Available online at http://scik.org
J. Math. Comput. Sci. 3 (2013), No. 1, 233-241
ISSN: 1927-5307

ON FUZZY DET - NORM MATRIX

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Abstract: In this paper we introduce fuzzy det-norm matrices using the structure of $M_n(F)$, the set of (n × n) fuzzy det-norm matrices is introduced. From this row and column,determinant of the fuzzy norm has been obtained by imposing an equivalence relation on $M_n(F)$. Also, we introduce the concept of fuzzy det-norm matrices, metricand equivalence fuzzy det-matrices.

Keywords: Fuzzy matrix, Fuzzy m-norm matrix, determinant of a square fuzzy matrix

2000 AMS Subject Classification: 03E72, 15B15

1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965. Nagoorgani A. and Kalyani G. [4] introduced the properties of fuzzy m-norm matrices. In 1995,Ragab.M. Z. and Emam E. G.[1] introduced the determinant and adjoint of a square fuzzy matrix. Nagoorgani A. and Kalyani G.[3] introduced the definition of fuzzy equivalence relation. Meenakshi A.R. and Cokilavany R. [2] introduced the concept of fuzzy 2-normed linear spaces.

In this paper, we introduce the concept of fuzzy det-norm matrices. The purpose of the introduction is to explaindet-norm and its properties for fuzzy matrices. In section 2, fuzzy det-norm is introduced in $M_n(F)$. In section 3, fuzzy norm equivalence matrix is discussed.

2. Preliminaries

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Received December 11, 2012

We consider F=[0,1] the fuzzy algebra with operation [+,·] and the standard order "≤" where a+b = max{a,b}, a·b = min{a,b} for all a,b in F.F is a commutative semi-ring with additive and multiplicative identities 0 and 1 respectively. Let M_{MN}(F) denote the set of all m×n fuzzy matrices over F. In short M_n(F) is the set of all fuzzy matrices of order n. define '+' and scalar multiplication in M_n(F) as $\mathrm{A} + \mathrm{B} = [a_{ij} + b_{ij}]$, where A = $[a_{ij}]$ and B = $[b_{ij}]$ and cA = $[ca_{ij}]$, where c is in [0,1], with these operations M_n(F) forms a linear space.

3. Fuzzy Matrices And Metric

Definition 2.1. An m×n matrix $A = [a_{ij}]$ whose components are in the unit interval [0,1] is called a fuzzy matrix.

Definition 2.2. The determinant $|A|$ of an n ×n fuzzy matrix A is defined as follows;

$$|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)}$$

Where S_n denotes the symmetric group of all permutations of the indices $(1,2,\cdots,n)$

Definition 2.3. Let M_n(F) be the set of all (n ×n) fuzzy matrices over F = [0,1], For every A in M_n(F), Define norm of A denoted by $\|A\|$ as

$\|A\| = det[A]$, where $A = [a_{ij}]$

Theorem 2.1. *If M_n(F) is the set of all (n×n) fuzzy matrices over F = [0,1] then for all fuzzy matrices A and B in $M_n(F)$ and any scalar α in [0,1], we have*

(i) $\|A\| = det[A] \geq 0$ *and* $\|A\| = 0$ *if and only if A=0*

(ii) $\|\alpha A\| = \alpha\, det[A]$ *for any α in [0,1]*

(iii) $\|A + B\| = det[A] + det[B]$ *for A,B in* $M_n(F)$

(iv) $\|AB\| = det[A] det[B]$ *for A,B in* $M_n(F)$

Proof.

Let $A = [a_{ij}]$ and $B = [b_{ij}]$be two fuzzy matrices.

First we prove

(i) If $\|A\|$ is a fuzzy matrix in M_n(F). Since all $a_{ij} \in [0,1]$,

$det[A] = \|A\| \geq 0$, for all A in M_n(F).

If $\|A\| = 0$then $det[A] = 0$, $a_{ij} = 0$ for all i and j, A=0.

Conversely, if A= 0 then $det[A] = 0, \|A\| = 0$

Therefore $\|A\|_m = 0$ if and only if A=0

(ii) If α in [0,1] then $\alpha A = [\alpha A]$,

$$\|\alpha A\| = det[\alpha A]$$

$$= \alpha det[A]$$

$$\|\alpha A\| = \alpha\|A\|$$

(iii) Let $\|A\| = det[A]$and$\|B\| = det[B]$

Now $\|A + B\| = \det[C]$, Where $c_{ij} = [a_{ij}] + [b_{ij}]$

$\|A + B\| = \det[[A] + [B]]$

$\|A + B\| = \det[A] + \det[B]$

$\|A + B\| = \|A\| + \|B\|$

(iv) Let $\|A\| = \det[A]$and$\|B\| = \det[B]$

If AB=D, then the entries of D are given by$d_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$

$$d_{ij} = \sum_{k=1}^{n}\{\min(a_{ik}b_{kj})\}$$

$$d_{ij} = \min(a_{i1}b_{1j}) + \min(a_{i1}b_{2j}) \cdots \min(a_{in}b_{in}) \qquad (2.1)$$

Case(1) If all $a_{ij} \le b_{ij}$ for j=1,2,…,n. Then we have $d_{ij} = a_{i1} + a_{i2} + \cdots + a_{in}$ (from (2.1))

$$d_{ij} = a_{i1} + a_{i2} + \cdots + a_{in}$$

$$d_{ij} = a_{ij}$$

$$\det[D] = \det[A]$$

$$\|AB\| = \|A\| = \|A\|\|B\|$$

Case(2) If all $b_{ij} \le a_{ij}$ for j=1,2,…,n. Then we have$d_{ij} = b_{i1} + b_{i2} + \cdots + b_{in}$ (from (2.1))

$$d_{ij} = b_{ij}$$

$$\det[D] = \det[B]$$

$$\|AB\| = \|B\| = \|A\|\|B\|$$

Case(3) Let some $a_{ij} \le b_{ij}$ and some other $b_{ij} \le a_{ij}$. Let us assume that $a_{im} < b_{im}$ for all n<m and$b_{im} < a_{im}$for all n ≥ m.

From(2.1),$d_{ij} = a_{ij} + \cdots + a_{im} + b_{i(m+1)} + \cdots + b_{ij}$

$$d_{ij} = \sum_{j=1}^{m} a_{ij} + \sum_{j=m+1}^{n} b_{ij} = a_{ij} + b_{ij}$$

$$d_{ij} = a_{ij} \text{if} a_{ij} \ge b_{ij}$$

$$d_{ij} = b_{ij} \text{if} a_{ij} \le b_{ij}$$

$$\det[D] = \det[A] = \|A\|$$

or $\det[D] = \det[B] = \|B\|$

$\|AB\| = \|A\|\|B\| = \det[A]\det[B]$

Example.

If $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$ and $B = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.4 \end{bmatrix}$

$$\|A\| = 0.8\begin{bmatrix} 0.9 & 0.6 \\ 0.7 & 0.7 \end{bmatrix} + 0.3\begin{bmatrix} 0.6 & 0.6 \\ 0.1 & 0.7 \end{bmatrix} + 0.2\begin{bmatrix} 0.6 & 0.9 \\ 0.1 & 0.7 \end{bmatrix}$$

$$= 0.8[0.7 \ + \quad 0.6] + 0.3[0.6 \ + \quad 0.1] + 0.2[0.6 \ + \quad 0.1]$$

= 0.7+0.3+0.2

$\|A\| = 0.7$

$$\|B\| = 0.6\begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.4 \end{bmatrix} + 0.2\begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.4 \end{bmatrix} + 0.1\begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$$

$$\|B\| = 0.6[0.3 + \quad 0.7] + 0.2[0.4 + \quad 0.6] + 0.1[0.4 + \quad 0.3]$$

= 0.6(0.7)+0.2(0.6)+0.1(0.4)

= 0.6+0.2+0.1

$\|B\| = 0.6$

$$\|A + B\| = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.7 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$$

= 0.8[0.7+0.7]+0.3[0.6+0.6]+0.2[0.6+0.6]

= 0.8(0.7)+0.3(0.6)+0.2(0.6)

= 0.7+0.3+0.2

$\|A + B\| = 0.7$

$\|A + B\| = \|A\| + \|B\|$

$\|A + B\| = det|A| + det|B| = 0.7 + 0.6 = 0.7$

Set α=0.5

$$\alpha A = 0.5\begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$$

$$\alpha A = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.5 & 0.5 \\ 0.1 & 0.5 & 0.5 \end{bmatrix}$$

$\|\alpha A\|$=0.5(0.5)+0.3(0.5)+0.2(0.5)

=0.5+0.3+0.2

$\|\alpha A\| = 0.5$

$\alpha\|A\| = (0.5)(0.7) = 0.5$

$\|\alpha A\| = \alpha\|A\| = 0.5$

$$\|AB\| = \begin{bmatrix} 0.6 & 0.3 & 0.3 \\ 0.6 & 0.6 & 0.7 \\ 0.6 & 0.7 & 0.7 \end{bmatrix}$$

= 0.6(0.6+0.7)+0.3(0.6+0.6)+0.3(0.6+0.6)

= 0.6+0.3+0.3

$\|AB\| = 0.6$

$\|AB\| = \|A\|\|B\| = 0.6$

4. Equivalence Fuzzy Matrices

Definition 3.1. A fuzzy matrix A is defined to be greater than B if $\|B\| \leq \|A\|$, A is strictly greater than B if $\|B\| < \|A\|$. We also say that B is smaller than A.

Example:

Let $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$ and $B = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.4 \end{bmatrix}$

$\|A\| = 0.7$ and $\|B\| = 0.6$

$\|B\| < \|A\| = 0.6 < 0.7$

Therefore, A is strictly greater than B.

Definition 3.2. Define a mapping d:$M_n(F) \times M_n(F) \to [0,1]$ as

$\mathrm{d}(\mathrm{A},\mathrm{B}) = \|A+B\| = \det[A,B]$ for all A,B in $M_n(F)$.

Theorem 3.1. *The above mapping d satisfies the following conditions for all A,B,C in* $M_n(F)$

(i) $d(A,B) \geq 0$ *and* $d(A,B) = 0$ *then* $A = B$

(ii) $d(A,B) = d(B,A)$

(iii) $d(A,B) \leq d(A,C) + d(B,C)$ *for all A,B,C in* $M_n(F)$

Then d is a pseudo-metric in $M_n(F)$

Proof.

(i) $\mathrm{d}(\mathrm{A},\mathrm{B}) = \|A+B\| = \det[A,B] \geq 0$for all A,B in $M_n(F)$

Therefore $\mathrm{d}(\mathrm{A},\mathrm{B}) \geq 0$

Suppose $\mathrm{d}(\mathrm{A},\mathrm{B}) = 0$ then $\|A+B\| = \det[A,B] = 0$

$\Longrightarrow \|A\| + \|B\| = \det[A] + \det[B] = 0$

$\Longrightarrow A = 0 \text{and} B = 0$

$\Longrightarrow A = B$

But$A = B$ imples $\|A\| = \|B\|$

$\Longrightarrow \|A+B\| = \|B\| + \|B\| = \det[B] + \det[B]$

$\Longrightarrow \|A+B\| = \|B\| = \det[B]$

$\Longrightarrow \mathrm{d}(\mathrm{A},\mathrm{B}) \neq 0$

Therefore, A=B need not implies $\det[A,B] = 0$

(ii) $\mathrm{d}(\mathrm{A},\mathrm{B}) = \|A+B\| = \|B+A\| = d(B,A)$

det[A,B] = det[B,A]

d(A,B) = d(B,A)

(iii) Let A,B,Cin $M_n(F)$ be such that $\|C\| \geq \|B\| \geq \|A\|$

$\mathrm{d}(\mathrm{A},\mathrm{B}) = \|A+B\|$

$= \det[A] + \det[B]$

$= \det[B] + \det[B]$

$= \det[B] = \|B\|$

$\mathrm{d}(\mathrm{A},\mathrm{C}) = \|A+C\|$

$= \det[A] + \det[C]$

$= \det[C] + \det[C]$

$= \det[C] = \|C\|$

$\mathrm{d}(\mathrm{B},\mathrm{C}) = \|B+C\|$

$= \det[B] + \det[C]$

$= \det[C] = \|C\|$

$\mathrm{d}(\mathrm{A},\mathrm{C}) + \mathrm{d}(\mathrm{B},\mathrm{C}) = \|C\| + \|C\| = \|C\|$

Therefore $\mathrm{d}(\mathrm{A},\mathrm{B}) \leq \mathrm{d}(\mathrm{A},\mathrm{C}) + \mathrm{d}(\mathrm{B},\mathrm{C})$

For the other cases also we have$\mathrm{d}(\mathrm{A},\mathrm{B}) \leq \mathrm{d}(\mathrm{A},\mathrm{C}) + \mathrm{d}(\mathrm{B},\mathrm{C})$. Thus in all cases $\mathrm{d}(\mathrm{A},\mathrm{B}) \leq \mathrm{d}(\mathrm{B},\mathrm{C}) + \mathrm{d}(\mathrm{C},\mathrm{A})$for all A,B,C in $M_n(F)$. Thus from (i), (ii) and (iii) we see that d is a pseudo-metric on $M_n(F)$.

Example 3.1.

If $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$,$B = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.4 \end{bmatrix}$ and $C = \begin{bmatrix} 0.8 & 0.4 & 0.6 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0.6 & 0.8 \end{bmatrix}$

(i) $\|A + B\| = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.7 \\ 0.1 & 0.7 & 0.7 \end{bmatrix} = 0.7,$

$\|A\| = 0.7 \text{and} \|B\| = 0.6$

$\|A + B\| = \|A\| + \|B\|$

$\|A + B\| = \|B\| + \|B\| = 0.6 + 0.6 = 0.6$

$\|A + B\| = \|B\| = 0.6$

(ii) $\|B + A\| = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.7 \\ 0.6 & 0.7 & 0.7 \end{bmatrix} = 0.7 \text{and} \|A + B\| = 0.7$

$\mathrm{d(A, B)} = \|A + B\| = \|B + A\| = 0.7$

(iii) $\Longrightarrow \|A + B\| = \|B\| = \det[B] = 0.6$

$\|A + C\| = \begin{bmatrix} 0.8 & 0.4 & 0.6 \\ 0.6 & 0.9 & 0.7 \\ 0.6 & 0.7 & 0.8 \end{bmatrix}$

$= 0.8[0.8 + 0.6] + 0.4[0.6 + 0.1] + 0.6[0.6 + 0.1]$

$= 0.8(0.8) + 0.4(0.6) + 0.6(0.6)$

$= 0.8 + 0.4 + 0.6$

$\|A + C\| = 0.8$

$\|B + C\| = \begin{bmatrix} 0.8 & 0.4 & 0.6 \\ 0.4 & 0.9 & 0.7 \\ 0.6 & 0.7 & 0.8 \end{bmatrix}$

$= 0.8[0.8 + 0.7] + 0.4[0.4 + 0.6] + 0.6[0.4 + 0.6]$

$= 0.8(0.8) + 0.4(0.6) + 0.6(0.6)$

$= 0.8 + 0.4 + 0.6$

$\|B + C\| = 0.8$

$\|B\| = 0.6 \text{and} \|C\| = 0.8$

$\Longrightarrow \|B + C\| = \|C\| = \det[C] = 0.8$

$\|A + C\| = 0.8, \|A\| = 0.7$ and $\|C\| = 0.8$

$\Longrightarrow \|A + C\| = \|C\| = \det[C] = 0.8$

$\|A + B\| \leq \|A + C\| + \|B + C\| = 0.7 \leq 0.8 + 0.8 = 0.7 \leq 0.8$

Therefore $\mathrm{d(A, B)} \leq \mathrm{d(A, C)} + \mathrm{d(B, C)}$

Theorem 3.2. *If* A, A', B, B' *in* $M_n(F)$. *Then* $d(A,B) + d(A',B') = d(A,A') + d(B,B')$

Proof.

$$\mathrm{d(A,B) + d(A',B') = det[A + B] + det[A' + B']}$$
$$= \mathrm{det}[A] + \mathrm{det}[B] + \mathrm{det[A'] + det[B']}$$
$$= \mathrm{det[A + A'] + det[B + B']}$$
$$= \|A + \mathrm{A'}\| + \|B + \mathrm{B'}\|$$
$$\mathrm{d(A,B) + d(A',B') = d(A,A') + d(B,B')}$$

Example 3.2.

If $A = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{bmatrix}$,$B = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.4 \end{bmatrix}$ and

$A' = \begin{bmatrix} 0.8 & 0.6 & 0.1 \\ 0.3 & 0.9 & 0.7 \\ 0.2 & 0.6 & 0.7 \end{bmatrix}$,$B' = \begin{bmatrix} 0.6 & 0.4 & 0.6 \\ 0.2 & 0.3 & 0.7 \\ 0.1 & 0.7 & 0.4 \end{bmatrix}$

$\|A\| = 0.7$and$\|B\| = 0.6$

$$\|A'\| = 0.8\begin{bmatrix} 0.9 & 0.7 \\ 0.6 & 0.7 \end{bmatrix} + 0.6\begin{bmatrix} 0.3 & 0.7 \\ 0.2 & 0.7 \end{bmatrix} + 0.1\begin{bmatrix} 0.3 & 0.9 \\ 0.2 & 0.6 \end{bmatrix}$$
$$= 0.8[0.7 \ + \quad 0.6] + 0.6[0.3 \ + \quad 0.2] + 0.1[0.3 \ + \quad 0.2]$$
= 0.7+0.3+0.1

$$\|A'\| = \ 0.7$$

$$\|B'\| = 0.6\begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.4 \end{bmatrix} + 0.4\begin{bmatrix} 0.2 & 0.7 \\ 0.1 & 0.4 \end{bmatrix} + 0.6\begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.4 \end{bmatrix}$$
$$\|B'\| = 0.6[0.3 + \quad 0.7] + 0.4[0.2 + \quad 0.1] + 0.6[0.3 + \quad 0.7]$$
= 0.6(0.7)+0.4(0.2)+0.6(0.7)

= 0.6+0.2+0.6

$$\|B'\| = \ 0.6$$

$$\|A' + B'\| = \begin{bmatrix} 0.8 & 0.6 & 0.6 \\ 0.3 & 0.9 & 0.7 \\ 0.2 & 0.7 & 0.7 \end{bmatrix}$$
= 0.8[0.7+0.7]+0.6[0.3+0.2]+0.6[0.3+0.2]

= 0.8(0.7)+0.6(0.3)+0.6(0.3)

= 0.7+0.3+0.3

$$\|A' + B'\| = \ 0.7$$

$$\|A + A'\| = \begin{bmatrix} 0.8 & 0.6 & 0.2 \\ 0.6 & 0.9 & 0.7 \\ 0.2 & 0.7 & 0.7 \end{bmatrix}$$

= 0.8[0.7+0.7]+0.6[0.6+0.2]+0.2[0.6+0.2]

= 0.8(0.7)+0.6(0.6)+0.2(0.6)

= 0.7+0.6+0.2

$\|A + A'\| = 0.7$

$$\|B + B'\| = \begin{bmatrix} 0.6 & 0.4 & 0.6 \\ 0.4 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.4 \end{bmatrix}$$

= 0.6[0.3+0.7]+0.4[0.4+0.6]+0.6[0.4+0.3]

= 0.6(0.7)+0.4(0.6)+0.6(0.4)

= 0.6+0.4+0.4

$\|B + B'\| = 0.6$

$$\|A + A'\| + \|B + B'\| = 0.7 + 0.6 = 0.7$$

$d(A, A') + d(B, B') = \|A + A'\| + \|B + B'\|$

Conclusion

In this paper, a new definition det-norm on fuzzy matrix and its properties are discussed. Numerical examples are given to clarify the developed theory and the proposed det-norm.

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