, and vitamins $A, B12, B6, C$, and E are essential for the maintenance of immune function system against COVID-19 during recovering stage of infected individual either asymptomatic, symptomatic or exposure of any sort recovers faster along side any other prescribed pharmaceutical approach. When individuals and health givers incorporate HD to the treatment routine, Figure 8 shows that the number of recovery is increased with regards to how dutiful one is the practice, Figure 9 also shows that their is a fall in the population of the asymptomatic cautioned by better response to treatment and increased in the population of susceptible class as shown in Figure 10 as both innate and acquired immunity is boosted within the community.

Individuals consuming well-balanced diets appear to be safer with better immune systems and lower incidence of chronic diseases and infections [49] which shows in Figure 11 and Figure 13 as recovery rate and susceptible population are increased respectively when well - balanced diet is practiced by symptomatic individuals. It can also be observed in Figure 12 that the effect of these diets are not automatic, it requires some time for the antibodies to build up because it

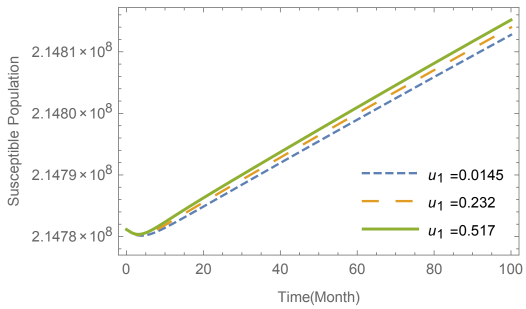

FIGURE 10. Effect of u_1 on the Susceptible Population.

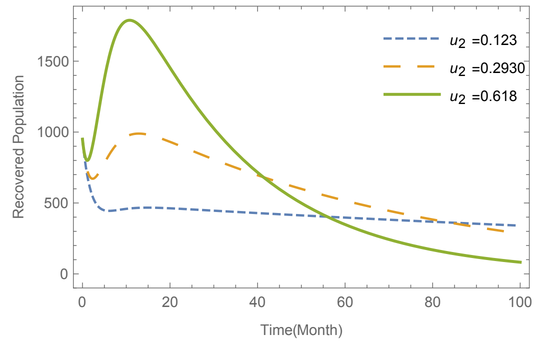

FIGURE 11. Effect of u_2 on the Recovered Population.

has been compromised by the pandemic, and the symptomatic population will start dwindling when they have developed to some point.

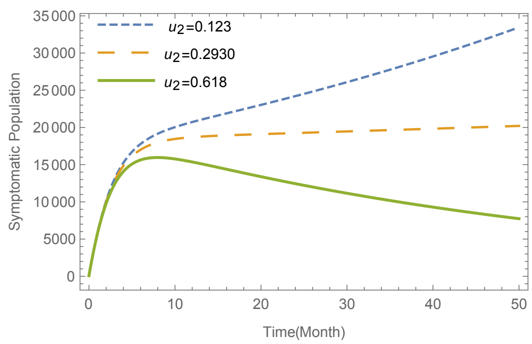

FIGURE 12. Effect of u_2 on the Symptomatic Population.

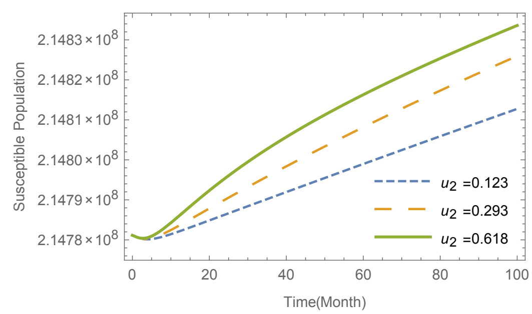

FIGURE 13. Effect of u_2 on the Recovered Population.

Consistent inculcation of healthy dietary habits help to maintain the physical, mental, anatomical, morphological and physiopsychogical health of individuals generally; which is shown by Figure 14 - 19. When v is increased considerably ie. when the community makes it habitual to take in balanced diet, the susceptible population as in Figure 14, often do not leave the compartment in the sense that even when they come in contact with either of the infected, their immune systems are strong enough to release immunoglobulins to wade - off the effect of COVID-19 in the body system. Consequently, there will be a gross decrease in population of the other compartments as shown in Figure 15, 16, 17, 18, and 19.

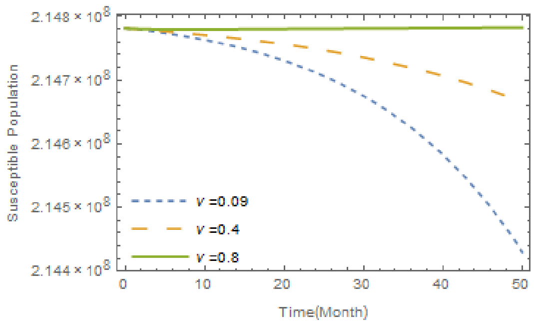

FIGURE 14. Effect of v on the Susceptible Population.

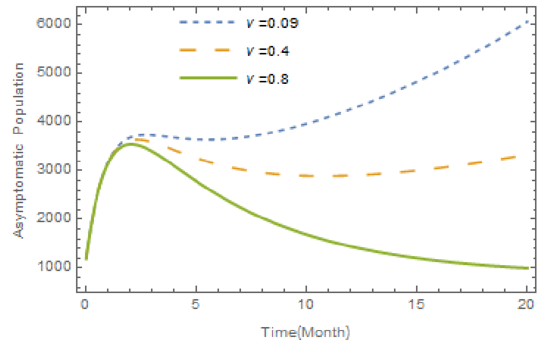

FIGURE 15. Effect of v on the Asymptomatic Population.

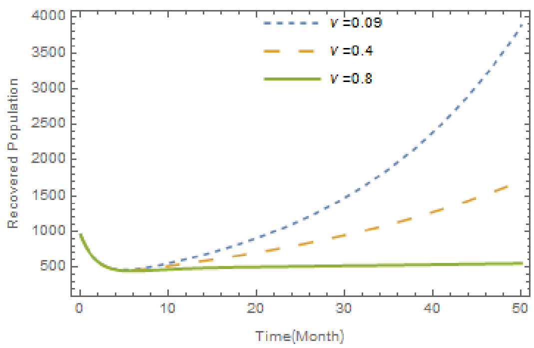

FIGURE 16. Effect of v on the Recovered Population.

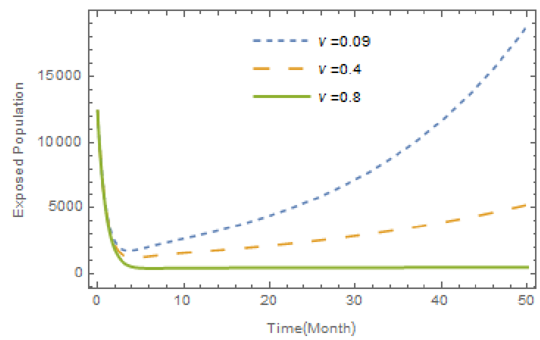

FIGURE 17. Effect of v on the Exposed Population.

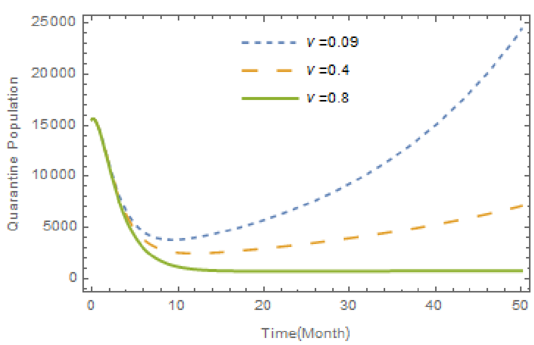

FIGURE 18. Effect of v on the Quarantine Population.

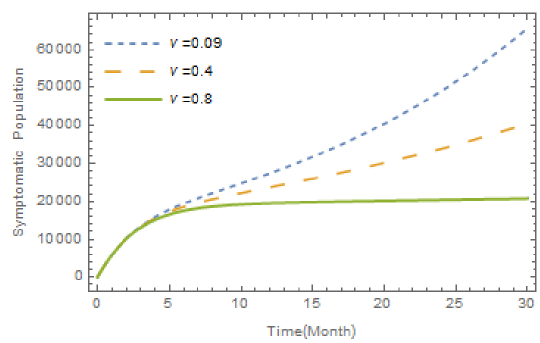

FIGURE 19. Effect of v on the Symptomatic Population.

In Figure 20 - 25, the effects of coefficient of transmission at different values are presented. It is observed that reducing the rate of transmission reduces the burden of infected individuals. Figure 20 shows that when the coefficient of transmission is reduced by individuals observance of all the control protocols enlisted by NCDC without waiting to be coerced the community remain at the susceptible level without progression to other red flag classes.

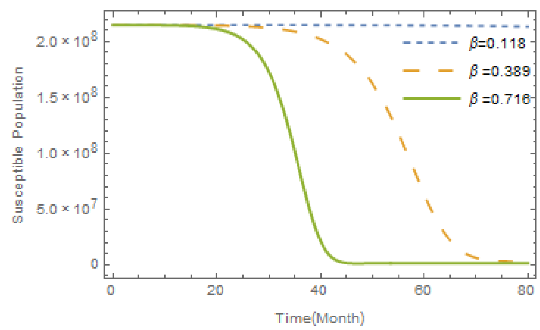

FIGURE 20. Effect of β on the Susceptible Population.

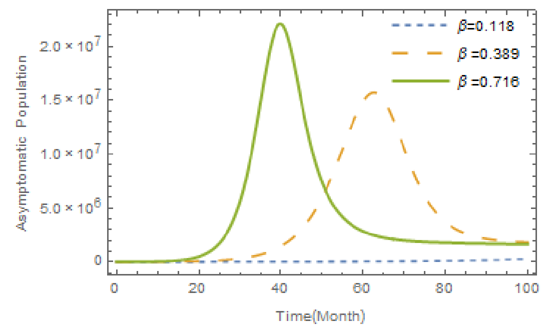

FIGURE 21. Effect of β on the Asymptomatic Population.

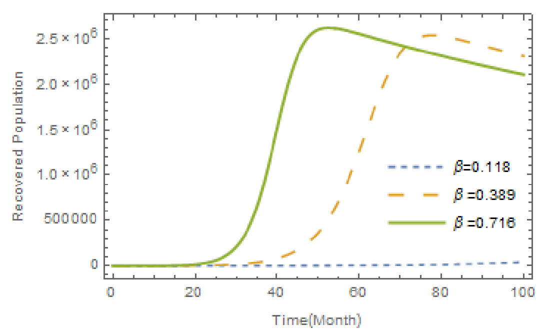

FIGURE 22. Effect of β on the Recovered Population.

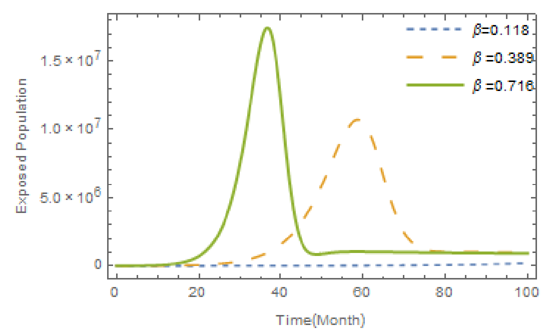

FIGURE 23. Effect of β on the Exposed Population.

From the Figures 21, 23 and 24 it is observed that asymptomatic class, Exposed class and Quarantined classes respectively will increase to a peak before it will start falling because it will take awhile for the community to adapt to the new wave of safety measures since Nigerians live a communal life. The population of Symptomatic and recovered will not fall like the aforementioned class because their systems have witnessed a great attack from the virus and the recuperation will be gradual as shown in Figure 22 and 25. Optimal usage of personal protective measures, taking full dose of vaccine and treatment are required to decrease the number of cases in the compartments except that the optimal application of the control measure and consuming balanced diet need to be maintained relatively for a longer period of time for optimal effect as shown in Figure 26 and 27. Hence combating the dreaded COVID-19 in Nigeria, the general

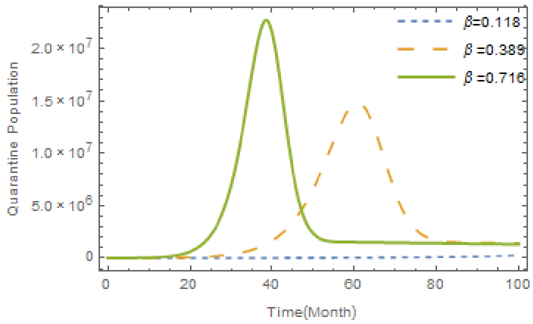

FIGURE 24. Effect of β on the Quarantine Population.

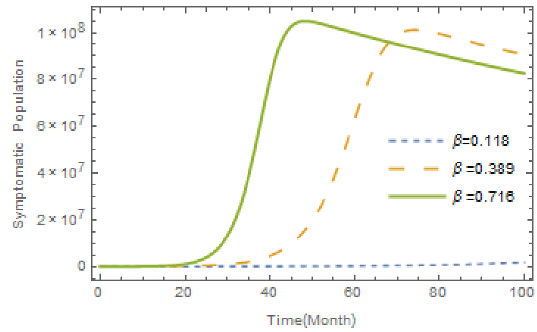

FIGURE 25. Effect of β on the Symptomatic Population.

public need to embrace diets that are able to reinvigorate the body system by assisting or inducing the production of immunoglobulins, along side with optimal usage of personal protective measures.

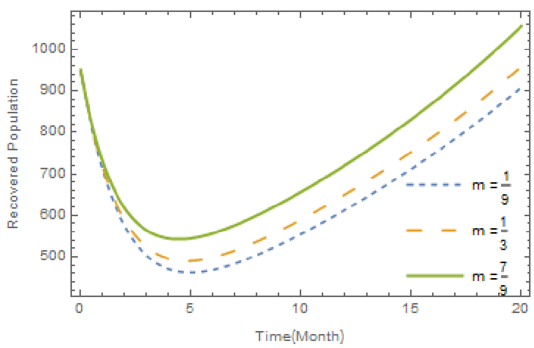

FIGURE 26. Effect of m on the Recovered Population.

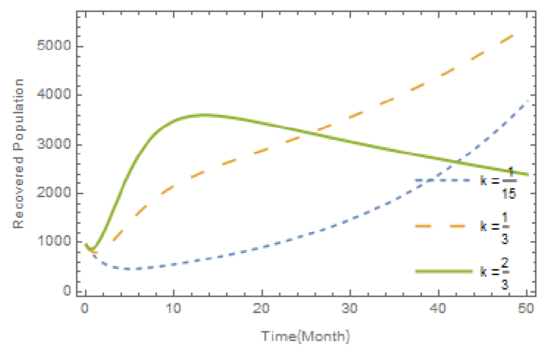

FIGURE 27. Effect of k on the Recovered Population.

6. CONCLUSION

We have modeled fitted data of COVID-19 in Nigeria using mathematical epidemiology technique that considers healthy diet as control and preventive measure. We used active cases data provided by NCDC from March, 2020 to March, 2022 to provide more accurate parameter estimation in our work. Non - pharmaceutical interventions like wearing of face mask, social distancing etc was assumed because of the rigorous public awareness, contact tracing and immediate isolation accompanied with prompt treatment witnessed in Nigeria, [41]. We analyzed the model by first obtaining the feasible region where the model is mathematically and epidemiologically reasonable; the effective reproduction number R_e was computed using next generation matrix. The local and global stability of the disease - free equilibrium based on R_e was analyzed, in addition with the global stability of the endemic equilibrium point. The analytical results shows that the DFE of COVID-19 is both locally and globally stable given that the $R_e < 1$ and unstable if $R_e > 1$. The bifurcation analysis using center manifold theory shows that the model exhibits forward bifurcation; which by implication means that the disease will die down gradually when $R_e < 1$ and persist when $R_e > 1$, and in the other hand, it means that the DFE does not coexist with the EEP hence the government, health workers, public and private sectors need to work collaboratively to reduce the disease burden within the country.

The normalized sensitivity indices of R_e show β, η, r_1, r_2 as the most sensitive parameters, suggesting that the disease load is reduced when the contact with the asymptomatic and symptomatic classes are reduced or avoided. It is also observed that$\alpha, \nu, k, m, u_1, u_2$ follows in the sensitivity level, which shows that the more the exposed is quarantined and treatment is administered appropriately to the infected the value of R_e is reduced, hence the disease burden shall be contained considerably. When the diet of the population is under consideration and balanced diet is cultivated, the R_e is reduced considerably because the boost of immunity as a result of optimal nutrition and dietary nutrient intake and production of immunoglobulin release to the body system. Along with the dietary management guidelines, food safety management with good food practices is compulsory boosting the immune system of the community. Although self - isolation, lockdown and social distancing are essential measures that enables the flattening of COVID-19 curve, they have severe repercussions on people lives [36, 47] especially in Nigeria that citizens live in clusters but a balanced diet will guarantee a strong immune system that is capable to withstand any assault unleashed by COVID-19. The numerical simulation result shows that COVID-19 pandemic is reduced considerably in a given population like Nigeria, if the transmission rate β is reduced by observance of personal hygiene and following of the control measures published by NCDC, reduction of the modified contact rate with the asymptomatic class, increase in the quarantine rate by self consciousness and active implementation of health caveats, using of nutrition and dietary intakes to invigorate our immune system. It also showed that these control measures are not automatic it requires consistent and collaborative efforts from the general public.

This work may be extend by using optimality theory to study eating of balanced diet as optimal intervention strategy, using non-integer fractional derivative to study the transmission dynamics and use of delayed differential system to consider the time lag between exposure and full development of the contagions.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] A.M. Mishra, S.D. Purohit, K.M. Owolabi, Y.D. Sharma, A nonlinear epidemiological model considering asymptotic and quarantine classes for SARS CoV-2 virus, Chaos Solitons Fractals. 138 (2020), 109953.

[2] M.O. Adewole, A.P. Okekunle, I.A. Adeoye, O.M. Akpa, Investigating the transmission dynamics of SARS-CoV-2 in Nigeria: A SEIR modelling approach, Sci. African. 15 (2022), e01116.

[3] I. Ahmed, G.U. Modu, A. Yusuf, P. Kumam, I. Yusuf, A mathematical model of Coronavirus Disease (COVID-19) containing asymptomatic and symptomatic classes, Results Phys. 21 (2021), 103776.

[4] World Health Organisation coronavirus (COVID-19) dashboard. Retrieve on 26/3/2022 from https://covid19.who.int.

[5] Nigeria centre for disease control (NCDC). covid-19 outbreak in Nigeria: Situation report. https://ncdc.gov.ng/diseases/coronavirus.

[6] Z.S. Kifle, L.L. Obsu, Mathematical modeling for COVID-19 transmission dynamics: A case study in Ethiopia, Results Phys. 34 (2022), 105191.

[7] Clinical Characteristics of COVID-19. European Centre for Disease Prevention and Control. Retrieved 25th February 2022 from https://ecdc.europa.eu/en/COVID-19/latest-evidence/clinical.

[8] O.D. Makinde, K.O. Okosun, Impact of chemo-therapy on optimal control of malaria disease with infected immigrants, Biosystems. 104 (2011), 32–41.

[9] J. Mohammed-Awel, A.B. Gumel, Mathematics of an epidemiology-genetics model for assessing the role of insecticides resistance on malaria transmission dynamics, Math. Biosci. 312 (2019), 33–49.

[10] S. Kim, A.A. de los Reyes V., E. Jung, Mathematical model and intervention strategies for mitigating tuberculosis in the Philippines, J. Theor. Biol. 443 (2018), 100–112.

[11] R. Anguelov, S.M. Garba, S. Usaini, Backward bifurcation analysis of epidemiological model with partial immunity, Computers Math. Appl. 68 (2014), 931–940.

[12] W. Guan, Z. Ni, Y. Hu, et al. Clinical characteristics of coronavirus disease 2019 in China, N. Engl. J. Med. 382 (2020), 1708–1720.

[13] World Population, worldpopulationreview.com/countries/Nigeria-population.

[14] C.T. Deressa, G.F. Duressa, Modeling and optimal control analysis of transmission dynamics of COVID-19: The case of Ethiopia, Alex. Eng. J. 60 (2021), 719–732.

[15] Y. Xie, E. Xu, B. Bowe, Z. Al-Aly, Long-term cardiovascular outcomes of COVID-19, Nat. Med. 28 (2022), 583–590.

[16] N.M. Ferguson, D. Laydon, G. Nedjati-Gilani, et al. Report 9 - Impact of Non-Pharmaceutical Interventions (NPIs) to Reduce COVID-19 Mortality and Healthcare demand, in COVID-19, Imperial College London, UK, 2020.

[17] Worldometer, Coronavirus incubation period, https://www.worldometers.info/coronavirus/coronavirus-incu bation-period.

[18] Cai L, Li X, Tuncer N, Martcheva M, Lashari A A. *Optimal control of a malaria model with asymptomatic class and superinfection.* Math Biosci, 288(2017), 94 – 108.

[19] Worldometer, World Population, https://www.worldometers.info/world-population/nigeria-population.

[20] C. Rothe, M. Schunk, P. Sothmann, et al. Transmission of 2019-nCoV infection from an asymptomatic contact in Germany, N. Engl. J. Med. 382 (2020), 970–971.

[21] Centres for Disease Control and Prevention (CDC), Symptoms of coronavirus, https://cdc.gov/coronavrirus /2019-nCoV/about/symptoms.html.

[22] V.P. Bajiya, S. Bugalia, J.P. Tripathi, Mathematical modeling of COVID-19: Impact of non-pharmaceutical interventions in India, Chaos. 30 (2020), 113143.

[23] World Health Organisation, Coronavirus Disease 2019 (COVID-19). Situation Report - 51, Retrieve on 26/3/2022 from https://www.who.int/emergencies/disease/npvel-coronavirus-2019/situation-reports.

[24] S. Ullah, M.A. Khan, Modeling the impact of non-pharmaceutical interventions on the dynamics of novel coronavirus with optimal control analysis with a case study, Chaos Solitons Fractals. 139 (2020), 110075.

[25] S. Olaniyi, O.S. Obabiyi, K.O. Okosun, A.T. Oladipo, S.O. Adewale, Mathematical modelling and optimal cost-effective control of COVID-19 transmission dynamics, Eur. Phys. J. Plus. 135 (2020), 938.

[26] O. Makinde, K. Björkqvist, K. Österman, Overcrowding as a risk factor for domestic violence and antisocial behaviour among adolescents in Ejigbo, Lagos, Nigeria, Glob. Ment. Health. 3 (2016), e16.

[27] R. Subramanian, Q. He, M. Pascual, Quantifying asymptomatic infection and transmission of COVID-19 in New York City using observed cases, serology, and testing capacity, Proc. Natl. Acad. Sci. U.S.A. 118 (2021), e2019716118.

[28] K.G. Varshney, Y.K. Dwivedi, Mathematical Modeling of influenza - Meningitis under the Quarantine effect of influanza, Turk. J. Computer Math. Educ. 12 (2021), 7214–7225.

[29] J.K. Hale, Ordinary differential equations. In: Pure and applied mathematics. New York: Wiley-Interscience, 1969.

[30] A.M. Mishra, S.D. Purohit, K.M. Owolabi, Y.D. Sharma, A nonlinear epidemiological model considering asymptotic and quarantine classes for SARS CoV-2 virus, Chaos Solitons Fractals. 138 (2020), 109953.

[31] S.S. Musa, S. Zhao, N. Hussaini, A.G. Habib, D. He, Mathematical modeling and analysis of meningococcal meningitis transmission dynamics, Int. J. Biomath. 13 (2020), 2050006.

[32] Diekmann O., Heesterbeek J.A. , Metz J.A. , On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations, J. Math. Biol. 28 (1990), 365–382.

[33] B.S. Kotola, T.T. Mekonnen, Mathematical model analysis and numerical simulation for codynamics of meningitis and pneumonia infection with intervention, Sci. Rep. 12 (2022), 2639.

[34] U.E. Michael, A.A. Offia, K.O. Elebute, Control model on transmission dynamic of conjunctivitis during harmattan in public schools, Appl. Comput. Math. 8 (2019), 29–36.

[35] S.M. Muhammad, E. Hincal, U.T. Mustapha, A. Yusuf, Mathematical modeling and analysis for the transmission dynamics of blinding trachoma with effect of awareness programs, Results Phys. 28 (2021), 104683.

[36] M. Farhan Aslam, S. Majeed, S. Aslam, J.A. Irfan, Vitamins: Key role players in boosting up immune response-a mini review, Vitam Miner. 6 (2017), 1.

[37] R. Arias, K. De Angeles, S. Maleki, R.R. Ahangar, Mathematical modeling of the HIV-AIDS epidemic, Open Access Lib. J. 9 (2022), e7972.

[38] D.K. Das, S. Khajanchi, T.K. Kar, The impact of the media awareness and optimal strategy on the prevalence of tuberculosis, Appl. Math. Comput. 366 (2020), 124732.

[39] C. Yang, J. Wang, A mathematical model for the novel coronavirus epidemic in Wuhan, China, Math. Biosci. Eng. 17 (2020), 2708–2724.

[40] L. Lemecha Obsu, S. Feyissa Balcha, Optimal control strategies for the transmission risk of COVID-19, J. Biol. Dyn. 14 (2020), 590–607.

[41] B.E. Bassey, J.U. Atsu, Global stability analysis of the role of multi-therapies and non-pharmaceutical treatment protocols for COVID-19 pandemic, Chaos Solitons Fractals. 143 (2021), 110574.

[42] M. Ali, S.T.H. Shah, M. Imran, A. Khan, The role of asymptomatic class, quarantine and isolation in the transmission of COVID-19, J. Biol. Dyn. 14 (2020), 389–408.

[43] S. Bugalia, V. Pal Bajiya, J. Prakash Tripathi, M.-T. Li, G.-Q. Sun, Mathematical modeling of COVID-19 transmission: the roles of intervention strategies and lockdown, Math. Biosci. Eng. 17 (2020), 5961–5986.

[44] P. Pandey, Y.-M. Chu, J.F. Gómez-Aguilar, H. Jahanshahi, A.A. Aly, A novel fractional mathematical model of COVID-19 epidemic considering quarantine and latent time, Results Phys. 26 (2021), 104286.

[45] D. Aldila, M. Z. Ndii, B. M. Samiadji, Optimal control on COVID-19 eradication program in Indonesia under the effect of community awareness, Math. Biosci. Eng. 17 (2020), 6355–6389.

[46] A. Atangana, Modelling the spread of COVID-19 with new fractal-fractional operators: Can the lockdown save mankind before vaccination?, Chaos Solitons Fractals. 136 (2020), 109860.

[47] M. Gleeson, D.C. Nieman, B.K. Pedersen, Exercise, nutrition and immune function, J. Sports Sci. 22(2004), 115-125.

[48] J.F. Gómez-Aguilar, T. Córdova-Fraga, T. Abdeljawad, et al. Analysis of fractal–fractional malaria transmission model, Fractals. 28 (2020), 2040041.

[49] F. Aman, S. Masood, How Nutrition can help to fight against COVID-19 Pandemic, Pak. J. Med. Sci. 36 (2020). https://doi.org/10.12669/pjms.36.COVID19-S4.2776.

[50] E.F. Doungmo Goufo, Y. Khan, Q.A. Chaudhry, HIV and shifting epicenters for COVID-19, an alert for some countries, Chaos Solitons Fractals. 139 (2020), 110030.

[51] M.A. Aba Oud, A. Ali, H. Alrabaiah, S. Ullah, M.A. Khan, S. Islam, A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation, and environmental viral load, Adv. Differ. Equ. 2021 (2021), 106.

[52] M.A. Hajji, Q. Al-Mdallal, Numerical Simulations of a Delay Model for Immune System-Tumor Interaction, SQU J. Sci. 23 (2018), 19.

[53] H. Khan, J.F. Gómez-Aguilar, A. Alkhazzan, A. Khan, A fractional order HIV-TB coinfection model with nonsingular Mittag-Leffler Law, Math. Meth. Appl. Sci. 43 (2020), 3786–3806.

[54] O. Diekmann, J.A. Heesterbeek, J.A. Metz, On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations, J. Math. Biol. 28 (1990), 365–382.

[55] F.M. Khan, A. Ali, Z.U. Khan, On existence and semi-analytical results to fractional order mathematical model of COVID-19, Arab J. Basic Appl. Sci. 29 (2022), 40–52.

[56] J.K. Hale, Ordinary differential equations. In: Pure and applied mathematics, New York: Wiley-Interscience, 1969.

[57] R. Subramanian, Q. He, M. Pascual, Quantifying asymptomatic infection and transmission of COVID-19 in New York City using observed cases, serology, and testing capacity, Proc. Natl. Acad. Sci. U.S.A. 118 (2021), e2019716118.