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SOME APPLICATION OF IDEALS OF NLA-RINGS

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Abstract. In this paper we define c-prime, 3-prime, equiprime and weakly prime ideal of nLA-ring which we will study relation of c-prime, 3-prime, weakly prime ideal and prime ideal.

Keywords: c-prime 3-prime; equiprime weakly prime ideal.

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1. Introduction

M.A. Kazim and MD. Naseeruddin [3] asserted that, in every LA-semigroups G a *medial law* hold

$$(ab)(cd) = (ac)(bd), \quad \forall a,b,c,d \in G.$$

Q. Mushtaq and M. Khan [5] asserted that, in every LA-semigroups G with left identity

$$(ab)(cd) = (db)(ca), \quad \forall a,b,c,d \in G.$$

Further M. Khan, Faisal, and V. Amjid [4], asserted that, if a LA-semigroup G with left identity the following law holds

$$a(bc) = b(ac), \quad \forall a,b,c \in G.$$

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M. Sarwar (Kamran) [7] defined LA-group as the following; a groupoid G is called a left almost group, abbreviated as LA-group, if (i) there exists $e \in G$ such that $ea = a$ for all $a \in G$, (ii) for every $a \in G$ there exists $a' \in G$ such that, $a'a = e$, (iii) $(ab)c = (cb)a$ for every $a, b, c \in G$.

Let $\langle G, \cdot \rangle$ be an LA-group and S be a non-empty subset of G and S is itself and LA-group under the binary operation induced by G, the S is called an LA-subgroup of G, then S is called an LA-subgroup of $\langle G, \cdot \rangle$.

S.M. Yusuf in [9] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and "·" is called a left almost ring, if $\langle R, + \rangle$ is an LA-group, $\langle R, \cdot \rangle$ is an LA-semigroup and distributive laws of "·" over "+" holds. T. Shah and I. Rehman [9, p.211] asserted that a commutative ring $\langle R, +, \cdot \rangle$, we can always obtain an LA-ring $\langle R, \oplus, \cdot \rangle$ by defining, for $a, b, c \in R$, $a \oplus b = b - a$ and $a \cdot b$ is same as in the ring. We can not assume the addition to be commutative in an LA-ring. An LA-ring $\langle R, +, \cdot \rangle$ is said to be LA-integral domain if $a \cdot b = 0$, $a, b \in R$, then $a = 0$ or $b = 0$. Let $\langle R, +, \cdot \rangle$ be an LA-ring and S be a non-empty subset of R and S is itself and LA-ring under the binary operation induced by R, the S is called an LA-subring of R, then S is called an LA-subring of $\langle R, +, \cdot \rangle$. If S is an LA-subring of an LA-ring $\langle R, +, \cdot \rangle$, then S is called a left ideal of R if $RS \subseteq S$. Right and two-sided ideals are defined in the usual manner.

By [6] a near-ring is a non-empty set N together with two binary operations "+" and "·" such that $\langle N, + \rangle$ is a group (not necessarily abelian), $\langle N, \cdot \rangle$ is a semigroup and one sided distributive (left or right) of "·" over "+" holds.

In [1] an ideal I of N is called c-prime if $a, b \in N$ and $ab \in I$ implies $a \in I$ or $b \in I$. N is called c-prime near ring if $\{0\}$ is a c-prime ideal of R.

In [2] an ideal I of N is called equiprime if $a \in N \backslash I$ and $x, y \in N$ with $anx - any \in I$ for all $n \in N$ implies $x - y \in I$.

An ideal I of N is called 3-prime if $a, b \in N$ and $anb \in I$ for all $n \in N$ implies $a \in I$ or $b \in I$.

The notions of c-ideal, 3-prime ideal and prime ideal coincide in near rings.

In [8] A proper ideal I of an naer ring N to be weakly prime if $0 \neq AB \subseteq I$ implies either $A \subseteq I$ or $B \subseteq I$ for any ideals A, B of N.

The following implications are well known in near rings:

(1) equiprime $\Rightarrow$ 3-prime ideal $\Rightarrow$ prime ideal;

(2) c-prime ideal $\Rightarrow$ 3-prime ideal $\Rightarrow$ prime ideal;

(3) prime ideal $\Rightarrow$ weakly prime ideal.

2. NEAR LEFT ALMOST RINGS

T. Shah, F. Rehman and M. Raees [10] introduces the concept of a near left almost ring (nLA-ring).

Definition 2.1. [10]. A non-empty set N with two binary operation "+" and "·" is called a near left almost ring (or simply an nLA-ring) if and only if

(1) $\langle N,+\rangle$ is an LA-group.

(2) $\langle N,\cdot\rangle$ is an LA-semigroup.

(3) Left distributive property of $\cdot$ over $+$ holds, that is $a(b+c)=ab+ac$ for all $a,b,c\in N$.

Definition 2.2. [10]. An nLA-ring $\langle N,+,\cdot\rangle$ with left identity 1, such that $1a=a$ for all $a\in N$, is called an nLA-ring with left identity.

Definition 2.3. [10]. A nonempty subset S of an nLA-ring N is said to be an nLA-subring if and only if S is itself an nLA-ring under the same binary operations as in N.

Theorem 2.1. [10, p.1106]. A non-empty subset S of an nLA-ring $\langle N,+\rangle$ is an nLA-subring if and only if $a-b\in S$ and $ab\in S$ for all $a,b\in S$.

Definition 2.4. [10]. An nLA-subring I of an nLA-ring N is called a left ideal of N if $NI\subseteq I$, and I is called a right ideal if for all $n,m\in N$ and $i\in I$ such that $(i+n)m-nm\in I$, and is called two sided ideal or simply ideal if it is both left and right ideal.

Definition 2.5. [10]. An ideal P of a near ring N is said to be prime if $IJ\subseteq P$ then $I\subseteq P$ or $J\subseteq P$ for all I,J ideal of N.

3. MAIN RESULTS

In this section we define c-prime, 3-prime and equiprime of nLA-ring and study relations of c-prime, 3-prime and equiprime ideal.

Definition 3.1. An ideal I of an nLA-ring N is called *c-prime* if $a,b \in N$ and $ab \in I$ implies $a \in I$ or $b \in I$.

N is called c-prime nearring if $\{0\}$ is a c-prime ideal of N.

Definition 3.2. An ideal I of an nLA-ring N is called *3-prime* if $a,b \in N$ and $arb \in I$ for all $r \in N$ implies $a \in I$ or $b \in I$.

Definition 3.3. An ideal I of N is called *equiprime* if $a \in N \backslash I$ and $x,y \in N$ with $anx - any \in I$ for all $n \in N$ implies $x - y \in I$.

The following lemmas and theorem we will study relation of c-prime, 3-prime, equiprime and prime ideals.

Lemma 3.1. Every equiprime ideal is a 3-prime ideal

Proof. Let I be an equiprime ideal of N. Suppose $aNb \subseteq I$, where $a,b \in N$. Since I is an equiprime ideal there exists $n \in N$ such that $anb - an0 \notin I$. Then $a - b \in I$ so $a \in I$ or $b \in I$. □

Lemma 3.2. Every c-prime ideal is a 3-prime ideal

Proof. Suppose that I is a c-prime ideal of nLA-ring N, let $a,b \in N$ and $anb \in I$ for all $n \in N$. Since I is a c-prime ideal we have $a \in I$ and $b \in I$. Then I is a 3-prime ideal of N. □

Lemma 3.3. Every 3-prime ideal is a prime ideal

Proof. Suppose that I is a 3-prime ideal of nLA-ring N, let $a,b \in I$ and $ab \in I$. Since I is a 3-prime ideal we have $a \in I$ and $b \in I$. Then I is a prime ideal of N. □

Lemma 3.4. Every equiprime ideal is a prime ideal

Proof. Suppose that I is an equiprime ideal of nLA-ring N. By Lemma 3.1 and Lemma 3.3 we have I is a prime ideal. □

Lemma 3.5. Every c-prime ideal is a prime ideal

Proof. Suppose that I is a c-prime ideal of nLA-ring N, let $a,b \in I$ and $ab \in I$. Since I is a c-prime ideal we have $a \in I$ and $b \in I$. Then I is a prime ideal of N. □

In [11]. studied if I is a prime ideal in nLA-ring N if and only if N/I is an nLA-integral domain. The following theorems are application by lemmas 3.3, 3.4 and 3.5

Theorem 3.6. Let N be an nLA-ring. Then I is a 3-prime ideal in N if and only if N/I is an nLA-integral domain.

Proof. $(\Rightarrow)$ Let I is a 3-prime ideal in N. By Lemma 3.3 then I is a prime ideal. Thus N/I is an nLA-integral domain.

$(\Leftarrow)$ Assume that N/I is an nLA-integral domain with $arb \in I$ for all $r \in N$. Then $I + arb = I$ so $(I+a)r(I+b) = I$. Since N/I is an nLA-integral domain we have $I + a = I$ or $I + b = I$. Then $a \in I$ or $b \in I$. Thus I is a 3-prime ideal of N. $\square$

Theorem 3.7. Let N be an nLA-ring. Then I is a equiprime ideal in N if and only if N/I is an nLA-integral domain.

Proof. $(\Rightarrow)$ Let I is a equiprime ideal in N. By Lemma 3.4 then I is a prime ideal. Thus N/I is an nLA-integral domain.

$(\Leftarrow)$ Assume that N/I is an nLA-integral domain then there exists $n \in N$ with $anx - any \in I$ for all $a, b \in N \backslash I$ and $x, y \in N$. Thus $I + (anx - any) = I$ so $I + an(x - y) = I$. Since N/I is an nLA-integral domain we have $I + anx = I$ or $I + any = I$. Then $x - y \in I$ Thus I is a equiprime ideal of N. $\square$

Theorem 3.8. Let N be an nLA-ring. Then I is a c-prime ideal in N if and only if N/I is an nLA-integral domain.

Proof. $(\Rightarrow)$ Let I is a c-prime ideal in N. By Lemma 3.5 then I is a prime ideal. Thus N/I is an nLA-integral domain.

$(\Leftarrow)$ Assume that N/I is an nLA-integral domain with $ab \in I$ for all $a, b \in N$. Then $I + ab = I$ so $(I+a)(I+b) = I$. Since N/I is an nLA-integral domain we have $I + a = I$ or $I + b = I$. Then $a \in I$ or $b \in I$. Thus I is a c-prime ideal of N. $\square$

4. Weakly prime

In this section we define weakly prime and study application of c-prime, 3-prime, equiprime and prime ideal in weakly prime ideal.

Definition 4.1. A proper ideal I of an nLA-ring N to be *weakly prime* if $0 \neq AB \subseteq I$ implies either $A \subseteq I$ or $B \subseteq I$ for any ideals A, B of N.

Clearly every prime ideal is weakly prime and $\{0\}$ is always weakly prime ideal of N. The following theorem we will study properties

Theorem 4.1. If I is weakly prime but not prime, then $I^2 = 0$.

Proof. Assume that $I^2 \neq 0$. We show that I is a prime ideal. Let A and B be an ideal of N such that $AB \subseteq I$. If $AB \neq 0$, then $A \subseteq I$ or $B \subseteq I$. If $AB = 0$. since $I^2 \neq 0$ there exists $p_0, q_0 \in P$ such that $\langle p_0 \rangle \langle q_0 \rangle \neq 0$. Then $(A + \langle p_0 \rangle)(B + \langle q_0 \rangle) \neq 0$. Suppose that $(A + \langle p_0 \rangle)(B + \langle q_0 \rangle) \nsubseteq p$. Then there exists $a \in A, b \in B$ and $p_0' \in \langle p_0 \rangle; q_0' \in \langle q_0 \rangle$ such that $(a + \langle p_0' \rangle)(b + \langle q_0' \rangle) \notin I$ which implies $a(b + q_0') \notin I$, but $a(b + q_0') = a(b + q_0') - ab \in I$ since $AB = 0$, a contradiction. So $0 \neq (A + \langle p_0 \rangle)(B + \langle q_0 \rangle) \subseteq I$ which implies $A \subseteq I$ and $B \subseteq I$. □

Corollary 4.2. Let N be an-nLA-ring and I is an ideal of N. If $I^2 \neq 0$ then I is a prime ideal if and only if I is a weakly prime.

The following theorem we will study relation of c-prime, 3-prime, equiprime and weakly prime ideals.

Theorem 4.3. Every c-prime ideal is a weakly prime ideal

Proof. Suppose that I is a c-prime ideal of nLA-ring N. By Lemma 3.2 we have I is a prime ideal. Since every prime ideal is weakly prime ideal we have I is a weakly prime ideal of N. □

Theorem 4.4. Every 3-prime ideal is a weakly prime ideal

Proof. Suppose that I is a 3-prime ideal of nLA-ring N. By Lemma 3.3 we have I is a prime ideal. Since every prime ideal is weakly prime ideal we have I is a weakly prime ideal of N. □

Theorem 4.5. Every equiprime ideal is a weakly prime ideal

Proof. Suppose that I is an equiprime idealof nLA-ring N. By Lemma 3.4 we have I is a prime ideal. Since every prime ideal is weakly prime ideal we have I is a weakly prime ideal of N. □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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