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INTEGRATED PROFIT ORIENTED SUPPLY CHAIN OF PERFECT QUALITY ITEMS WITH MULTIPLE IMPRECISE GOALS IN AN UNCERTAIN ENVIRONMENT

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Abstract: The major issues of supply chain network are the product quality, cost and delivery time. In this paper, we investigate the effects of product quality and transportation flow in uncertain environment. A tradeoff between raw material quality, inspection cost, purchasing cost and reprocessing costs is considered. The total delivery time in transporting goods at all levels is minimized. A supply chain with multiple suppliers, manufacturers, distribution centers is taken and a possibilistic mixed integer linear programming with multiple imprecise goals is applied to accomplish the above mentioned tradeoffs. The transportation costs, delivery time, selling price are considered in uncertain environment and are taken as triangular numbers. At each echelon, the decision maker specifies the most possible value in the possibility distribution of each imprecise data as a precise number.

Keywords: Supply chain, multiobjective mixed integer linear programming, triangular number, profit maximization, inspection cost, delivery time, LINDO software.

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1. Introduction

The major issues of supply chain network are linked to procurement, production, distribution, transportation and supplier's selection etc., Integration of manufacturing and distribution becomes essential for a successful supply chain. Supplier's selection criteria includes factors related to organizational infrastructure, quality, cost and delivery time besides technology and manufacturing capability (Shin et al., 2009). Both quality of raw material and finished product affect the agreements with suppliers and retailers, since bad quality raw materials lead to imperfect quality finished products which require reworking. Instead a contract is made considering product quality options which supplier present, as quality is an important factor in the value adding process involved in the production (Bulgak et al., 2008) To decrease the total cost, the decision maker has to choose a supplier who supplies good quality items, if not, improves by reprocessing the raw material (Turan Packsoy et al., 2012). Reworking process adds cost and not value to the product in manufacturers (EI Saadany and Jaber 2008).

The production of defect-free components and parts that meet the requirements of customers along the supply chain is critical for the quality of the final products. Sustaining quality efforts throughout the chain also has significant implications for reducing costs (Bulgak et al., 2008). Turan Packsoy et al., 2012 considered raw materials to be of high, low or bad quality used option contracts to hedge against the loss.

In this paper, we consider a decision maker at each echelon. In addition after the production at manufacturer's level, all items are subject to inspection which ensures good and perfect quality products which finally reach the retailers and hence consumers. We have also taken into consideration labour hours and maximum machine hours at manufacturer's level. Possibilistic mixed integer programming problem is formulated and solved using LINGO Software.

2. Network Structure and Basic Features

The model represents four echelons, multi-suppliers, multi-manufacturers, multi-distribution centres, multi-retailers and multi-quality raw material options problem. Manufacturers deals with transportation functions by which items will be delivered on time from suppliers with an engagement contract. Products which are produced with multi-quality optional raw materials supplied from suppliers are moved to distribution centre and retailers from manufacturers respectively. Here the decision maker faces a tradeoff between low-mid-high quality raw materials, their purchasing costs and reprocessing costs according to quality options in manufacturers and try to minimize the delivery time. Such a supply chain network problem can include all transportation costs, between all echelons, raw material cost, reprocessing cost and total supplier engagement fees which would be less than the total budget.

2.1. Assumptions

1. The supplier selection is binary.
2. The demand of each retailer must be satisfied.
3. The flow is only allowed to be transferred between two consecutive echelons.
4. The capacities of facilities are limited.
5. The reprocessing rates of each quality level and estimated selling amounts of each retailer are given.
6. To supply perfect quality items, each item is subject to inspection which incurs inspection cost.
7. Labour time required and maximum machine time available for q^{th} quality product at each manufacturer is given.

2.2. Indices

$i \in I$	Set of potential suppliers.
$j \in J$	Set of manufacturers.
$k \in K$	Set of distribution centres.
$l \in L$	Set of retailers.
$q \in Q$	Set of quality options.

2.3. Variables

X_{ijk} -Quantity of q^{th} raw material shipped from supplier i to manufacturer j.

Y_{jk} -Quantity of product shipped from manufacturer j to distribution centre kK.

Z_{kl} -Quantity of quality product distributed from distribution centre k to retailer l.

S_l -Estimated quantity product sold from retailer l.

P_{iq} -Purchased quantity of qth quality raw material from supplier i.

RP_{jq} -Quantity of q^{th} quality product reprocessed in manufacturer j.

Δ_i -If an agreement is signed mutually, 1 ; otherwise, 0.

Y_{jq} -Quantity of q^{th} quality raw material inspected at manufacturer j.

2.4. Parameters

δ_i -Agreement contract fee of supplier i.

C_{aiq} -Capacity of supplier i to supply q^{th} quality raw material for manufacturers.

C_{aj} -Capacity of j^{th} manufacturer.

C_{ak} -Distributor Capacity of DC k.

E_{lnax},E_{lmin} -Estimated maximum and minimum demand at retailer l.

B -Maximum allowable money budget for dealing with suppliers.

$\tilde{P}_{rl}$ -Unit selling price at retailer l.

$\tilde{C}_{ijq}$ -Unit transportation cost of q^{th} quality raw material between supplier i and manufacturer j.

$\tilde{C}_{jk}$ -Unit transportation cost between manufacturer j and DC k.

$\tilde{T}_{jk}$ -Unit delivery time between manufacturer j and DC k.

$\tilde{T}_{kl}$ -Unit delivery time between DC k to retailer l.

$\tilde{T}_{ijq}$ -Unit delivery time between supplier i to manufacturer j.

$\tilde{C}_{kl}$ -Unit transportation cost between DC k and retailer l.

$\tilde{G}_i$ -Contract fees of supplier i.

$\tilde{C}_{iq}$ -Purchased price of q^{th} quality raw material from supplier i.

$\tilde{C}_{jq}$ -Reprocessing cost of q^{th} quality product at manufacturer j.

λ_q -Percent value of total q^{th} quality raw material which needs reprocessing.

M_q -Machine hours for q^{th} quality product.

M_{jq} -Labour hours to process q^{th} quality product by j^{th} manufacturer.

$\tilde{I}_{jq}$ -Unit Inspection cost of q^{th} quality raw material at manufacturer j.

3. Mathematical Formulation

Because of the supplier selection's binary situation, the model can be defined a mixed integer and also linear programming model. The mixed integer linear programming model is formulated by including aforementioned indices, variables, parameters, objective function and constraints as follows.

Objective Function

Maximise

$$\sum_{l} S_l \tilde{P}_{rl} - \sum_{i}\sum_{j}\sum_{q} X_{ijq} \cdot \tilde{C}_{ijkq} - \sum_{j}\sum_{q} P_{iq} \cdot \tilde{C}_{iq} - \sum_{j}\sum_{q} RP_{jq}\tilde{C}_{jq} - \sum_{j}\sum_{q} Y_{jk}\tilde{C}_{jk}$$

$$- \sum\sum Z_{kl}\tilde{C}_{kl} - \sum_{j}\sum_{q} Y_{JK} I_{jk} - \sum_{i} \delta_i \Delta_i \qquad \ldots (1)$$

Constraints

$$\sum_{j} X_{ijq} \leq C_{aiq} \cdot \Delta_i \qquad \forall_{i,q} \qquad \ldots (2)$$

$$\sum_{k} Y_{jk} \leq C_{aj} \qquad \forall_{j} \qquad \ldots (3)$$

$$\sum_{l} Z_{kl} \leq C_{ak} \qquad \forall_{k} \qquad \ldots (4)$$

$$E_{lmin} \leq S_i \leq E_{lmax} \qquad \forall_{l} \qquad \ldots (5)$$

$$\lambda_q \cdot \sum_{k} X_{ijk} = RP_{jq} \ \forall_{j,q} \qquad \ldots (6)$$

$$\sum_{j} X_{ijq} = P_{iq} \qquad \forall_{i,q} \qquad \ldots (7)$$

$$\sum_{i}\sum_{q} X_{ijq} = \sum_{k} Y_{jk} \qquad \forall_{i,} \qquad \ldots (8)$$

$$\sum_{j} Y_{jk} = \sum_{l} Z_{kl} \qquad \forall_{k} \qquad \ldots (9)$$

$$\sum Z_{kl} = S_{i} \qquad \forall_{i} \qquad \ldots (10)$$

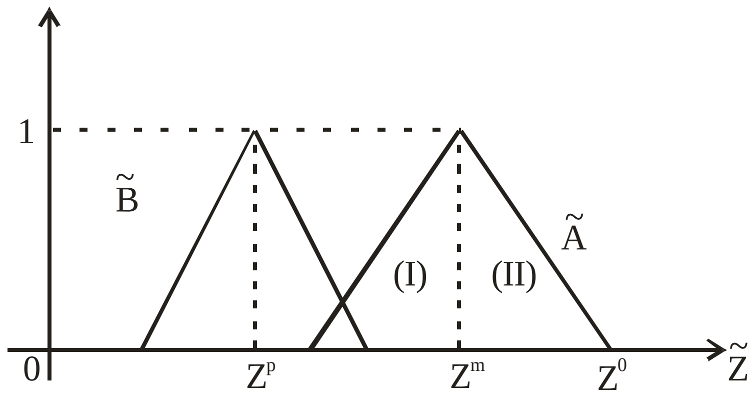
$$\sum_{j}\sum_{q} M_{jq} Y_{jq} \le M_{q} \qquad \ldots (11)$$

$$\sum_{i} \delta_{i} \Delta_{i} \le B \qquad \ldots (12)$$

$$\Delta_{i} \in (0, 1) \qquad \forall_{i} \qquad \ldots (13)$$

$$X_{ijq}, Y_{jk}, Z_{kl}, P_{iq}, RP_{jq} \ge 0 \quad \forall_{i,j,k,l,q} \qquad \ldots (14)$$

4. MODEL THE IMPRECISE DATA



The possibility distribution can be stated as the degree of occurrence of an event with imprecise data. Here DM adopted triangular possibility distribution for all imprecise numbers as they are flexible for fuzzy arithmetic operations. For example C_{ij} is based on three prominent data as follows.

(i) The most pessimistic value (C_{ij}^{p}) that has a very low likelihood of belonging to the set of available values (possibility degree = 0 if normalized).

(ii) The most possible value (C_{ij}^{m}) that definitely belongs to the set of available values (possibility degree = 1 if normalized).

(iii) The most optimistic value (C_{ij}^{0}) that has a very low likelihood of belonging to the set of available values (possibility degree = 0 if normalized)

The strategy involves simultaneously minimizes the most possible goal of the imprecise objective function Z_1^m, maximizes the possibility of obtaining lower goal value (region I in Fig.1), $\left(Z_1^m - Z_1^p\right)$ and minimizing the risk of obtaining higher goal value (region II in Fig.1), $\left(Z_1^0 - Z_1^m\right)$. The last two goals are relative measures from Z_1^m, the most possible value of the imprecise total net cost. In this paper, at each echelon we have transportation cost, delivery time to be minimized, reprocessing cost, inspection cost to be minimized at manufacturer's level. For example in the case of supplier, if $\tilde{Z}_1$ is the total transportation cost incurred while transporting products to manufacturers then the new objective function is given by

$$\text{Min } Z_{11} = Z_1^m = \sum\sum C_{ijq}^m X_{ijq} \qquad \ldots (15)$$

$$\text{Max } Z_{12} = \left(Z_1^m - Z_1^p\right) = \sum\sum\left(C_{ijq}^m - C_{ijq}^p\right) X_{ijq} \qquad \ldots (16)$$

$$\text{Min } Z_{13} = \left(Z_1^0 - Z_1^m\right) = \sum\sum\left(C_{ijq}^0 - C_{ijq}^m\right) X_{ijq} \qquad \ldots (17)$$

Similarly equations (17 – 19) list this result for the new objective function of total delivery time

$$\text{Min } Z_{21} = Z_2^m = \sum_i\sum_j T_{ij}^m X_{ijq} \qquad \ldots (18)$$

$$\text{Max } Z_{22} = \left(Z_2^m - Z_2^p\right) = \sum_i\sum_j\left(T_{ij}^m - T_{ij}^p\right) X_{ijq} \qquad \ldots (19)$$

$$\text{Min } Z_{23} = \left(Z_2^0 - Z_2^m\right) = \sum_i\sum_j\left(T_{ij}^0 - T_{ij}^m\right) X_{ijq} \qquad \ldots (20)$$

4.1. Solving the auxiliary MOLP problem

The auxiliary MOLP problem can be converted into equivalent single goal LP problem using Linear Membership Function of Zimmerman (1976, 78) to represent imprecise goal of DM together with fuzzy decision making concept of Bellman and Zadeh (1970), first specifies the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of the objective functions (13) to (21) of the auxiliary MOLP problem as follows

$$Z_{g_1}^{PIS} = \text{Min } Z_g^m \qquad Z_{g_1}^{NIS} = \text{Max } Z_g^m \quad g = 1, 2, 3 \qquad \ldots (21)$$

$$Z_{g_2}^{PIS} = \text{Max }\left(Z_g^m - Z_g^p\right) \qquad Z_{g_2}^{NIS} = \text{Min}\left(Z_g^m - Z_g^p\right) \quad g = 1, 2, 3 \qquad \ldots (22)$$

$$Z_{g_3}^{PIS} = \text{Min }\left(Z_g^0 - Z_g^m\right) \qquad Z_{g_3}^{NIS} = \text{Max}\left(Z_g^0 - Z_g^m\right) \quad g = 1, 2, 3 \qquad \ldots (23)$$

Furthermore the corresponding linear membership functions for each objective function is defined by

$$fg_1(Zg_1) = \begin{cases} 1 & \text{if } Z_{g_1} < Z_{g_1}^{PIS} \\ \dfrac{Z_{g_1}^{NIS} - Z_{g_1}}{Z_{g_1}^{NIS} - Z_{g_1}^{PIS}} & \text{if } Z_{g_1}^{PIS} \le Z_{g_1} \le Z_{g_1}^{NIS} \\ 0 & \text{if } Z_{g_1} > Z_{g_1}^{PIS} \qquad g = 1, 2, 3 \end{cases} \qquad \ldots (24)$$

$$fg_2(Zg_2) = \begin{cases} 1 & \text{if } Z_{g_2} > Z_{g_2}^{PIS} \\ \dfrac{Z_{g_2} - Z_{g_2}^{NIS}}{Z_{g_2}^{PIS} - Z_{g_2}^{NIS}} & \text{if } Z_{g_2}^{NIS} \le Z_{g_2} \le Z_{g_2}^{PIS} \\ 0 & \text{if } Z_{g_2} < Z_{g_2}^{NIS} \qquad g = 1, 2, 3 \end{cases} \qquad \ldots (25)$$

$$fg_3(Zg_3) = \begin{cases} 1 & \text{if } Z_{g_3} < Z_{g_3}^{PIS} \\ \dfrac{Z_{g_3}^{NIS} - Z_{g_3}}{Z_{g_3}^{NIS} - Z_{g_3}^{PIS}} & \text{if } Z_{g_3}^{PIS} \le Z_{g_3} \le Z_{g_3}^{NIS} \\ 0 & \text{if } Z_{g_3} > Z_{g_3}^{NIS} \qquad g = 1, 2, 3 \end{cases} \qquad \ldots (26)$$

The maximum operator of the fuzzy decision making concept of Bellman and Zadeh (1970) is used to aggregate all fuzzy sets. Introducing the auxiliary variable L enables the auxiliary MOLP into single goal LP, which can be solved efficiently using standard simplex method. Consequently the complete ordinary LP model for solving the MDPD problems with multiple imprecise goals can be formulated as follows.

Maximize L

subject to $L \le \dfrac{Z_{g_1}^{NIS} - Z_{g_1}}{Z_{g_1}^{NIS} - Z_{g_1}^{PIS}} \quad g = 1, 2, 3$

$$L \le \frac{Z_{g_2} - Z_{g_2}^{NIS}}{Z_{g_2}^{PIS} - Z_{g_2}^{NIS}} \quad g = 1, 2, 3$$

$$L \le \frac{Z_{g_3}^{NIS} - Z_{g_3}}{Z_{g_3}^{NIS} - Z_{g_3}^{PIS}} \quad g = 1, 2, 3$$

Equations (2) to (14)

$Q_{ij} \geq 0, \forall i, \forall j$

where L value ($0 \leq L \leq 1$) represents the overall DM satisfaction with the determined goal values.

4.2. Solution Procedure Algorithm

Step 1: Formulate the original imprecise multiobjective PLP model according to equations 1 to 13.

Step 2: Model the imprecise coefficients and right hand side values using the triangular possibility distributions.

Step 3: Develop the new objective functions of auxiliary MOLP problem for each of the imprecise objective functions for suppliers, distributors, manufacturers and retailers (Given in Appendix).

Step 4: Specify the corresponding linear membership functions for each of the new objective functions in the auxiliary MOLP problem and then aggregates the auxiliary MOLP problem into an equivalent ordinary single goal LP model by the minimum operator.

Step 6: Solve and modify the model interactively.

5. NUMERICAL EXAMPLE

In this section we present a numerical example for a logical data to illustrate the proposed model. The supply chain network contains fine supplies, three distribution centres and four retailers for selling. The network is structured to supply raw materials and transport products from suppliers to end users is constituted from multi echelon, multi quality raw material, supplier contract fees, labour hours and maximum machine working hours and capacitated elements of network.

In this example we consider decision maker at each echelon. When the profit is maximized, decision maker has to minimize the total transportation costs and total delivery times between all echelons, reprocessing costs, inspection cost in manufacturers, purchasing rates of raw material and contract fees, keeping in mind the quality of raw materials to find equity between cost and quality.

In the example, the maximum allowable budget B for dealing with suppliers is given 2000 TL and percent values (λ, 1, 2, 3) of total qth quality raw material which needs reprocessing are 5, 10 and 30% respectively. The other values are given in tables (1) to (7).

We used possibilistic mixed integer programming model and got decision maker's satisfaction level at each echelon and solution got using LINDO 13 package. The solution is tabulated and results of sub objective functions are also given.

Table 1 – Unit transportation cost and delivery time between supplier and manufacturer CTL

Supplier	Manufacturer 1	Manufacturer 1	Manufacturer 1
1	(0.85, 0.9, 0.94)*/ (4.8, 6, 7)**	(0.65, 0.7, 0.74)*/ (8.6, 10, 11.2)**	(0.75, 0.8, 0.84)*/ (10.6, 12, 13)**
2	(1.04, 1.1, 1.16)/ (34, 40, 44)	(0.95, 1, 1.04)/ (27, 32, 36)	(1.05, 1.1, 1.16)/ (26, 30, 33.2)
3	(0.75, 0.8, 0.84)/ (10.6, 12, 13)	(0.85, 0.9, 0.96)/ (13.2, 15, 16.2)	(0.95, 1, 1.05)/ (16.2, 18, 19.4)
4	(0.65, 0.7, 0.74)/ (14, 16, 17)	(0.70, 0.75, 0.8)/ (20, 22, 23.6)	(0.75, 0.8, 0.84)/ (8.6, 10, 11)
5	(1.04, 1.1, 1.16)/ (11.6, 13, 14)	(0.85, 0.9, 0.96)/ (27, 31, 34.2)	(0.95, 1, 1.05)/ (9.6, 11, 12)

* denotes transportation cost per unit ($) ** denotes delivery time to carry 100 units (hours)

Table 2 – Unit transportation costs between manufacturers – DCs, delivery times and capacities of manufacturer

Manufacturer	DC_1	DC_2	DC_3	Capacity
1	(1.15, 1.2, 1.25)*/ (6, 7, 8)**	(1.24, 1.3, 1.36)*/ (1.8, 20, 22)**	(1.05, 1.1, 1.16)*/ (14, 16, 17)**	(9350, 9400, 9460)
2	(1.35, 1.4, 1.45)/ (26, 30, 33)	(1.24, 1.3, 1.36)/ (27, 32, 35)	(1.45, 1.5, 1.56)/ (20, 22, 24)	(7900, 8100, 8300)
3	(1.05, 1.1, 1.16)/ (10, 12, 13)	(1.45, 1.5, 1.56)/ (8, 9, 10)	(1.35, 1.4, 1.45)/ (12, 13, 14)	(7750, 7800, 7852)

* denotes transportation cost per unit ($) ** denotes delivery time to carry 100 units (hours)

Table 3 – Unit transportation costs, delivery times between DCs and retailers, selling prices and demand of retailers, capacities of DCs (TL)

Retailers	DC_1	DC_2	DC_3	Selling Price	Demand
1	(1.15, 1.6, 1.62)*/ (4.8, 6, 7)**	(1.26, 1.3, 1.35)/ (8.6, 10, 11.2)	(1.16, 1.2, 1.24)/ (10.6, 12, 13)	(14.5, 15, 15.6)	4200 < . . . < 5000
2	(1.25, 1.3, 1.33)/ (34, 40, 44)	(1.54, 1.6, 1.65)/ (27, 32, 36)	(1.44, 1.5, 1.55)/ (26, 30, 33.2)	(13.6, 14, 14.4)	4000 < . . . < 4500
3	(1.37, 1.4, 1.45)/ (10.6, 12, 13)	(1.12, 1.2, 1.26)/ (13.2, 15, 16.2)	(1.08, 1.1, 1.15)/ (16.2, 8, 19.4)	(15.6, 16, 16.5)	4200 < . . . < 4700
4	(1.45, 15, 1.54)/ (14, 16, 17)	(1.36, 1.4, 1.45)/ (6.2, 18, 19.4)	(1.64, 1.7, 1.75)/ (8.6, 10, 11)	(16.5, 17, 17.4)	4400 < . . . < 4900

* denotes transportation cost per unit ($) ** denotes delivery time to carry 100 units (hours)

Table 4 – Purchasing values of unit raw material from suppliers and reprocessing costs of a unit product at manufacturer for each quality (TL)

Quality Options	Supplier					Manufacturer		
	1	2	3	4	5	1	2	3
High	(1.98, 2, 2.02)	(2.18, 2.2, 2.22)	(2.05, 2.1, 2.15)	(1.85, 1.9, 1.95)	(2.25, 2.3, 2.35)	(0.08, 1, 1.02)	(0.85, 0.9, 0.95)	(0.75, 0.8, 0.85)
Moderate	(1.75, 1.8, 1.85)	(1.95, 2, 2.05)	(1.85, 1.9, 1.95)	(1.75, 1.8, 1.85)	(1.98, 2, 2.02)	(1.15, 2, 2.05)	(1.15, 2, 2.05)	(1.05, 1.1, 1.15)
Low	(1.55, 1.6, 1.65)	(1.75, 1.8, 1.85)	(1.55, 1.6, 1.65)	(1.55, 1.6, 1.65)	(1.15, 1.8, 1.85)	(1.35, 1.4, 1.45)	(1.25, 1.3, 1.35)	(1.20, 1.25, 1.3)

Table 5 – Raw Material Capacities and Contract Fees of Suppliers (Unit)

Quality Options	Suppliers				
	1	2	3	4	5
High	(1550, 1600, 1650)	(1850, 1900, 1950)	(1750, 1800, 1850)	(1500, 1550, 1600)	(1800, 1850, 1900)
Mid	(1550, 1600, 1650)	(1850, 1900, 1950)	(1750, 1800, 1850)	(1500, 1550, 1600)	(1800, 1850, 1900)
High	(1550, 1600, 1650)	(1850, 1900, 1950)	(1750, 1800, 1850)	(1500, 1550, 1600)	(1800, 1850, 1900)
Contract Fees	(475, 500, 525)	(525, 550, 575)	(460, 480, 500)	(430, 450, 470)	(510, 530, 550)

Table 6 – Unit Inspection Cost at each Manufacturer for each Quality (TL) Manufacturer

Quality Options	Manufacturers		
	1	2	3
High	(.015, .02, .025)	(0.025, 0.03, 0.035)	(0.055, 0.06, 0.065)
Mid	(0.055, 0.06, 0.065)	(0.035, 0.04, 0.045)	(0.065, 0.07, 0.075)
Low	(0.045, 0.05, 0.055)	(0.065, 0.07, 0.075)	(0.075, 0.08, 0.085)

Table 7 – Labour hours for the reprocessing of a unit product at Manufacturer's for each quality (hours) and available machine hours

Quality Options	Suppliers			Machine Hours
	1	2	3	4
High	3	3.2	3	38000
Mid	2	2.5	2.8	39000
Low	2.5	2.8	3.2	42000

Table 8 – The results obtained by LINDO Package Program (without L values)

Variable	Value	Variable	Value	Variable	Value
X_{121}	1600	Z_{12}	4500	RP_{12}	180
X_{122}	650	Z_{14}	400	RP_{13}	540
X_{132}	950	Z_{21}	2000	RP_{21}	80
X_{133}	1600	Z_{24}	4500	RP_{22}	65
X_{311}	1800	Z_{31}	3000	RP_{32}	95
X_{312}	1800	Z_{33}	4700	RP_{33}	480
X_{313}	1800	P_{11}	1600	L_1	1
X_{411}	900	P_{12}	1600	L_3	1
X_{412}	1530	P_{13}	1600	L_4	1
X_{413}	1530	P_{31}	1800	L_5	1
X_{431}	650	P_{32}	1800	S_1	5000

X_{521}	550	P_{41}	1550	S_2	4500
X_{522}	1850	P_{42}	1550	S_3	4700
X_{523}	1850	P_{43}	1550	S_4	4900
Y_{11}	1700	P_{51}	550		
Y_{13}	7700	P_{52}	1850		
Y_{22}	6500	P_{53}	1850		
Y_{31}	3200	RP_{11}	90		

Table 9 – Optimal Solution (without L values)

Supplier Case Transportation Cost		**Min**	**Max**	**Min**	**PIS**	**NIS**	**L**
Min Z11	17460	17460			17000	19000	0.77
Max Z12	1125		1125		1200	875	0.769231
Min Z13	985			985	900	1300	0.7875
Delivery Time							
Min Z21	3541.5	3541.5			3500	3700	0.79
Max Z21	449.5		449.5		480	350	0.77
Min Z23	311.8			311.8	300	400	0.88
Contract Fees							
Min Z31	1960	1960			1800	2500	0.77
Max Z32	85		85		90	70	0.75
Min Z33	85			85	80	100	0.75
Distribution Center Transportation Cost							
Min Z11	26000	26000			25000	29000	0.75

Max Z12	719		719		730	690	0.73
Min Z13	831			831	800	950	0.79
Delivery Time							
Min Z21	4260	4260			4150	4500	0.69
Max Z22	522.6		522.6		532	500	0.71
Min Z23	697.6			697.6	650	750	
Manufacturer Case							
Transportation Cost							
Min Z11	22480	22480			21000	26000	0.704
Max Z12	1020		1020		800	1100	0.733333
Min Z13	1129			1129	1075	1300	0.76
Manufacturer Case							
Delivery Time							
Min Z21	3815	3815			3750	4100	0.814286
Max Z22	333		333		360	320	0.675
Min Z23	321			321	310	350	0.725
Reprocessing Cost							
Min Z31	1916.5	1916.5			1850	2100	0.734
Max Z32	91.8		91.8		97	90	0.742857
Min Z33	73.8			73.8	70	85	0.746667
Purchasing Value							
Min Z41	32350	32350			31000	37000	0.775
Max Z42	761.5		761.5		800	750	0.77
Min Z43	761.5			73.8	70	85	0.746667

Inspection Cost							
Min Z51	871	871			850	1000	0.86
Max Z52	95.5		95.5		110	90	0.725
Min Z53	95.5			95.5	90	120	0.816667
Retailer Case							
Selling Price							
Max Z11	296500	296500			350000	280000	0.764286
Min Z12	7160		7160		7000	7700	0.771429
Max Z13	9110			9110	11000	9000	0.816667

Table 10 – Optimal Values of Variables (with L values)

Variable	Value	Variable	Value	Variable	Value
X_{121}	1600	Z_{22}	4000	P_{23}	1900
X_{122}	1600	Z_{23}	600	P_{32}	1800
X_{123}	1600	Z_{31}	2300	P_{33}	1800
X_{211}	1200	S_1	4200	P_{41}	1550
X_{221}	700	S_2	4000	P_{42}	1150
X_{222}	1900	S_3	4200	L_1	1
X_{213}	1900	S_4	4400	L_2	1
X_{411}	1530	RP_{11}	60	L_3	1
X_{412}	1150	RP_{12}	180	L_4	1
X_{312}	1800	RP_{13}	1110		
X_{313}	1800	RP_{21}	115		
Y_{11}	600	RP_{22}	350		
Y_{12}	6500	RP_{23}	480		
Y_{13}	2300	P_{11}	1600		
Y_{21}	7400	P_{12}	1600		

Z_{13}	3600	P_{13}	1600		
Z_{14}	4400	P_{21}	1900		
Z_{21}	1900	P_{22}	1900		

Table 11 – Optimal Solution (with L values)

Retailer						
Selling Price (L = 0.82)						
Min Z11	261000		$\tilde{Z}_1$	253840	261000	268160
Max Z12	7160					
Min Z13	7980					
Supplier Case (L = 0.76)						
Transportation Cost						
Min Z11	14140		$\tilde{Z}_1$	13269	14140	15011
Max Z12	871					
Min Z13	788					
Min Delivery Time						
Min Z21	3416		$\tilde{Z}_2$	2928.4	3416	3903.6
Max Z22	487.6					
Min Z23	348.6					
Min Contract fees						
Min Z31	1980		$\tilde{Z}_3$	1890	1980	2070
Max Z32	90					
Min Z33	90					

Manufacturer Case (L = 0.75)						
Transportation Cost						
Min Z11	22060		$\tilde{Z}_1$	21155	22060	22965
Max Z12	905					
Min Z13	928					
Delivery Time						
Min Z21	3930		$\tilde{Z}_2$	3452	3930	4408
Max Z22	478					
Min Z23	381					
Manufacturer Case (L = 0.75)						
Purchasing Value			$\tilde{Z}_3$	30611	31355	32099
Min Z31	31355					
Max Z32	744					
Min Z33	744					
Inspection Cost						
Min Z41	789		$\tilde{Z}_4$	705	789	873
Max Z42	84					
Min Z43	84					
Distributor Case (L = 0.75)						
Transportation Cost						
Max Z11	23990		$\tilde{Z}_1$	23206	23990	29774

Min Z12	784					
Max Z13	779					
Delivery Time						
Min Z21	2972		$\tilde{Z}_2$	2564	2972	3380
Max Z22	408					
Min Z23	3106					
Reprocessing Cost		2977.5				
Total Profit		163708.5				

Table 12 – Optimal Solution (with L values)

Total Sale of Retailers	261000
Total Transportation Costs incurred by Suppliers	14140
Total Transportation Costs incurred by Manufacturers	22060
Total Transportation Costs incurred by Distribution Centres	23990
Total Purchasing Costs	31355
Total Reprocessing Costs	2977.50
Total Inspection Costs	789
Total Contract Fees	1980
Total Profit	163708.50
Minimum Total Delivery Time Between Supplier and Manufacturer	3416
Minimum Total Delivery Time Between Manufacturer and Distribution Center	3930
Minimum Total Delivery Time Between Distribution Center and Retailer	2972

CONCLUSION

In this paper, in addition to product quality analysis within the context of supply chain network, delivery time between different echelons is also minimized. The transportation costs, delivery times, inspection costs are taken as imprecise numbers to formulate the given problem as multi objective linear programming problem. This paper assumes that the decision maker at each level specifies the most possible value in the possibility distribution of each imprecise data as the precise number. The results are tabulated in Tables 8-12. If there is no decision maker at any level, the total selling price increases as suppliers 1, 3, 4 and 5 are selected. If the decision maker is included he selects suppliers 1, 2, 3 and 4 which reduces the total selling price. We have introduced inspection cost and labour time at manufacturer's level to satisfy the ultimate end user. The entire supply chain network aimed at maximizing the total profit while minimizing total transportation cost, production costs, raw material costs, reprocessing costs, inspection costs and supplier's contract fees. From Tables (9) and (11) it is seen that transportation cost, delivery time, inspection cost, purchasing cost are less when decision maker is present, but reprocessing cost alone increases.

APPENDIX

Imprecise Objective Functions of Supplier, Manufacturers and Distributors

I. Supplier

(i) To minimize the transportation cost

$$\text{Min } Z_{11} = Z_1^m = \sum_i \sum_j \sum_q C_{ijq}^m X_{ijq}$$

$$\text{Max } Z_{12} = \left(Z_2^m - Z_1^p\right) = \sum\sum\sum\left(C_{ijq}^m - C_{ijq}^p\right)X_{ijq}$$

$$\text{Min } Z_{13} = \left(Z_2^0 - Z_2^m\right) = \sum\sum\sum\left(C_{ijq}^0 - C_{ijq}^m\right)X_{ijq}$$

(ii) To minimize the total delivery time

$$\text{Min } Z_{21} = Z_2^m = \sum_i \sum_j \sum_q T_{ijq}^m X_{ijq}$$

$$\text{Max } Z_{22} = \left(Z_2^m - Z_2^p\right) = \sum\sum\sum\left(T_{ijq}^m - T_{ijq}^p\right)X_{ijq}$$

$$\text{Min } Z_{23} = \left(Z_2^0 - Z_2^m\right) = \sum\sum\sum\left(T_{ijq}^0 - T_{ijq}^m\right)X_{ijq}$$

(iii) To minimize the contract fees

$$\text{Min } Z_{31} = Z_3^m = \sum_i G_i^m L_i$$

$$\text{Max } Z_{32} = \left(Z_3^m - Z_3^p\right) = \sum_i \left(G_i^m - G_i^p\right) L_i$$

$$\text{Min } Z_{33} = \left(Z_3^0 - Z_3^m\right) = \sum_i \left(G_i^0 - G_i^m\right) L_i$$

II. Manufacturer

(i) To minimize the transportation cost

$$\text{Min } Z_{11} = Z_1^m = \sum_j \sum_q C_{jk}^m \; Y_{jk}$$

$$\text{Max } Z_{12} = \left(Z_2^m - Z_1^p\right) = \sum_j \sum_k \left(C_{jk}^m - C_{jk}^p\right) Y_{jk}$$

$$\text{Min } Z_{13} = \left(Z_2^0 - Z_2^m\right) = \sum_j \sum_k \left(C_{jk}^0 - C_{jk}^m\right) Y_{jk}$$

(ii) To minimize the total delivery time

$$\text{Min } Z_{21} = Z_2^m = \sum_j \sum_k T_{jk}^m \; Y_{jk}$$

$$\text{Max } Z_{22} = \left(Z_2^m - Z_2^p\right) = \sum_j \sum_k \left(T_{jk}^m - T_{jk}^p\right) Y_{jk}$$

$$\text{Min } Z_{23} = \left(Z_2^0 - Z_2^m\right) = \sum_j \sum_k \left(T_{jk}^0 - T_{jk}^m\right) Y_{jk}$$

(iii) To minimize the purchasing value of unit raw material

$$\text{Min } Z_{31} = Z_3^m = \sum_i \sum_q C_{iq}^m \; P_{iq}$$

$$\text{Max } Z_{32} = \left(Z_3^m - Z_3^p\right) = \sum_i \sum_q \left(C_{iq}^m - C_{iq}^p\right) P_{iq}$$

$$\text{Min } Z_{33} = \left(Z_3^0 - Z_3^m\right) = \sum_i \sum_q \left(C_{iq}^0 - C_{iq}^m\right) P_{iq}$$

(iv) To minimize the reprocessing cost

$$\text{Min } Z_{41} = Z_4^m = \sum_j \sum_q C_{jq}^m \; RP_{jq}$$

$$\text{Max } Z_{42} = \left(Z_4^m - Z_4^p\right) = \sum_j \sum_q \left(C_{jq}^m - C_{jq}^p\right) RP_{jq}$$

$$\text{Min } Z_{43} = \left(Z_3^0 - Z_3^m\right) = \sum_j \sum_q \left(C_{jq}^0 - C_{jq}^m\right) RP_{jq}$$

(v) To minimize the inspection cost

$$\text{Min } Z_{51} = Z_5^m = \sum_j \sum_q I_{jq}^m \ Y_{jq}$$

$$\text{Max } Z_{52} = \left(Z_5^m - Z_5^p\right) = \sum_j \sum_q \left(I_{jq}^m - I_{jq}^p\right) Y_{jq}$$

$$\text{Min } Z_{53} = \left(Z_5^0 - Z_5^m\right) = \sum_j \sum_q \left(I_{jq}^0 - I_{jq}^m\right) Y_{jq}$$

II. Distribution Centres

(i) To minimize the total transportation cost

$$\text{Min } Z_{11} = Z_1^m = \sum_k \sum_l C_{kl}^m \ Z_{kl}$$

$$\text{Max } Z_{12} = \left(Z_1^m - Z_1^p\right) = \sum_k \sum_l \left(C_{kl}^m - C_{kl}^p\right) Z_{kl}$$

$$\text{Min } Z_{13} = \left(Z_1^0 - Z_1^m\right) = \sum_k \sum_l \left(C_{kl}^0 - C_{kl}^m\right) Z_{kl}$$

(ii) To minimize the total delivery time

$$\text{Min } Z_{21} = Z_2^m = \sum_k \sum_l T_{kl}^m \ Z_{kl}$$

$$\text{Max } Z_{22} = \left(Z_2^m - Z_2^p\right) = \sum_k \sum_l \left(T_{kl}^m - T_{kl}^p\right) Z_{kl}$$

$$\text{Min } Z_{23} = \left(Z_2^0 - Z_2^m\right) = \sum_k \sum_l \left(T_{kl}^0 - T_{kl}^m\right) Z_{kl}$$

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