Available online at http://scik.org
J. Math. Comput. Sci. 2022, 12:185
https://doi.org/10.28919/jmcs/7480
ISSN: 1927-5307

SOFT D_s^*-METRIC SPACES AND A FIXED POINT THEOREM OF SOFT CONTINUOUS MAPPINGS ON SOFT D_s^*-METRIC SPACES

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Abstract. The extent of soft Ds^*- metric space is mostly explained in this study. We define the soft Ds^*-metric space and provide fundamental definitions. We have included examples to support the definition. For the situation of soft Ds^*-metric space, we also introduced soft Δs^*-distance. We also prove the fixed point theorem for soft continuous mappings on soft Ds^*- metric space.

Keywords: soft D-metric; D^*-metric space; soft D_s^*-metric space; soft Δ_s^*-distance; fixed point theorem.

2010 AMS Subject Classification: 47H10, 54H25.

1. INTRODUCTION

Diverse generalizations and classifications of metric spaces are being studied for decades due to its wide range of applications in several disciplines of mathematics and other areas of science. Bapure Dhage [3], Das and Samanta [2], Aras et al. [1] introduced the generalizations called D-metric space, soft metric space and Soft D-metric spaces. In 2007 Sedghi et al. [4, 5] introduced and investigated the importance and properties of D^*-metric spaces which tackled the limitations of D-metric spaces in fixed point theory. In this paper we are introducing the definition of soft D_s^*- metric space which is a combination of generalized metric spaces which

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Received May 08, 2022

can be seen in [4] and [1] and investigates its properties and applications in connection with fixed point theory.

2. PRELIMINARIES

This section is devoted to detail only the most necessary definitions and properties connected to follow up the results obtained here. Further explanations are included in the cited references.

Definition 2.1. [4] Let X be a non empty set. A generalized metric (or D^*-metric) on X is a function, $D^* : X^3 \longrightarrow [0,\infty)$, that satisfies the following conditions. For each $x,y,z,a \in X$:

(1) $D^*(x,y,z) \geq 0$,
(2) $D^*(x,y,z) = 0$ if and only if $x = y = z$,
(3) $D^*(x,y,z) = D^*(p\{x,y,z\})$, (symmetry) where p is a permutation function,
(4) $D^*(x,y,z) \leq D^*(x,y,a) + D^*(a,z,z)$.

The pair (X,D^*) is called a generalized metric (or D^*-metric) space.

Definition 2.2. [1] A mapping $D : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E^*)$ is called a soft D- metric on the soft set $\tilde{X}$ that satisfies the following conditions, for each soft points $x_a, y_b, z_c, u_d \in SP(\tilde{X})$,

(1) $D(x_a,y_b,z_c) \geq \tilde{0}$ and equality holds if and only if $x_a = y_b = z_c$ $(coincidence)$
(2) $D(x_a,y_b,z_c) = D(y_b,x_a,z_c) = D(x_a,z_c,y_b) = \ldots (Symmetry)$
(3) $D(x_a,y_b,z_c) \leq D(x_a,y_b,u_d) + D(x_a,u_d,z_c) + D(u_d,y_b,z_c)$

Then the set of $\tilde{X}$ with a soft D- metric is called a soft D- metric space and denoted by $(\tilde{X},D,E)$.

Definition 2.3. [1] Let $\tilde{X}$ be a soft D-metric space with soft metric D. Then a mapping $\Delta :$ $(SP(\tilde{X}))^3 \longrightarrow R(E)^*$ is called a soft Δ-distance on the soft set $\tilde{X}$ if the following conditions are satisfied:

(1) $\Delta(x_a,y_b,z_c) \leq \Delta(x_a,y_b,u_d) + \Delta(x_a,u_d,z_c) + \Delta(u_d,y_b,z_c)$ for all soft points $x_a,y_b,z_c,u_d \in SP(\tilde{X})$,
(2) for any $x_a,y_b \in SP(\tilde{X}), \Delta(x_a,y_b,.) : SP(\tilde{X}) \longrightarrow R(E)^*$ is soft continuous,
(3) for any $\tilde{\varepsilon} > \tilde{0}$, there exists $\tilde{\delta} > \tilde{0}$ such that $\Delta(u_d,x_a,y_b) \leq \tilde{\delta}$, $\Delta(u_d,x_a,z_c) \leq \tilde{\delta}$ and $\Delta(u_d,y_b,z_c) \leq \tilde{\delta}$ imply that $D(x_a,y_b,z_c) \leq \tilde{\varepsilon}$.

Lemma 2.1. [1] *Let $(\tilde{X}, D, E)$ be a soft D-metric space and Δ-be a soft distance on the soft set $\tilde{X}$. Let $\{x_{a_n}^n\}$ and $\left\{y_{b_n}^n\right\}$ be two soft sequences in $\tilde{X}$ and $\{p_n\}, \{q_n\}$ and $\{t_n\}$ be sequences in $R(E)^*$ converging to $\tilde{0}$ and assume that soft points $x_a, y_b, z_c, u_d \in SP(\tilde{X})$. Then we have the following statements:*

(a) *If $\Delta(x_{a_n}^n, p_n, y_{b_n}^n) \leq p_n$, $\Delta(x_{a_n}^n, p_n, z_c) \leq q_n$ and $\Delta(x_{a_n}^n, y_{b_n}^n, z_c) \leq t_n$ for any $n \in N$, then $D(p_n, y_{b_n}^n, z_c) \longrightarrow \tilde{0}$.*

(b) *If $\Delta(x_{a_n}^n, x_{a_m}^m, x_{a_k}^k) \leq p_n$ for any $n, m, k \in N$ with $m > n > k$, then $\{x_{a_n}^n\}$ is a Cauchy sequence in $(\tilde{X}, D, E)$.*

Theorem 2.1. *Let $(\tilde{X}, D, E)$ be a complete D metric space and Δ-be a distance on $\tilde{X}$, $(f, \varphi) : (\tilde{X}, D, E) \longrightarrow (\tilde{X}, D, E)$ be a soft mapping. Let $\tilde{X}$ be Δ-bounded. Suppose that there exists a soft real number $\tilde{r} \in R(E)$; $\tilde{0} < \tilde{r} < \tilde{1}$*

$$\Delta((f,\varphi)(x_a), (f,\varphi)^2(x_a), (f,\varphi)(y_b)) \leq \tilde{r}\Delta(x_a, (f,\varphi)(x_a), y_b)$$

for all $x_a, y_b \in SP(\tilde{X})$. Then there exist $z_c \in SP(\tilde{X})$ such that $z_c = (f,\varphi)(z_c)$. In addition to, if $v_s = (f,\varphi)(v_s)$, then $\Delta(v_s, v_s, v_s) = \tilde{0}$.

3. Main Results

Definition 3.1. Let $\tilde{X}$ be the absolute soft set, E be a non empty set of parameters and $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $R(E)^*$ denote the set of all non negative soft real numbers. A mapping $D_s^* : (SP(\tilde{X}))^3 \longrightarrow R(E)^*$ is called a soft D_s^*-metric on the soft set $\tilde{X}$ that D_s^* satisfies the following conditions, for each soft points $\tilde{x}_a, \tilde{y}_b, \tilde{z}_c, \tilde{u}_d \in SP(\tilde{X})$,

(1) $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \geq \tilde{0}$

(2) $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \tilde{0}$ if and only if $\tilde{x}_a = \tilde{y}_b = \tilde{z}_c$

(3) $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = D_s^*(\tilde{y}_b, \tilde{x}_a, \tilde{z}_c) = D_s^*(\tilde{x}_a, \tilde{z}_c, \tilde{y}_b) = \ldots (Symmetry)$

(4) $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \leq D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{u}_d) + D_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c)$

Then the soft set $\tilde{X}$ with a soft D_s^*-metric is called a soft D_s^*-metric space and denoted by $(\tilde{X}, D_s^*, E)$.

Example 3.1. $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \begin{cases} \tilde{0} & if\ \tilde{x}_a = \tilde{y}_b = \tilde{z}_c \\ \tilde{1} & otherwise \end{cases}$

Example 3.2. *Let (X, d^*) be an ordinary metric on X such that*

$$d_s^*(\tilde{x}_a, \tilde{y}_b) = |a-b| + d^*(x,y)$$

is a soft metric on $SP(\tilde{X})$. Then the soft D_s^*-metric,

$$D_s^* : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E)^*$$

can be defined as

$$D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = d_s^*(\tilde{x}_a, \tilde{y}_b) + d_s^*(\tilde{y}_b, \tilde{z}_c) + d_s^*(\tilde{x}_a, \tilde{z}_c)$$

for all $\tilde{x}_a, \tilde{y}_b, \tilde{z}_c, \in SP(\tilde{X})$

The conditions (1),(2),(3) are clear since d_s^* is a soft metric on $SP(\tilde{X})$. We need to verify condition (4) only.

$$\begin{aligned}
&D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c)\\
&= d_s^*(\tilde{x}_a, \tilde{y}_b) + d_s^*(\tilde{y}_b, \tilde{z}_c) + d_s^*(\tilde{x}_a, \tilde{z}_c)\\
&= d_s^*(\tilde{x}_a, \tilde{y}_b) + |b-c| + d^*(y,z) + |a-c| + d^*(x,z)\\
&\leq d_s^*(\tilde{x}_a, \tilde{y}_b) + |b-d| + |d-c| + d^*(y,u) + d^*(u,z) + |a-d| + |d-c| + d^*(x,u) + d^*(u,z)\\
&= d_s^*(\tilde{x}_a, \tilde{y}_b) + |b-d| + d^*(y,u) + |a-d| + d^*(x,u) + 2\,[|d-c| + d^*(u,z)]\\
&= d_s^*(\tilde{x}_a, \tilde{y}_b) + d_s^*(\tilde{y}_b, \tilde{u}_d) + d_s^*(\tilde{x}_a, \tilde{u}_d) + 2d_s^*(\tilde{u}_d, \tilde{z}_c)\\
&= D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{u}_d) + D_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c)
\end{aligned}$$

Thus D_s^* is a soft D_s^* metric on $SP(\tilde{X})$.

Example 3.3. *Let (X, d^*) be an ordinary metric on X such that*

$$d_s^*(\tilde{x}_a, \tilde{y}_b) = |a-b| + d^*(x,y)$$

is a soft metric on $SP(\tilde{X})$. Then the soft D_s^* metric,

$$D_s^* : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E)^*$$

can be defined as

$$D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \max\{d_s^*(\tilde{x}_a, \tilde{y}_b), d_s^*(\tilde{y}_b, \tilde{z}_c), d_s^*(\tilde{x}_a, \tilde{z}_c)\}$$

As in above example, the conditions (1),(2),(3) are clear since d_s^* is a soft metric on $SP(\tilde{X})$. We need to verify condition (4) only. It can be verify easily as in Example 3.2.

Remark 3.1. *If $(\tilde{X}, D_s^*, E)$ is a soft D_s^*-metric space, then $(\tilde{X}, D_a, E)$ is a soft D_a-metric space for each $a \in E$, where D_a stands for the soft D_a-metric for single parameter a. So every soft D_s^*-metric space is a family of parametrized D_a-metric space.*

Remark 3.2. *In a soft D_s^*-metric space we prove that $D_s^*(\tilde{x}_a, \tilde{x}_a, \tilde{y}_b) = D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b)$.*

Proof. For,

i. $D_s^*(\tilde{x}_a, \tilde{x}_a, \tilde{y}_b) \leq D_s^*(\tilde{x}_a, \tilde{x}_a, \tilde{x}_a) + D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b) = D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b)$

ii. $D_s^*(\tilde{y}_b, \tilde{y}_b, \tilde{x}_a) \leq D_s^*(\tilde{y}_b, \tilde{y}_b, \tilde{y}_b) + D_s^*(\tilde{y}_b, \tilde{x}_a, \tilde{x}_a) = D_s^*(\tilde{y}_b, \tilde{x}_a, \tilde{x}_a)$

Using symmetry we have $D_s^*(\tilde{x}_a, \tilde{x}_a, \tilde{y}_b) = D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b)$ □

Definition 3.2. Let $(\tilde{X}, D_s^*, E)$ be a soft D_s^* metric space.

(a) A soft sequence $\{x_{a_n}^n\}$ in $(\tilde{X}, D_s^*, E)$ converges to a soft point $\tilde{x}_{a_p} \in SP(\tilde{X})$ if for each $\tilde{\varepsilon} > \tilde{0}$, there exist $n_0 \in N$ such that, for all $m, n \geq n_0$,

$$D_s^*(\tilde{x}_{a_n}^n, \tilde{x}_{a_m}^m, \tilde{x}_{a_p}) < \tilde{\varepsilon}$$

(b) A soft sequence $\{\tilde{x}_{a_n}^n\}$ in $(\tilde{X}, D_s^*, E)$ is called a Cauchy sequence if for $\tilde{\varepsilon} > \tilde{0}$, there exist $n_0 \in N$ such that, for all $m, n > n_0$,

$$D_s^*(\tilde{x}_{a_n}^n, \tilde{x}_{a_n}^n, \tilde{x}_{a_m}^m) < \tilde{\varepsilon}.$$

(c) The soft D_s^*-metric space $(\tilde{X}, D_s^*, E)$ is said to be complete if every Cauchy sequence is convergent.

Definition 3.3. Let $\tilde{X}$ be a soft D_s^*-metric space with soft metric D_s^*. Then a mapping $\Delta_s^* : (SP(\tilde{X}))^3 \longrightarrow R(E)^*$ is called a soft Δ_s^*-distance on the soft set $\tilde{X}$ if the following conditions are satisfied:

(1) $\Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \leq \Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{u}_d) + \Delta_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c)$ for all soft points $\tilde{x}_a, \tilde{y}_b, \tilde{z}_c, \tilde{u}_d \in SP(\tilde{X})$,

(2) for any $\tilde{x}_a, \tilde{y}_b \in SP(\tilde{X}), \Delta_s^*(\tilde{x}_a, \tilde{y}_b, .) : SP(\tilde{X}) \longrightarrow R(E)^*$ is soft continuous,

(3) for any $\tilde{\varepsilon} > \tilde{0}$, there exists $\tilde{\delta} > \tilde{0}$ such that $\Delta_s^*(\tilde{u}_d, \tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c) \leq \tilde{\delta}$ imply that $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \leq \tilde{\varepsilon}$.

Example 3.4. *Let (X, d^*) be an ordinary metric on X such that*

$$d_s^*(\tilde{x}_a, \tilde{y}_b) = |a - b| + d^*(x, y)$$

is a soft metric on $SP(\tilde{X})$. Then the soft D_s^* metric,

$$D_s^* : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E)^*$$

can be defined as

$$D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \max\{d_s^*(\tilde{x}_a, \tilde{y}_b), d_s^*(\tilde{y}_b, \tilde{z}_c), d_s^*(\tilde{x}_a, \tilde{z}_c)\}$$

. Then $\Delta_s^* = D_s^*$ is a soft Δ_s^*-distance on the soft set $\tilde{X}$.

Conditions (1) and (2) are clear. We just want to prove the condition (3) only. Let $\tilde{\varepsilon} > \tilde{0}$ be given and put $\tilde{\delta} = \tilde{\varepsilon}$. If $\Delta_s^*(\tilde{u}_d, \tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c) \leq \tilde{\delta}$, we have $d_s^*(\tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}, d_s^*(\tilde{y}_b, \tilde{z}_c) \leq \tilde{\delta}$ and $d_s^*(\tilde{x}_a, \tilde{z}_c) \leq \tilde{\delta}$, which implies that

$$D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \max\{d_s^*(\tilde{x}_a, \tilde{y}_b), d_s^*(\tilde{y}_b, \tilde{z}_c), d_s^*(\tilde{x}_a, \tilde{z}_c)\} \leq \tilde{\delta} = \tilde{\varepsilon}.$$

Example 3.5. *Consider Example 3.2. The mapping $\Delta_s^* : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E)^*$ defined by $\Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \tilde{r}$, a non negative soft real number.*

For the soft D_s^*-metric conditions (1) and (2) are trivial. To show the condition (3), for arbitrary $\tilde{\varepsilon} > \tilde{0}$, take $\tilde{\delta} = \frac{\tilde{\varepsilon}}{3}$, then $\Delta_s^*(\tilde{u}_d, \tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c) \leq \tilde{\delta}$ we have $d_s^*(\tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}, d_s^*(\tilde{y}_b, \tilde{z}_c) \leq \tilde{\delta}$ and $d_s^*(\tilde{x}_a, \tilde{z}_c) \leq \tilde{\delta}$, which implies that

$$D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) = \{d_s^*(\tilde{x}_a, \tilde{y}_b) + d_s^*(\tilde{y}_b, \tilde{z}_c) + d_s^*(\tilde{x}_a, \tilde{z}_c)\} \leq \frac{\tilde{\varepsilon}}{3} + \frac{\tilde{\varepsilon}}{3} + \frac{\tilde{\varepsilon}}{3} = \tilde{\varepsilon}.$$

Lemma 3.1. *Let $(\tilde{X}, D_s^*, E)$ be a soft D_s^*-metric space and Δ_s^*-be a soft distance on the soft set $\tilde{X}$. Let $\{\tilde{x}_{a_n}^n\}$ and $\left\{\tilde{y}_{b_n}^n\right\}$ be two soft sequences in $\tilde{X}$ and $\{\tilde{p}_n\}$ and $\{\tilde{q}_n\}$ be sequences in $R(E)^*$ converging to $\tilde{0}$ and assume that soft points $\tilde{x}_a, \tilde{y}_b, \tilde{z}_c, \tilde{u}_d \in SP(\tilde{X})$. Then we have the following statements:*

(a) *If $\Delta_s^*(\tilde{x}_{a_n}^n, \tilde{p}_n, \tilde{y}_{b_n}^n) \leq \tilde{p}_n$ and $\Delta_s^*(\tilde{x}_{a_n}^n, \tilde{z}_c, \tilde{z}_c) \leq \tilde{q}_n$ for any $n \in N$, then $D_s^*(\tilde{p}_n, \tilde{y}_{b_n}^n, \tilde{z}_c) \longrightarrow \tilde{0}$.*

(b) *If* $\Delta_s^*(\tilde{x}_{a_n}^n, \tilde{x}_{a_n}^n, \tilde{x}_{a_m}^m) \leq \tilde{p}_n$ *for any* $n, m \in N$ *with* $m > n$, *then* $\left\{\tilde{x}_{a_n}^n\right\}$ *is a Cauchy sequence in* $(\tilde{X}, D_s^*, E)$.

Proof. (a). Let $\tilde{\varepsilon} > \tilde{0}$ be arbitrary. The definition of Δ_s^*-distance provides a $\tilde{\delta} > \tilde{0}$ such that $\Delta_s^*(\tilde{u}_d, \tilde{x}_a, \tilde{y}_b) \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{u}_d, \tilde{z}_c, \tilde{z}_c) \leq \tilde{\delta}$ imply that $D_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \leq \tilde{\varepsilon}$. Choose $n_0 \in N$ such that $\tilde{p}_n \leq \tilde{\delta}$ and $\tilde{q}_n \leq \tilde{\delta}$ for every $n \geq n_0$. Then for any $n \geq n_0$ we have $\Delta_s^*(\tilde{x}_{a_n}^n, \tilde{p}_n, \tilde{y}_{b_n}^n) \leq \tilde{p}_n \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{x}_{a_n}^n, \tilde{z}_c, \tilde{z}_c) \leq \tilde{q}_n \leq \tilde{\delta}$ and hence $D_s^*(\tilde{p}_n, \tilde{y}_{b_n}^n, \tilde{z}_c) \leq \tilde{\varepsilon}$. If we replace $\{\tilde{p}_n\}$ with $\left\{\tilde{y}_{b_n}^n\right\}$, then $\left\{\tilde{y}_{b_n}^n\right\}$ converges to $\tilde{z}_c$.

(b). Let $\tilde{\varepsilon} > \tilde{0}$ be arbitrary. As in the proof of (a), choose $\tilde{\delta} > \tilde{0}$, $n_0 \in N$, $m > n > n_0$, $\Delta_s^*(\tilde{x}_{a_{n_0}}^{n_0}, \tilde{x}_{a_n}^n, \tilde{x}_{a_n}^n) \leq \tilde{p}_n \leq \tilde{\delta}$ and $\Delta_s^*(\tilde{x}_{a_{n_0}}^{n_0}, \tilde{x}_{a_m}^m, \tilde{x}_{a_m}^m) \leq \tilde{q}_n \leq \tilde{\delta}$ and hence $D_s^*(\tilde{x}_{a_n}^n, \tilde{x}_{a_n}^n, \tilde{x}_{a_m}^m) \leq \tilde{\varepsilon}$ implies $\left\{\tilde{x}_{a_n}^n\right\}$ is a Cauchy sequence in $(\tilde{X}, D_s^*, E)$. □

Definition 3.4. Let $\tilde{X}$ be an absolute soft set. $\tilde{X}$ is said to be Δ_s^*-bounded if there is a constant $\tilde{M}^*$ such that $\Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{z}_c) \leq \tilde{M}^*$ for all $\tilde{x}_a, \tilde{y}_b, \tilde{z}_c \in SP(\tilde{X})$.

Theorem 3.1. *Let* $(\tilde{X}, D_s^*, E)$ *be a complete* D_s^* *metric space and* Δ_s^**-be a distance on* $\tilde{X}$, $(\tilde{f}, \tilde{\varphi}) : (\tilde{X}, D_s^*, E) \longrightarrow (\tilde{X}, D_s^*, E)$ *be a soft mapping. Let* $\tilde{X}$ *be* Δ_s^**-bounded. Suppose that there exists a soft real number* $\tilde{r} \in R(E) : \tilde{0} < \tilde{r} < \tilde{1}$

$$\Delta_s^*((\tilde{f}, \tilde{\varphi})(\tilde{x}_a), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_a), (\tilde{f}, \tilde{\varphi})(\tilde{y}_b)) \;\leq\; \tilde{r}\, \Delta_s^*(\tilde{x}_a, (\tilde{f}, \tilde{\varphi})(\tilde{x}_a), \tilde{y}_b)$$

for all $\tilde{x}_a, \tilde{y}_b \in SP(\tilde{X})$. *Then there exist* $\tilde{z}_c \in SP(\tilde{X})$ *such that* $\tilde{z}_c = (\tilde{f}, \tilde{\varphi})(\tilde{z}_c)$. *In addition to, if* $\tilde{v}_s = (\tilde{f}, \tilde{\varphi})(\tilde{v}_s)$, *then* $\Delta_s^*(\tilde{v}_s, \tilde{v}_s, \tilde{v}_s) = \tilde{0}$.

Proof. We claim that

$$\inf\left\{\Delta_s^*(\tilde{x}_a, (\tilde{f}, \tilde{\varphi})(\tilde{x}_a), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_a)) + \Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b) : \tilde{x}_a \in SP(\tilde{X})\right\} > \tilde{0},$$

for all $\tilde{y}_b \in SP(\tilde{X})$ with $\tilde{y}_b \neq (\tilde{f}, \tilde{\varphi})(\tilde{y}_b)$. Suppose that the claim is true. Let $\tilde{u}_d \in SP(\tilde{X})$ and define a soft sequence $\left\{\tilde{u}_{d_n}^n\right\}$ in $\tilde{X}$ by $\tilde{u}_{d_n}^n = (\tilde{f}, \tilde{\varphi})^n(\tilde{u}_d)$, for all $n \in N$. Then for all $n, t \in N$, we have

$$\Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_n}^n, \tilde{u}_{d_{n+t}}^{n+t}) \;\leq\; \Delta_s^*(\tilde{u}_{d_{n-1}}^{n-1}, \tilde{u}_{d_{n-1}}^{n-1}, \tilde{u}_{d_{n+t-1}}^{n+t-1}) \;\leq\; \dots \;\leq\; \tilde{r}^n \Delta_s^*(\tilde{u}_d, \tilde{u}_d, \tilde{u}_{d_t}^t)$$

Thus for any $m > n$ for which $m = n + k$ $(k \in N)$, we have

$$\begin{aligned}\Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_n}^n, \tilde{u}_{d_m}^m) &\leq \Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_n}^n, \tilde{u}_{d_{n+1}}^{n+1}) + \ldots + \Delta_s^*(\tilde{u}_{d_{m-1}}^{m-1}, \tilde{u}_{d_{m-1}}^{m-1}, \tilde{u}_{d_m}^m) \\ &\leq \frac{\tilde{r}^n}{\tilde{1} - \tilde{r}} 2\tilde{M}^*\end{aligned}$$

By part (b) of Lemma 3.1, the soft sequence $\left\{\tilde{u}_{d_n}^n\right\}$ converges to a soft point $\tilde{z}_c \in SP(\tilde{X})$. Let $n \in N$ be fixed. Then by soft continuous of Δ_s^*, we have

$$\Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_n}^n, \tilde{z}_c) \leq \lim_{m \longrightarrow \infty} \Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_n}^n, \tilde{u}_{d_m}^m) \leq \frac{\tilde{r}^n}{\tilde{1} - \tilde{r}} 2\tilde{M}^*.$$

Assume that $\tilde{z}_c \neq (\tilde{f}, \tilde{\varphi})(\tilde{z}_c)$. Then by hypothesis, we have

$$\begin{aligned}\tilde{0} &< \inf\left\{\Delta_s^*(\tilde{x}_a, (\tilde{f}, \tilde{\varphi})(\tilde{x}_a), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_a)) + \Delta_s^*(\tilde{x}_a, \tilde{z}_c, \tilde{z}_c)\right\} \\ &\leq \inf\left\{\Delta_s^*(\tilde{u}_{d_n}^n, \tilde{u}_{d_{n+1}}^{n+1}, \tilde{u}_{d_{n+2}}^{n+2}) + \Delta_s^*(\tilde{u}_{d_n}^n, \tilde{z}_c, \tilde{z}_c)\right\} \\ &\leq \inf\left\{\tilde{r}^n \tilde{M}^* + \frac{\tilde{r}^n}{\tilde{1} - \tilde{r}} 2\tilde{M}^* : n \in N\right\} \\ &= \tilde{0}\end{aligned}$$

This is a contradiction. Therefore we have, $\tilde{z}_c = (\tilde{f}, \tilde{\varphi})(\tilde{z}_c)$. Now, if $\tilde{v}_s = (\tilde{f}, \tilde{\varphi})(\tilde{v}_s)$, we have

$$\begin{aligned}\Delta_s^*(\tilde{v}_s, \tilde{v}_s, \tilde{v}_s) &= \Delta_s^*\left((\tilde{f}, \tilde{\varphi})(\tilde{v}_s), (\tilde{f}, \tilde{\varphi})^2(\tilde{v}_s), (\tilde{f}, \tilde{\varphi})^3(\tilde{v}_s)\right) \\ &\leq \tilde{r}\Delta_s^*\left(\tilde{v}_s, (\tilde{f}, \tilde{\varphi})(\tilde{v}_s), (\tilde{f}, \tilde{\varphi})^2(\tilde{v}_s)\right) \\ &= \tilde{r}\Delta_s^*(\tilde{v}_s, \tilde{v}_s, \tilde{v}_s)\end{aligned}$$

and so $\Delta_s^*(\tilde{v}_s, \tilde{v}_s, \tilde{v}_s) = \tilde{0}$.

Now we prove the claim. Assume that there exists $\tilde{y}_b \in SP(\tilde{X})$, $\tilde{y}_b \neq (\tilde{f}, \tilde{\varphi})(\tilde{y}_b)$ and

$$\inf\left\{\Delta_s^*(\tilde{x}_a, (\tilde{f}, \tilde{\varphi})(\tilde{x}_a), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_a)) + \Delta_s^*(\tilde{x}_a, \tilde{y}_b, \tilde{y}_b)\right\} = \tilde{0}$$

There exists a sequence $\left\{\tilde{x}_{a_n}^n\right\}$ in $\tilde{X}$ such that

$$\lim_{n \longrightarrow \infty} \left\{\Delta_s^*(\tilde{x}_{a_n}^n, (\tilde{f}, \tilde{\varphi})(\tilde{x}_{a_n}^n), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_{a_n}^n)) + \Delta_s^*(\tilde{x}_{a_n}^n, \tilde{y}_b, \tilde{y}_b)\right\} = \tilde{0}$$

Thus we have

$$\lim_{n \longrightarrow \infty} \Delta_s^*\left(\tilde{x}_{a_n}^n, (\tilde{f}, \tilde{\varphi})(\tilde{x}_{a_n}^n), (\tilde{f}, \tilde{\varphi})^2(\tilde{x}_{a_n}^n)\right) = \tilde{0}$$

$$\lim_{n \longrightarrow \infty} \Delta_s^*(\tilde{x}_{a_n}^n, \tilde{y}_b, \tilde{y}_b) = \tilde{0}$$

and hence by part (a) of Lemma 3.1, we have

$$\lim_{n\longrightarrow\infty} D_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),\tilde{y}_b\right)=\tilde{0},$$

and by soft continuity of D_s^*-metric,

$$\lim_{n\longrightarrow\infty}(\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n)=\lim_{n\longrightarrow\infty}(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n)=\tilde{y}_b$$

We have

$$\Delta_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})(\tilde{y}_b)\right)\ \leq\ \tilde{r}\lim_{n\longrightarrow\infty}\Delta_s^*\left(\tilde{x}_{a_n}^n,(\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),\tilde{y}_b\right)=\tilde{0}$$

$$\begin{aligned}\lim_{n\longrightarrow\infty}\Delta_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),\tilde{y}_b,(\tilde{f},\tilde{\varphi})(\tilde{y}_b)\right)\ &\leq\ \lim_{n\longrightarrow\infty}\inf\Delta_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})(\tilde{y}_b)\right)\\ &\leq\tilde{r}\lim_{n\longrightarrow\infty}\Delta_s^*\left(\tilde{x}_{a_n}^n,(\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),\tilde{y}_b\right)=\tilde{0}\end{aligned}$$

and

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\Delta_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})(\tilde{y}_b)\right)\\ &\leq\lim_{n\longrightarrow\infty}\inf\Delta_s^*\left((\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n)\right)\\ &\leq\tilde{r}\lim_{n\longrightarrow\infty}\Delta_s^*\left(\tilde{x}_{a_n}^n,(\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})(\tilde{x}_{a_n}^n)\right)\\ &\leq\tilde{r}\lim_{n\longrightarrow\infty}\Delta_s^*\left(\tilde{x}_{a_n}^n,(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),(\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n)\right)\\ &=\tilde{0}\end{aligned}$$

By part (a) of Lemma 3.1, we have

$$\lim_{n\longrightarrow\infty}D_s^*\left((\tilde{f},\tilde{\varphi})^2(\tilde{x}_{a_n}^n),\tilde{y}_b,(\tilde{f},\tilde{\varphi})(\tilde{y}_b)\right)=\tilde{0}$$

and thus $\tilde{y}_b=(\tilde{f},\tilde{\varphi})(\tilde{y}_b)$. This is a contradiction. This completes the proof. □

4. CONCLUSION

In this paper we have introduced soft D_s^*-metric space and soft Δ_s^*-distance for soft points of soft sets and proved fixed point theorem of continuous type mappings on soft D_s^*-metric space.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] C.G. Aras, S. Bayramov, M.I. Yazar. Soft d-metric spaces, Bol. Soc. Paran. Mat. 38 (2020), 137–147.

[2] S. Das, S. Samanta. Soft metric, Ann. Fuzzy Math. Inform. 6 (2013), 77–94.

[3] B.C. Dhage. Generalised metric space and mappings with fixed point, Bull. Calc. Math. Soc. 84 (1992), 329-336.

[4] S. Sedghi, N. Shobe, H. Zhou. A common fixed point theorem in D*-metric spaces, Fixed Point Theory Appl. 2007 (2007), 27906.

[5] S. Sedghi, D. Turkoglu, N. Shobe, S. Sedghi, Common fixed point theorems for six weakly compatible mappings in D*-metric spaces, Thai J. Math. 7 (2012), 381–391.