Available online at http://scik.org
J. Math. Comput. Sci. 2022, 12:187
https://doi.org/10.28919/jmcs/7503
ISSN: 1927-5307

SIMPLE OF NEAR LEFT ALMOST RINGS

THITI GAKETEM*

Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand

Abstract. In this pager, we define simple of nLA-rings. Finally we will study properties of simple of nLA-rings and some properties of ideal of nLA-rings.

Keywords: nLA-ring; simple.

2010 AMS Subject Classification: 16Y60, 03E72.

1. Introduction

M.A. Kazim and MD. Naseeruddin defined LA-semigroup as the following; a groupoid S is called a left almost semigroup, abbreviated as LA-semigroup if

$$(ab)c = (cb)a, \quad \forall a,b,c \in S.$$

We called properties above left invertive law.

M.A. Kazim and MD. Naseeruddin [1] asserted that, in every LA-semigroups G a *medial law* hold

$$(ab)(cd) = (ac)(bd), \quad \forall a,b,c,d \in G.$$

*Corresponding author

E-mail address: thiti.ga@up.ac.th

Received May 16, 2022

Q. Mushtaq and M. Khan [3] asserted that, in every LA-semigroups G with left identity

$$(ab)(cd) = (db)(ca), \quad \forall a,b,c,d \in G.$$

Further M. Khan, Faisal, and V. Amjid [2], asserted that, if a LA-semigroup G with left identity the following law holds

$$a(bc) = b(ac), \quad \forall a,b,c \in G.$$

In this note we prefer to called left almost semigroup (LA-semigroup) as Abel-Grassmann's groupoid (abbreviated as an "AG-groupoid").

M. Sarwar (Kamran) [5] defined LA-group as the following; a groupoid G is called a left almost group, abbreviated as LA-group, if (i) there exists $e \in G$ such that $ea = a$ for all $a \in G$, (ii) for every $a \in G$ there exists $a' \in G$ such that, $a'a = e$, (iii) $(ab)c = (cb)a$ for every $a,b,c \in G$. We called (iii) left invertive law.

Let $\langle G, \cdot \rangle$ be an LA-group and S be a non-empty subset of G and S is itself and LA-group under the binary operation induced by G, the S is called an LA-subgroup of G, then S is called an LA-subgroup of $\langle G, \cdot \rangle$.

S.M. Yusuf in [8] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and "·" is called a left almost ring, if $\langle R, + \rangle$ is an LA-group, $\langle R, \cdot \rangle$ is an LA-semigroup and distributive laws of "·" over "+" holds. T. Shah and I. Rehman [8] asserted that a commutative ring $\langle R, +, \cdot \rangle$, we can always obtain an LA-ring $\langle R, \oplus, \cdot \rangle$ by defining, for $a,b,c \in R$, $a \oplus b = b - a$ and $a \cdot b$ is same as in the ring. We can not assume the addition to be commutative in an LA-ring. An LA-ring $\langle R, +, \cdot \rangle$ is said to be LA-integral domain if $a \cdot b = 0$, $a,b \in R$, then $a = 0$ or $b = 0$. Let $\langle R, +, \cdot \rangle$ be an LA-ring and S be a non-empty subset of R and S is itself and LA-ring under the binary operation induced by R, the S is called an LA-subring of R, then S is called an LA-subring of $\langle R, +, \cdot \rangle$. If S is an LA-subring of an LA-ring $\langle R, +, \cdot \rangle$, then S is called a left ideal of R if $RS \subseteq S$. Right and two-sided ideals are defined in the usual manner.

By [4] a near-ring is a non-empty set N together with two binary operations "+" and "·" such that $\langle N, + \rangle$ is a group (not necessarily abelian), $\langle N, \cdot \rangle$ is a semigroup and one sided distributive (left or right) of "·" over "+" holds.

In this page we will defined of simple of nLA-ring such that we will defined the same as simple of nearring.

2. Near Left Almost Rings

T. Shah, F. Rehman and M. Raees [7] introduces the concept of a near left almost ring (nLA-ring). And we study some properties of nLA-ring

Definition 2.1. [7]. A non-empty set N with two binary operation "+" and "·" is called a *near left almost ring* (or simply an nLA-ring) if and only if

(1) $\langle N,+\rangle$ is an LA-group.
(2) $\langle N,\cdot\rangle$ is an LA-semigroup.
(3) Left distributive property of $\cdot$ over $+$ holds, that is $a\cdot(b+c)=a\cdot b+a\cdot c$ for all $a,b,c\in N$.

Definition 2.2. [7]. An nLA-ring $\langle N,+\rangle$ with left identity 1, such that $1\cdot a=a$ for all $a\in N$, is called an *nLA-ring with left identity*.

Let N be an nLA-ring. If A,B are non-empty subset of N, we denote by AB the subset of N consisting of all finite sums of the form $\sum a_ib_i$ where $a_i\in A$ and $b_i\in B$, i.e.,

$$AB=\left\{\sum_{i=1}^{m}a_ib_i \mid n\in\mathbb{N}, a_i\in A, b_j\in B\right\} \text{ for all } i$$

Definition 2.3. [7]. A non-empty subset S of an nLA-ring N is said to be an *nLA-subring* if and only if S is itself an nLA-ring under the same binary operations as in N.

Theorem 2.1. [7]. A non-empty subset S of an nLA-ring $\langle N,+,\cdot\rangle$ is an nLA-subring if and only if $a-b\in S$ and $ab\in S$ for all $a,b\in S$.

Corollary 2.2. Let A,B be non-empty subset of nLA-ring N. Then AB is an nLA-subring of N.

Proof. Since $A,B\neq\emptyset$ we have $AB\neq\emptyset$.

Let $x=\sum_{i=1}^{m}r_is_i,\ y=\sum_{j=1}^{n}u_jv_j\in AB$. Then

$$x-y=\sum_{i=1}^{m}r_is_i-\sum_{j=1}^{n}u_jv_j\in AB$$

and

$$xy = \left(\sum_{i=1}^{m} r_i s_i\right)\left(\sum_{j=1}^{n} u_j v_j\right) \in AB.$$

Thus AB is an nLA-subring of N. □

Theorem 2.3. Let N be an nLA-ring. If A, B and C are non-empty subset of N. Then $(AB)C \subseteq (CB)A$

Proof. Let $x \in (AB)C$ then $x = \sum_{i=1}^{n}\left(\sum_{j=1}^{m} a_j b_j\right) c_i$ where $m, n \in \mathbb{N}$, $a_j \in A, b_j \in B, c_i \in C$ for all i, j. Thus

$$\begin{aligned} x &= \sum_{i=1}^{n}\left(\sum_{j=1}^{m} a_j b_j\right) c_i = \sum_{i=1}^{n}\sum_{j=1}^{m}(a_j b_j)c_i \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m}(c_i b_j)a_i = \sum_{i=1}^{n}\sum_{j=1}^{m}(c_i b_j)a_i \in (CB)A \end{aligned}$$

Hence $(AB)C \subseteq (CB)A$ □

Next we will defined of zero and we will study properties of zero of nLA-ring.

Definition 2.4. Let N be an nLA-ring. An element $x \in N$ is called a *left (right) zero* if $xy = x(yx = x)$ for all $x, y \in N$. Furthermore if x is both a left and right zero of N then x is called a *zero* of N

Theorem 2.4. Let N be an nLA-ring. Then the following statement hold

(1) If N has a left zero and right zero, then N has a zero.
(2) If N has zero, then that zero is unique.

Proof. (1) Let e and f be a left zero and right zero of N, respectively. Then $e = ef = f$ N. Thus N has a zero.

(2) This is obvious from the proof of (1) that the zero is unique.

□

Definition 2.5. An nLA-ring N with additive identity 0 is called *zero-symmetric* if $0x = 0 = x0$ for all $x \in N$

Definition 2.6. [7]. An nLA-subring I of an nLA-ring N is called a *left ideal* of N if $NI \subseteq I$, and I is called a *right ideal* if for all $n, m \in N$ and $i \in I$ such that $(i+n)m - nm \in I$, and is called *two sided ideal* or simply *ideal* if it is both left and right ideal.

Theorem 2.5. Let N be an nLA-ring. Then N is a zero-symmetric if and only $\{0\}$ is an ideal.

Proof. $(\Rightarrow)$ Assume that N is a zero-symmetric. Then $\{0\}$ is an LA-subsemigroup of N, since 0 is the zero of N. Thus $\{0\}$ is an ideal of N.

$(\Leftarrow)$ Assume that $\{0\}$ is an ideal of N. By Definition 2.5 then N is a zero-symmetric. □

3. Simple of Near Left Almost Rings

Next we will defined of simple in nLA-ring and study properties of simple in nLA-ring.

Definition 3.1. An nLA-ring N is called *left (right) simple* if left (right) ideal of N it self. Furthermore N is called a *simple* if the only ideal of N is self.

Theorem 3.1. Let N be an nLA-ring. If N is left (right) zero then N is a left (right) simple.

Proof. Let N be a left (right) zero, A is left ideal of N and $x \in N$. Then $x = xa \in A$. Thus $N \subseteq A$ Since A is a left ideal of N. Then $A \subseteq N$. Thus $N = A$. Hence N is a left simple. Similarly we can show that N is a right simple. □

Theorem 3.2. Let N be an nLA-ring. Then the following statement hold.

(1) If $Nx = N$ for all $x \in N$, then N a left simple.
(2) $xN = N$ for all $x \in N$ if and only if N is a right simple.
(3) If $(Nx)N = N$ for all $x \in N$ then N is a simple.

Proof. (1) Assume that $Nx = N$ for all $x \in N$. Let L be a left ideal of N and $x \in L$. Then $L \subseteq N$. Consider

$$N = Nx \subseteq NL \subseteq L.$$

Then $N \subseteq L$ .Thus $N = L$. Hence N a left simple.

(2) $(\Rightarrow)$ Assume that $xN = N$ for all $x \in N$. Let A be a right ideal and $x \in A$ Then $A \subseteq N$. Consider

$$N = xN \subseteq AN \subseteq A.$$

Then $N \subseteq A$ .Thus $N = A$. Hence N a right simple.

$(\Leftarrow)$ Assume that $N$is a right simple. For each $x \in N$ Then xN is a right of N. Thus $Nx = N$

(3) Assume that $(Nx)N = N$ for all $x \in N$. Let I be an ideal of N and $x \in I$ Then $I \subseteq N$. Consider

$$N = (Nx)N \subseteq IN \subseteq I.$$

Then $N \subseteq I$. Thus $N = I$. Hence N is a simple.

□

ACKNOWLEDGEMENTS

The author is very grateful to the anonymous referee for stimulating comments and improving presentation of the paper.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] M.A. Kazim, M.D. Naseeruddin, On almost semigroup, Port. Math. 36 (1977), 41-47.

[2] M. Khan, Faisal, V. Amjid, On some classes of Abel-Grassmann's groupoids, (2010). http://arxiv.org/abs/1010.5965.

[3] Q. Mushtaq, M. Khan, M-System in LA-semigroups, Southeast Asian Bull. Math. 33 (2009), 321-327.

[4] G. Pilz, Near-rings, North Holland, Amsterdam-New York, 1997.

[5] M. Sarwar (Kamran), Conditions for LA-semigroup to resemble associative structures, PhD thesis, Quaid-i-Azam University, 1993.

[6] P. Sajee, Simple Γ-seminearrings, J. Math. Res. 1 (2009), 124-129.

[7] T. Shah, F. Rehman, M. Raees, On near left alomst rings, Int. Math. Forum, 6 (2011), 1103-1111.

[8] T. Shah, I. Rehman, On LA-rings of finitely nonzero function, Int. J. Contemp. Math. Sci. 5 (2010), 209-222.