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MODELLING THE IMPACT OF FARMING AWARENESS AND CONTROL MEASURES ON FUSARIUM WILT DISEASE IN CASHEW PLANTS

FATU CHILINGA[1,*], ALFRED W. MANYONGE[2], ALFRED K. HUGO[1]

[1]Department of Mathematics, The University of Dodoma, P. O. Box 338, Dodoma, Tanzania

[2]Department of Mathematics, Maseno University, P. O. Box 333, Maseno, Kenya

Abstract: This research is based on a mathematical model involving farming awareness with optimal control and cost-effectiveness analysis of Fusarium wilt on cashew plants. The optimal strategy for lowering spread and the cost of applying control measures was determined by introducing time-dependent control. In addition, the forward-backwards sweep method of the fourth-order Runge-Kutta scheme, which is based on the forward solution of the state equation, was used. According to the findings of the cost-effectiveness analysis, fungicides, cutting and burning severely infected plants, and public advertising are the most cost-effective strategies for managing Fusarium wilt disease with limited resources. As a result, the Fusarium wilt disease can be managed if farmers use fungicides correctly and raise farming awareness through advertisements that help identify the severely infected plant for cutting and burning.

Keywords: fusarium wilt; awareness; optimal control; cost-effectiveness.

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[*]Corresponding author
E-mail address: fchilinga@yahoo.com
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1. INTRODUCTION

Cashew plants are grown in more than 32 countries in Asia, Africa, South America, and Australia. Africa accounts for over 40% of global cashew nut production, whereas Tanzania is the continent's third-largest producer, accounting for roughly 20% of total African output [1]. Cashew nuts are a source of vital nutrients that reduce human disease risk when regularly consumed. It includes crude fat (47.8g), carbohydrate (29.9g), protein (16.8g), and Kcal (574g) per 100g of the consumption intake [2]–[4]. Regular cashew nut consumption lowers blood pressure, diabetes, coronary heart disease, and cholesterol levels [3], [4].

Also, the cashew nut is the main cash crop grown in the south-eastern regions of Tanzania and is the major source of income for most families[5]–[7]. Regardless of its contribution to income generation at an individual, family, and national level for around 10% of the overall export value, its production is affected by several factors such as decreasing soil fertility, drought, insect pests and diseases [5], [8]. Diseases, among other factors, are the main factor affecting cashew nut productivity. The Fusarium wilt disease seems to play a considerable role in frustrating the production of cashew nuts [9]. The Fusarium wilt disease is a significant, deadly disease in cashew plants. It has been fast spreading in the south-eastern region of Tanzania and causing severe loss of production [7], [9], [10].

The farmers where farms are affected by Fusarium wilt have been struggling with how to overcome the challenge. Based on the experience of the farmers and works of literature, the proposed and practised control strategies are intercropping cashew plants with leguminous crops, chemical control, and cutting and burning infected plants [7], [11]–[18]. In addition, farmers need to be aware of disease and control strategies for proper selection and application of control during a disease outbreak.

Farming-related awareness campaigns through mobile telephone, television, radio, magazines, etc., can aid in spreading correct and relevant farming information to producers about Fusarium wilt disease and how to handle the control approach [19]–[21]. Thus, yield increases as the farming awareness campaign increases and disease prevalence decrease [22]. Early-stage plant disease identification helps the recovery of the disease when control is applied than when the treatment is delayed. The farming awareness helped to communicate to the farmers about the symptoms and transmission of fusarium wilt disease, appropriate control strategies, and how to apply it to minimize the disease.

Numerous studies have shown that farming awareness significantly impacts the transmission and management of infectious diseases [23]–[25]. Mbasa and others [17] show that farming awareness help to control Fusarium wilt disease through early disease detection and proper choice of control methods. Mosaic disease transmission can be minimized or eliminated by raising farming awareness [26]. Abraha, Al Basir, et al. [22] observed that self-aware individuals would successfully implement the available control strategies. Also, raising knowledge among individuals about acceptable time delays may be essential in achieving optimal pest control within crop fields and reducing pesticide adverse effects on the environment and human health.

Anggriani [27] presents a mathematical model of plant-fungal disease transmission dynamics by describing the interaction between susceptible plants and the fungal. The result indicates the right proportion of fungicides for plant protection and fungi treatment. The fungicides increase plant protection, but self-awareness is crucial for health protection and reducing environmental hazards [28].

They use optimal control theory to minimize pest management costs due to bio-pesticide. Furthermore, several researchers use the optimal control theory to reduce the cost of implementing control strategies and infection [29]–[31]. A mathematical model of farming awareness based on optimal interventions for crop pest control was presented by [32]. The findings show that pesticides and advertisements are the best options. Also, a model for using biological insecticides in pest management has been developed [33]. The optimal control theory was proposed to eradicate the parasites in agroecosystems [34]. However, as far as we know, the Fusarium wilt disease model gained little attention on optimal control.

In this study, we apply the theory of optimal control to the actions involved in preventing Fusarium wilt disease. The control measures implemented aimed to decrease the transmission of Fusarium wilt infection while minimizing the cost of adopting control strategies and boosting the economy on an individual, family, and national level. Specifically, the control strategies investigated are public awareness through advertisement, cutting and burning of severely infected cashew plants due to the low chance of recovery and proper use of fungicide for treatment of infected cashew plants.

2. MODEL FORMULATION

Following the farming awareness on disease control, the mathematical model is formulated to evaluate its impact on controlling Fusarium wilt in cashew plants based on the basic fusarium wilt mathematical model by Chilinga et al. [35]. The model includes two populations: cashew plants and Fusarium Oxysporum fungus. The cashew plant population consists of the susceptible cashew plants S, the exposed plants E, infected plants I and recovered plants R. The cashew plant's total population is represented by $N = S + E + I + R$. The Fusarium Oxysporum population is divided into two compartments: Chlamydospores C and Macroconidia spores M.

Cashew plant population expansion is governed by a logistic function based on cashew plants' growth rate r and carrying capacity k_1. The interactions between the plants and the fungus are claimed to follow the standard mass action principle with h β the force of infection. The effective contact rates among susceptible cashew plants to Chlamydospores fungus from the land and susceptible Cashew plant to infected Cashew plant were τ_1 and τ_2 respectively [11]. The parameter d represents the saturation rate of Chlamydospores in the soil, resulting in disease risk. The term $\frac{C}{d+C}$ refers to the probability of a susceptible individual contracting fusarium wilt disease from a single interaction, and the force of infection will be expressed as $\beta = \left(\frac{\tau_1 C}{d+C} + \tau_2 I \right)$. The exposed cashew plants join the infected cashew plants after a progression rate ω. The infected cashew plant has a chance of dying naturally at a rate of α. After 120 days, the treated infected cashew plants may recover at a rate ρ and return to the susceptible compartment at a rate of φ [17]. The most severely affected cashew plants reduce the survival rate [17]. Farmer's awareness plays a significant role in mitigating. In modelling, the aspect δ is defined as the additional mortality rate of the infected plant due to farming awareness modelled through a logistic function $\frac{\delta QI}{1+Q}$.

The growth rate a and carrying capacity k_2 of macroconidia spores follow a logistic function. The macroconidia spores then changed into Chlamydospores at a rate of η [36]. Finally, Chlamydospores decay at a rate of μ [37]. The farming awareness on fungicides added chlamydospores mortality rate b and modelled through logistic function represented by $\frac{bQI}{1+Q}$.

The disease's farming awareness Q is growing at a rate g via global sources (mobile telephone, radio, television, magazine and social media) campaigns. The number of infected plants seen in the field will increase the farmer's local awareness at a rate h. As a result of the decline in importance, farming awareness levels may drop at a rate of z.

The following model equations are derived from the assumptions made:

$$
\begin{aligned}
\frac{dS}{dt} &= rS\left(1-\frac{S}{k_1}\right)+\varphi R-\left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)S \\
\frac{dE}{dt} &= \left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)S-(\alpha+\omega)E \\
\frac{dI}{dt} &= \omega E-(\alpha+\rho+\sigma)I-\frac{\delta QI}{1+Q} \\
\frac{dR}{dt} &= \rho I-(\alpha+\varphi)R \\
\frac{dM}{dt} &= \upsilon E+\gamma I+aM\left(1-\frac{M}{k_2}\right)-\eta M \\
\frac{dC}{dt} &= \eta M-\mu C-\frac{bQC}{1+Q} \\
\frac{dQ}{dt} &= g+hI-zQ
\end{aligned} \tag{1}
$$

Initial biological conditions: $S \geq 0, E \geq 0, I \geq 0, R \geq 0, M \geq 0, C \geq 0, Q \geq 0$

3. MODEL ANALYSIS

3.1.Positivity of the Solution

Assume the initial value of the model variable in the equation is $S(t) > 0, E(t) \geq 0, I(t) \geq 0, R(t) \geq 0, M(t) \geq 0, C(t) \geq 0, Q(t) \geq 0$. The solution set is then created as $\{S(t) > 0, E(t) \geq 0, I(t) \geq 0, R(t) \geq 0, M(t) \geq 0, C(t) \geq 0, Q(t) \geq 0\} \in \Re_+^7$ is positive for all time t.

Proof: Let us consider equation (1a) in the model

$$\frac{dS(t)}{dt} = rS\left(1 - \frac{S}{k_1}\right) + \varphi R - \left(\frac{\tau_2 C}{d + C} + \tau_1 I\right)S$$

$$\frac{dS(t)}{dt} \geq -\left(\frac{\tau_2 C}{d + C} + \tau_1 I\right)S$$

By separation of variables, we have

$$\frac{dS(t)}{S(t)} \geq -\left(\frac{\tau_2 C}{d + C} + \tau_1 I\right)dt$$

Integrate both sides for t we have

$$\int_0^t \frac{dS(t)}{S(t)} \geq -\int_0^t \left(\frac{\tau_2 C}{d + C} + \tau_1 I\right)dt$$

$$S(t) \geq S(0)e^{-\int_0^t \left(\frac{\tau_2 C}{d+C} + \tau_1 I\right)dt}$$

Thus as $t \to \infty$ then, it follows that $S(t) \geq S(0)e^{-\int_0^t \left(\frac{\tau_2 C}{d+C} + \tau_1 I\right)dt} \geq 0$ since $\frac{\tau_2 C}{d + C} + \tau_1 I \geq 0$

The same as in another equation, we have

$$\frac{dE(t)}{dt} = \left(\frac{\tau_2 C}{d + C} + \tau_1 I\right)S - (\alpha + \omega)E$$

$$\frac{dE(t)}{dt} \geq -(\alpha + \omega)E$$

Integrate both sides for t we have

$$\int_0^t \frac{dE(t)}{E(t)} \geq -\int_0^t (\alpha+\omega)\,dt$$

Hence, $E(t) \geq E(0)e^{-(\alpha+\omega)t}$

Equally $t \to \infty$ then, it follows $E(t) \geq E(0)e^{-(\alpha+\omega)t} \geq 0, E(t) \geq 0$

The same as in another equation, we have

$$\frac{dI(t)}{dt} = \omega E - (\alpha+\rho+\sigma)I - \frac{\delta QI}{1+Q}$$

$$\frac{dI(t)}{dt} \geq -(\alpha+\rho+\sigma)I - \frac{\delta QI}{1+Q}$$

Integrate both sides for t we have

$$\int_0^t \frac{dI(t)}{I(t)} \geq -\int_0^t \left(\alpha+\rho+\sigma+\frac{\delta QI}{1+Q}\right)dt$$

Hence, we get $I(t) \geq I(0)e^{-\int_0^t \left(\alpha+\rho+\sigma+\frac{\delta QI}{1+Q}\right)dt}$

Thus as $t \to \infty$ then, it follows that $I(t) \geq I(0)e^{-\int_0^t \left(\alpha+\rho+\sigma+\frac{\delta QI}{1+Q}\right)t} \geq 0, I(t) \geq 0$

The same as in another equation, we have

$$\frac{dR(t)}{dt} = \rho I - (\alpha+\varphi)R$$

$$\frac{dR(t)}{dt(t)} \geq -(\alpha+\varphi)R$$

Integrate both sides for t $\int_0^t \frac{dR(t)}{R(t)} \geq -\int_0^t (\alpha+\varphi)\,dt$ then,

$$\int_0^t \frac{dR(t)}{R(t)} \geq -\int_0^t (\alpha+\varphi)\,dt$$

Hence, we get $R(t) \geq R(0)e^{-\int_0^t (\alpha+\varphi)(t)}$

Thus as $t \to \propto$ then, it follows that $R(t) \geq R(0)e^{-\int_0^t (\alpha+\varphi)(t)} \geq 0, R(t) \geq 0$

The same as in another equation, we have

$$\frac{dM(t)}{dt} = \upsilon E + \gamma I + aM\left(1+\frac{M}{k_2}\right)M - \eta M$$

$$\frac{dM(t)}{dt(t)} \geq -\eta M$$

Integrate both sides for t we have

$$\int_0^t \frac{dM(t)}{M(t)} \geq -\int_0^t \eta dt$$

$M(t) \geq M(0)e^{-\int_0^t \eta dt}$ Hence, we get $M(t) \geq M(0)e^{-\int_0^t \eta dt}$

Thus as $t \to \propto$ then, it follows that $M(t) \geq M(0)e^{-\int_0^t \eta dt} \geq 0, M(t) \geq 0$

The same as in another equation, we have

$$\frac{dC(t)}{dt} = \eta M - \left(\mu + \frac{bQ}{1+Q}\right)C$$

$$\frac{dC(t)}{dt(t)} \geq -\left(\mu + \frac{bQ}{1+Q}\right)C$$

Integrate both sides for t we get

$$\int_0^t \frac{dC(t)}{C(t)} \geq -\int_0^t \left(\mu + \frac{bQ}{1+Q}\right)dt$$

Hence, we get $C(t) \geq C(0)e^{-\int_0^t \left(\mu+\frac{bQ}{1+Q}\right)dt}$

Thus as $t \to \propto$ then, it follows that $C(t) \geq C(0)e^{-\int_0^t \left(\mu+\frac{bQ}{1+Q}\right)dt} \geq 0, C(t) \geq 0$

The same as in another equation, we have

$$\frac{dQ(t)}{dt} = g + hI - zQ$$

$$\frac{dQ(t)}{dt(t)} \geq -zQ$$

Integrate both sides for t we have

$$\int_0^t \frac{dQ(t)}{Q(t)} \geq -\int_0^t zdt$$

Hence, we get $Q(t) \geq Q(0)e^{-\int_0^t zdt}$

Thus as $t \to \infty$ then, it follows that $Q(t) \geq Q(0)e^{-\int_0^t zdt} \geq 0, Q(t) \geq 0$

In conclusion, the model equation is epidemiologically significant in the $\Re_+^7$ region and may be utilized to research Fusarium Wilt in the cashew plant.

3.2 Model Equilibria

Model (1) has the disease-free equilibrium DFE as

$$\phi^0 = \left(S^0, 0,0,0,0,0, Q^0\right) = \left(k_1, 0,0,0,0,0, \frac{g}{z}\right)$$ and endemic equilibrium point

$E_* = \left(S^*, E^*, I^*, R^*, M^*, C^*, Q^*\right)$ are

$$S^* = \frac{\left(M\eta + d\mu + bI^*\right)\left(\alpha + \omega\right)E^*}{\left(M\eta + bI^*\right)\tau_1 + \left(M\eta + d\mu + bI^*\right)\tau_2 I^*},$$

$$E^* = \frac{\left(z\left(\alpha + \rho + \sigma\right) + \left(\alpha + \rho + \sigma + \delta\right)\left(g + hI^*\right)\right)I^*}{\left(g + hI^* + z\right)\omega}, \quad R^* = \frac{\rho I^*}{\alpha + \varphi}, \quad Q = \frac{g + hI^*}{z}$$

$$C^* = \frac{M\eta + bI^*}{\mu}$$ and

$$M = \frac{k_2\left(\eta - a + \sqrt{\left(\eta - a\right)^2 + 4aI^*\left(\gamma + \frac{\upsilon\left(z\left(\alpha + \rho + \sigma\right) + \left(\alpha + \rho + \sigma + \delta\right)\left(g + hI^*\right)\right)}{\left(g + hI^* + z\right)\omega k_2}\right)}\right)}{2a}$$

3.3 The Basic Reproduction Number, R_0

Reproductive number estimates the probability of an outbreak and the vital need to restrict its spread ability [38]. The spectral radius of the matrix is then chosen as the most significant eigenvalue in absolute terms FV^{-1} and hence gives the basic reproduction number as

$$R_0 = \frac{k_1 z\omega\tau_1}{(\alpha+\omega)\left(g\delta + z(\alpha+\rho+\sigma)\right)} + \frac{k_1\eta\tau_2(g+z)\left(g\delta\upsilon + z\upsilon(\alpha+\rho+\sigma) + z\gamma\omega\right)}{d(\eta-a)\left(bg+\mu(g+z)\right)(\alpha+\omega)\left[g\delta + z(\alpha+\rho+\sigma)\right]} \tag{2}$$

The basic reproductive number will be positive since we expect to have a greater transformation rate from Macroconidia to Chlamydospores than the growth rate of Macroconidia because the transformation rate depends on three parameters, including Macroconidia growth rate.

3.4 Local Stability of Fusarium Wilt Disease

The Routh Hurwitz criterion is used to investigate the stability analysis of the Fusarium wilt disease-free equilibrium point ϕ^0 of the model system (1) as done in [39]. By differentiating the individual equation of the model system with respect to its state variable ϕ^0, the Jacobian Matrix J_{ϕ^0} is obtained. Therefore, the Jacobian matrix of the model system ϕ^0, is then given by

$$J_{\phi^0} = \begin{bmatrix} -r & -k_1\tau_2 & 0 & \varphi & 0 & 0 & \frac{-k_1\tau_1}{d} \\ 0 & -(\alpha+\omega) & k_1\tau_2 & 0 & 0 & 0 & \frac{k_1\tau_1}{d} \\ 0 & \omega & -(\alpha+\sigma+\rho+\frac{g\delta}{\left(1+\frac{g}{z}\right)z}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & -(\alpha+\varphi) & 0 & 0 & 0 \\ 0 & \upsilon & \gamma & 0 & -(\eta-a) & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta & -\left(\mu+\frac{bg}{\left(1+\frac{g}{z}\right)z}\right) & 0 \\ 0 & 0 & h & 0 & 0 & 0 & -z \end{bmatrix} \tag{3}$$

Equation (3) gives two eigenvalues which are $-r, -\left(\mu + \dfrac{bg}{\left(1+\dfrac{g}{z}\right)z}\right)$, $-\eta + a$ and $-(\alpha+\varphi)$ then reduced to Matrix H as

$$T = B_{ij} = \begin{bmatrix} -(\alpha+\omega) & k_1\tau_2 & \dfrac{k_1\tau_1}{d} \\ \omega & -(\alpha+\sigma+\rho+\dfrac{g\delta}{\left(1+\dfrac{g}{z}\right)z}) & 0 \\ 0 & h & -z \end{bmatrix} \tag{4}$$

The characteristic equation of (4) is

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0 \tag{5}$$

with

$$c_1 = \frac{1}{z+g}\left(2\alpha(z+g) + g(\delta+z) + z^2 + (z+g)(\rho+\sigma+\omega)\right)$$

$$c_2 = \frac{1}{z+g}\left((\alpha+z)^2(z+g) + \omega(z+g)(\delta+\sigma+z) + \alpha g(\delta+\sigma+\rho+\omega) + zg(\sigma+\rho) - \omega k_1\tau_2(z+g)\right)$$

$$c_3 = \frac{1}{d(z+g)}\begin{pmatrix} dz\alpha^2(z+g) + \left(d\alpha zg + \alpha dz^2\right)(\omega+\rho+\sigma) + dgz\omega(\delta+\rho+\sigma) + \\ dz^2\omega(\rho+\sigma) - \left(k_1\tau_2 d\omega z + k_1\tau_1 h\omega\right)(z+g) \end{pmatrix}$$

The local asymptotic stability of the disease-free equilibrium point depends on whether the major leading diagonal of T_n are all positive for $n = 1,2,3$. Then we have

$$\Delta T_1 = z + 2\alpha + \rho + \sigma + \omega > 0.$$

$$\Delta T_2 = \begin{bmatrix} c_1 & 1 \\ c_3 & c_2 \end{bmatrix} = c_1c_2 - c_3 > 0,$$

$$\Delta T_3 = \begin{bmatrix} c_3 & 1 & 0 \\ c_1 & c_2 & c_3 \\ 0 & c_0 & c_1 \end{bmatrix} = c_3\left(c_2c_1 - c_0c_3\right) - c_1^2 > 0$$

Therefore the disease-free equilibrium point ϕ^0 is locally asymptotically stable if and only if ΔT_1, ΔT_2 and $\Delta T_3 > 0$. Then $\Delta T_1 > 0$ since we have

$$\frac{1}{z+g}\left(2\alpha(z+g)+g(\delta+z)+z^2+(z+g)(\rho+\sigma+\omega)\right)>0$$

Then, for $\Delta T_2 > 0$ if and only if $c_1c_2 - c_3 > 0$ and $\Delta T_3 > 0$ if $c_3\left(c_2c_1 - c_0c_3\right) - c_1^2 > 0$, Thus gives the theorem one as follows:

Theorem 1. $T(\lambda)$ is stable if and only if the leading principal major diagonals of $T_n\left(n \in \Re^+\right)$ are all positive, and thus the disease-free equilibrium point is locally asymptotically stable

3.5 Global Stability of the Disease-Free Equilibrium Point

The following result is presented to understand the global stability of the disease-free equilibrium.

Theorem 2. The disease-free equilibrium is theoretically asymptotically stable if $\Re_0 < 1$ otherwise, it is unstable.

Proof: Adopt the comparison theorem to determine the global stability of a disease-free equilibrium by considering the rate of change for infected classes from a model (1).

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \\ \frac{dM}{dt} \\ \frac{dC}{dt} \end{pmatrix} = (F-V)\begin{pmatrix} E \\ I \\ M \\ C \end{pmatrix} - \begin{pmatrix} 0 & k_1\tau_2 & 0 & \frac{k_1\tau_1}{d} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} E \\ I \\ M \\ C \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \\ \frac{dM}{dt} \\ \frac{dC}{dt} \end{pmatrix} \le (F-V)\begin{pmatrix} E \\ I \\ M \\ C \end{pmatrix} \tag{7}$$

Where F and V are Jacobian matrices as defined as

$$F = \begin{pmatrix} 0 & k_1\tau_2 & 0 & \dfrac{k_1\tau_1}{d} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

and

$$V = \begin{pmatrix} \alpha+\omega & 0 & 0 & 0 \\ -\omega & \left(\alpha+\sigma+\rho+\dfrac{g\delta}{\left(1+\dfrac{g}{z}\right)z}\right) & 0 & 0 \\ -\upsilon & -\gamma & \eta-a & 0 \\ 0 & 0 & -\eta & \mu+\dfrac{bg}{\left(1+\dfrac{g}{z}\right)z} \end{pmatrix} \text{ respectively.} \tag{9}$$

Since the eigenvalues of the matrix $\left(F-V\right)$ have a negative real part, then model (1) is stable whenever $\Re_0 < 1$. Then $\left(S,E,I,R,M,C\right) \rightarrow \left(S^0,0,0,0,0,0,\right)$ and $S \rightarrow S^0$ as $t \rightarrow \infty$. Considering the comparison theorem presented by Shaban [40] gives $\left(S,E,I,R,M,C\right) \rightarrow E_0$ as $t \rightarrow \infty$. Hence, disease-free equilibrium is globally asymptotically stable.

4. APPLICATION OF CONTROL MEASURES

The control strategies for controlling Fusarium wilt disease in the model (1) have been introduced to minimize the disease and management cost. This study will focus on the traditional controlling approach, chemical approach, and farming awareness of disease transmission. Specifically, the study employs the strategy of cutting and burning the severely infected plants represented by $u_1(t)$ the fungicide, which follows under the chemical control approach represented by $u_2(t)$, awareness through advertisements defined by $u_3(t)$ the strategy for disease control.

In model (1), we incorporate the time depending on the control parameter for investigating the effect of cutting and burning severely infected cashew plants, fungicide treatment, and the impact of public advertisement, respectively. The model with control is presented by modifying a model (1) under the stated assumption as follows:

$$\begin{aligned}
\frac{dS}{dt} &= rS\left(1-\frac{S}{k_1}\right)+\varphi R-u_2\left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)S \\
\frac{dE}{dt} &= u_2\left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)S-(\alpha+\omega)E \\
\frac{dI}{dt} &= \omega E-\left(\alpha+\sigma+u_2\rho+\frac{u_1\delta Q}{1+Q}\right)I \\
\frac{dR}{dt} &= u_2\rho I-(\alpha+\varphi)R \\
\frac{dM}{dt} &= \upsilon E+\gamma I+aM\left(1-\frac{M}{k_2}\right)-(u_2+\eta)M \\
\frac{dC}{dt} &= \eta M-\left(\mu+\frac{u_2 bQ}{1+Q}\right)C \\
\frac{dQ}{dt} &= (1-u_3)g+hI-zQ
\end{aligned} \tag{10}$$

The admissible control is restricted $0 \le u_i(t) \le 1$ since it is the management percentages. When the control takes the minimal value 0, it indicates no additional steps to reduce the disease. At the same time, the peak amount relates to one hundred per cent successful application of Fusarium wilt disease prevention.

The control measure plays a significant role in providing strategies to minimize the farmers' burden. Model system (10) is reformulated as an optimal control problem to lower the number of infections while maintaining the control cost to a minimum [29]. Considering the three time-dependent control mechanisms used in the model system: cutting and burning, badly infected cashew plants, fungicide treatment, and advertisement. An objective cost function is indicated by the

$J\left(u_1,u_2,u_3\right)$ and defined as

$$J=\min\int_0^T\left(P_1I+P_2C+\frac{G_1}{2}u_1^2+\frac{G_2}{2}u_2^2+\frac{G_3}{2}u_3^2\right)dt \tag{11}$$

Where T is the ultimate time, P_1 and P_2 are positive weight associated with the number of infective cashew plants and Chlamydospores in the soil. G_1, G_2 and G_3 are the positive weight of control relevant to its implication cost. The cost related to any control set-up is assumed to be non-linear and takes a quadratic form. The quadratic expressions $\frac{G_1}{2}u_1^2$, $\frac{G_2}{2}u_2^2$ and $\frac{G_3}{2}u_3^2$ indicate the cost of cutting and burning control efforts, fungicides treatment, and farming awareness advertising, respectively.

The goal was to reduce the objective function J to its smallest possible value; hence the optimal control must be found. Then we have J as

$$J\left(u^*\right)=\min J\left(u|u\in U\right),$$

Where $U=\{\left(u_1,u_2,u_3\right)|u_i$ is Lebesgue quantifiable, such that $0\le U\le 1$ and $t=[0,T], i=1,2,3\}$ is the acceptable control set. The optimal control problem's basic idea is to detect the occurrence and exclusivity of optimal control and identify solutions.

4.1 Optimal Control Existence

The contemplating strategy of Lukes [41] can be utilized to determine if there is an optimal control.

Theorem 3. Assumed $J(u)$ subjected to the model (10) such that $\left(S^0,E^0,I^0,R^0,M^0,C^0,Q^0\right)\ge\left(0,0,0,0,0,0,0\right)$ then, the best management strategy u^* associated with $\left(S^*,E^*,I^*,R^*,M^*,C^*,Q^*\right)$, which reduces $J(u)$ in U.

Proof: The presence of optimal control was obtained using a result [42]. The following properties must be examined:

1. The controls set and state variables aren't empty.
2. The measurable control set is convex and closed

3. Every right-hand side of the state system is continuous, limited above the total of the restricted control and the state, and may be expressed as a linear function with values that depend on time and condition.
4. The integrand $\ell(f,u)$ of the objective function is convex.
5. Must be coefficients $B_1, B_2 > 0,$ and $B \geq 1$ whereby the objective functional partial derivative fulfils functional satisfies $\ell \geq B_1\left(\|u_1\|^2 + \|u_2\|^2 + \|u_3\|^2\right)^{\frac{B}{2}} - B_2$.

The state system presence in Lukes [41] verifies that the first property is met. The control set U is convex and closed according to the definition of a convex set. Hence the second property holds. The right-hand side of a linear state system u_i is bounded by a linear function unless the state solution is bounded. Therefore, the constant $B_1, B_2 > 0,$ and $B \geq 1$satisfying

$$P_1 I + P_2 C + \frac{G_1}{2}u_1^2(t) + \frac{G_2}{2}u_2^2(t) + \frac{G_3}{2}u_3^2(t) \geq B_1\left(\|u_1\|^2 + \|u_2\|^2 + \|u_3\|^2\right)^{\frac{B}{2}} - B_2$$

because the state variable is fixed, as a result, the optimal control existence is confirmed by the presence of optimal control [42].

4.2 Optimal Control Characterization

Pontryagin's Maximum Principle depicts optimal control [29]. To accomplish this goal, one must first convert the optimal control problem into a problem of minimizing a Hamilton, H with respect to u. Let χ be the set of the state variable, U be the set of control, K be the set of adjoint variables, and f be the right-hand side of the differential of the i^{th} state variable. The Lagrangian function of our problem is the integrand of the objective functional, the inner product of the right-hand side of the state equation, and the adjoint variable$\left(\mathrm{K}_1, \mathrm{K}_2, \mathrm{K}_3, \mathrm{K}_4, \mathrm{K}_5, \mathrm{K}_6, \mathrm{K}_7\right)$.

The Lagrangian is written in a more compact way by

$$P_1 I + P_2 C + \frac{G_1}{2}u_1^2 + \frac{G_2}{2}u_2^2 + \frac{G_3}{2}u_3^2 + \mathrm{K}f\left(t, x(t), u_i(t)\right)$$

The Lagrangian expanded form is then provided by

$$\begin{aligned}\mathrm{H} &= P_1 I + P_2 C + \frac{G_1}{2}u_1^2 + \frac{G_2}{2}u_2^2 + \frac{G_3}{2}u_3^2 + \mathrm{K}_1\left(rS\left(1-\frac{S}{k_1}\right) + \varphi R - u_2\left(\frac{\tau_1 C}{d+C} + \tau_2 I\right)S\right)\\ &+\mathrm{K}_2\left(u_2\left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)S - (\alpha+\omega)E\right) + \mathrm{K}_3\left(\omega E - \left(\alpha+\sigma+u_2\rho+\frac{u_1\delta Q}{1+Q}\right)I\right)\\ &+\mathrm{K}_4\left(\rho I - (\alpha+\varphi)R\right) + \mathrm{K}_5\left(\upsilon E + \gamma I + aM\left(1-\frac{M}{k_2}\right) - \eta M\right)\\ &+\mathrm{K}_6\left(\eta M - \mu C - \frac{u_2 bQC}{1+Q}\right) + \mathrm{K}_7\left((u_3-1)g + hI - zQ\right)\end{aligned} \tag{12}$$

Theorem 4. Since u_i^* is the set of optimal control and x^* the corresponding set of solutions for the state system (2) that minimizes J over Π; then there exists an adjoint variable K such that

$\frac{d\mathrm{K}}{dt} = -\frac{d\mathrm{H}}{dx}$, adjoint condition (13)

$\mathrm{K}(T) = 0$, transversality conditions (14)

$\frac{d\mathrm{H}}{du} = 0$, at u^*, optimality conditions. (15)

Proof: The adjoint system can be attained by considering the Lagrangian partial derivative concerning the state variables. Then we obtain the following adjoint equations:

$$\begin{aligned}\frac{d\mathrm{K}_1}{dt} &= \left(r\left(\frac{S}{k_1}-1\right) + u_2\left(\tau_2 I + \frac{\tau_1 C}{d+C} + \frac{rS}{k_1}\right)\right)\mathrm{K}_1 - u_2\left(\frac{\tau_1 C}{d+C}+\tau_2 I\right)\mathrm{K}_2\\ \frac{d\mathrm{K}_2}{dt} &= (\alpha+\omega)\mathrm{K}_2 - \omega\mathrm{K}_3 - \upsilon\mathrm{K}_5\\ \frac{d\mathrm{K}_3}{dt} &= (u_2 S\tau_2)\mathrm{K}_1 - (u_2 S\tau_2)\mathrm{K}_2 + \left(\alpha+\sigma+u_2\rho+\frac{u_1\delta Q}{1+Q}\right)\mathrm{K}_3 - \rho u_2\mathrm{K}_4 - \gamma\mathrm{K}_5 - h\mathrm{K}_7 - P_1\\ \frac{d\mathrm{K}_4}{dt} &= (\alpha+\varphi)\mathrm{K}_4 - \varphi\mathrm{K}_1\\ \frac{d\mathrm{K}_5}{dt} &= \left(\eta + u_2 + a\left(\frac{M}{k_2}-1\right) + \frac{aM}{k_2}\right)\mathrm{K}_5 - \eta\mathrm{K}_6\\ \frac{d\mathrm{K}_6}{dt} &= \left(u_2 S\left(\frac{\tau_1}{(d+C)} - \frac{\tau_1 C}{(d+C)^2}\right)\right)\mathrm{K}_1 - \left(u_2 S\left(\frac{\tau_1}{(d+C)} - \frac{\tau_1 C}{(d+C)^2} + \right)\right)\mathrm{K}_2 + \left(\mu - \frac{u_2 bQ}{1+Q}\right)\mathrm{K}_6 - P_2\end{aligned} \tag{16}$$

$$\frac{dK_7}{dt} = \left(\frac{u_1\delta}{1+Q} - \frac{u_1\delta Q}{(1+Q)^2}\right)IK_3 + \left(\frac{u_2 b}{1+Q} - \frac{u_2 bQ}{(1+Q)^2}\right)CK_6 + zK_7$$

The control problem's optimality is determined as follows:

$$u_i^*(t) = \frac{\partial H}{\partial u_i} \quad \text{where } \; i = 1,2,3.$$

Then

$$\frac{\partial H}{\partial u_1} = G_1 u_1 - \frac{\delta QI}{1+Q}K_3$$

$$\frac{\partial H}{\partial u_2} = G_2 u_2 - \left(I\tau_2 + \frac{C\tau_1}{C+d}\right)SK_1 + \left(I\tau_2 + \frac{C\tau_1}{C+d}\right)SK_2 - \rho IK_3 + \rho IK_4 - MK_5 - \frac{bQC}{1+Q}K_6$$

$$\frac{\partial H}{\partial u_3} = G_3 u_3 - gK_7 \tag{17}$$

The solution $u_1^*(t), u_2^*(t)$ and $u_3^*(t)$ delivered in short form as

Then, we have

$$u_1^* = \max\left\{0, \min\left\{\frac{\delta QI}{G_1(1+Q)}K_3\right\}\right\}$$

$$u_2^* = \max\left\{0, \min\left\{\frac{\left(S\left(I\tau_2 + \frac{C\tau_1}{C+d}\right)K_1 - S\left(I\tau_2 + \frac{C\tau_1}{C+d}\right)K_2 + \frac{\rho I}{B_2}K_3 - \frac{\rho I}{B_2}K_4 - \frac{M}{B_2}K_5 + \frac{bQC}{(1+Q)}K_6\right)}{G_2}\right\}\right\}$$

$$u_3^* = \max\left\{0, \min\left\{\frac{(gK_7)}{G_3}\right\}\right\} \tag{18}$$

5. Numerical Simulation

This section provides numerical simulations for systems (1) and (3). The numerical impacts of farming awareness are well illustrated as disease transmission responds to the increase in farming awareness. The forward-backwards sweep method is used to solve the optimality system of state and adjoint systems. The adjoint system was solved by a fourth-order Runge-Kutta scheme using the forward solution of the state equation. The symmetrical iteration of the initial control values satisfies the optimality requirement.

5.1 Impact of Farming Awareness on Disease Transmission

Figure 1 shows that as the additional mortality rate of infected plants increases, the infected plant decreases. The additional mortality rate of infected plants may occur when farmers know the effect of severely infected plants. The severely infected plant has a low chance of recovery with a high rate of disease transmission through root contact. Farming awareness helps farmers to identify the severely infected plant and remove it to reduce disease transmission.

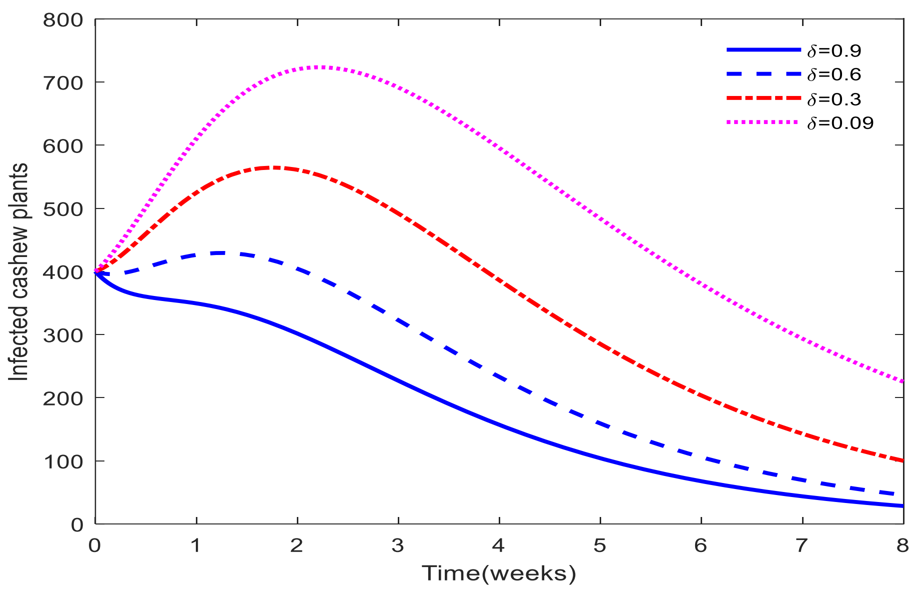

Figure 1: Impact of the additional mortality rate of infected plants due to farming awareness

Figure 2 shows that as the additional mortality rate of Chlamydospores due to farming awareness increases, the Chlamydospores population decreases, hence decreasing disease transmission. The additional mortality rate of Chlamydospores may occur when farmers know about the effect of disease transmission and the proper fungicide to use. The fungus disseminates to the new area through machinery, tools or footwear [11], [37]. All tools should be sanitized using fire or fungicide. Farming awareness helps farmers to minimize fungus dissemination to the new area by increasing the fungus mortality rate to reduce disease transmission.

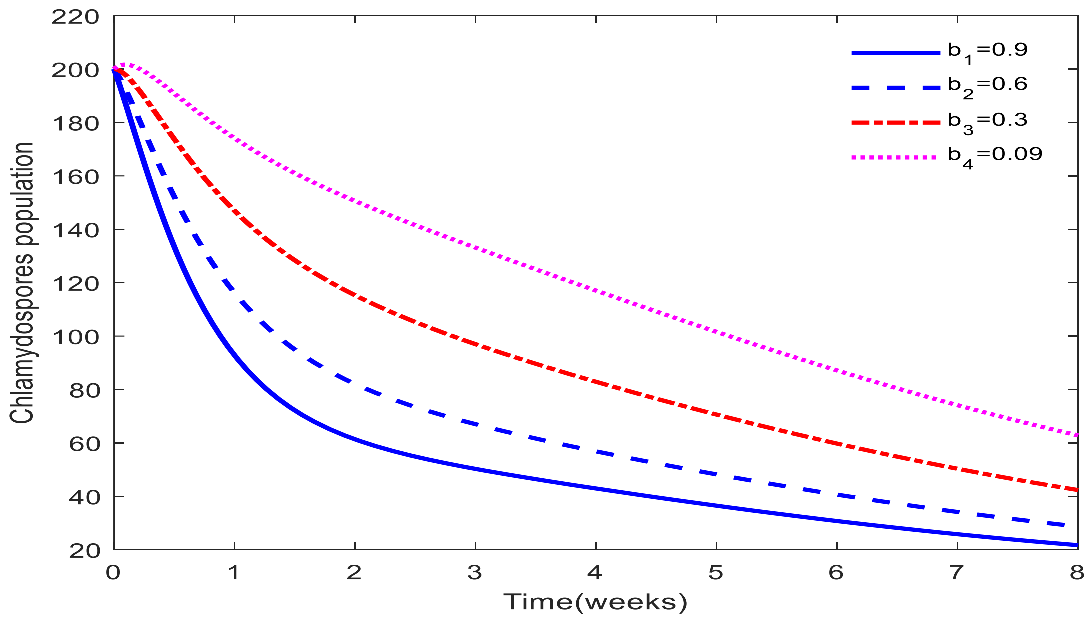

Figure 2: Effect of the additional mortality rate of Chlamydospores due to awareness

Figure 3 illustrates the effect of farming awareness on exposed plants. Exposed plants increase at the beginning when the farming awareness level is lower. An increase in farming awareness indicates the rapid decrease of the exposed plant, which suggests that at a time, more farmers are aware of the Fusarium wilt disease infection decreases.

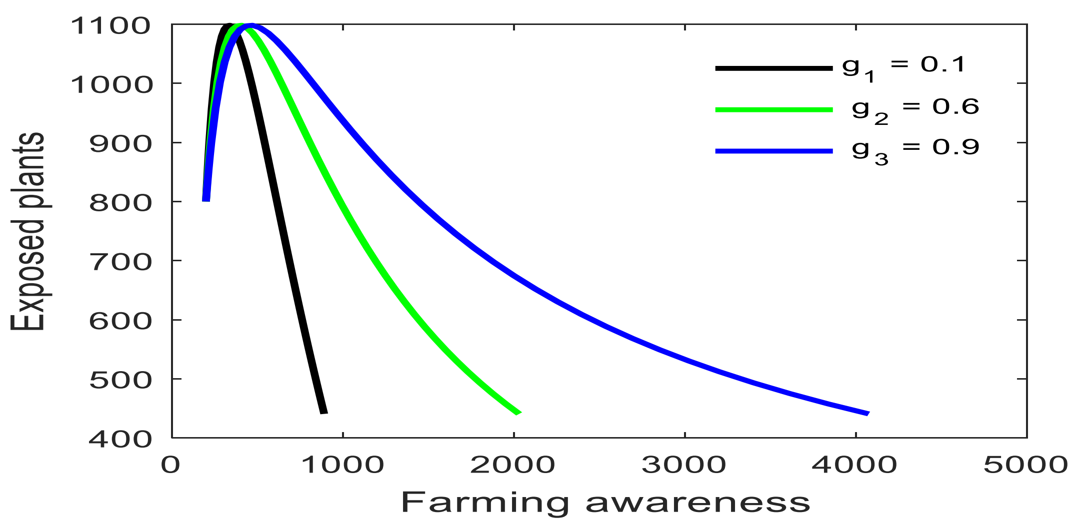

Figure 3: Effect of farming awareness on the transmission of infection

Figure 4 depicts how the number of infected plants rises in the early stages of the disease when few farmers are aware of the problem. However, as awareness grows, the number of infected plants diminishes. This is because farmers adopt proper disease management precautions.

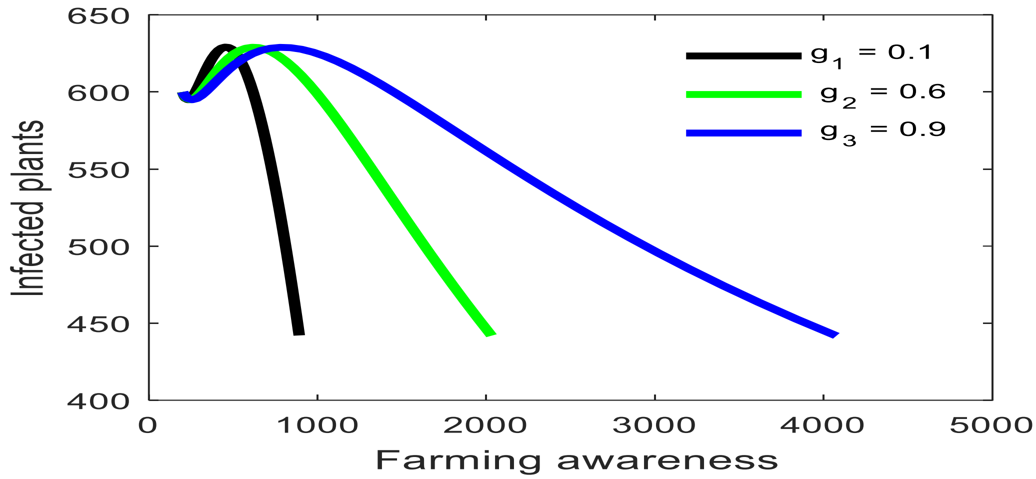

Figure 4. Effect of farming awareness on the transmission of infection

Figure 5 shows that as farming awareness increases, the number of plants that recover from the disease increases because farmers can control the disease at the right time using appropriate methods.

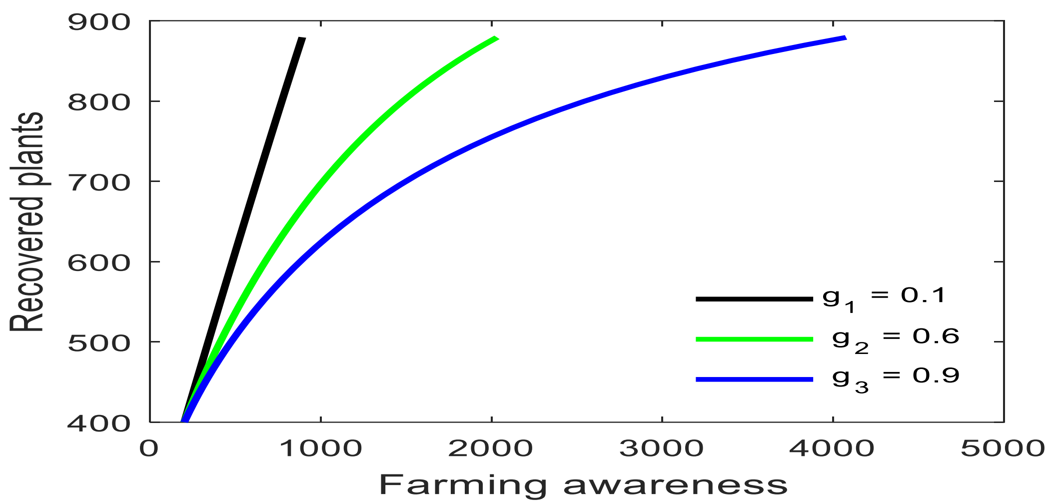

Figure 5: Effect of farming awareness on controlling transmission

Figure 6 shows that the number of Macroconidia spores decreases as more farmers become aware of the disease and its appropriate control measures. This is owing to the use of suitable disease prevention and control measures.

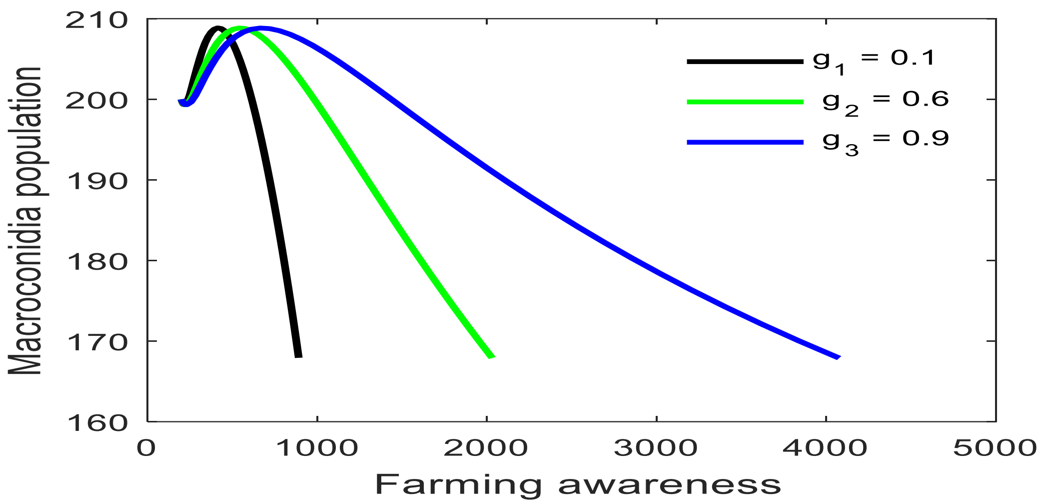

Figure 6: Effect of farming awareness on infection transmission

Figure 7 indicates that the number of Chlamydospores diminishes when more farmers become aware of disease transmission and the proper application of control techniques. This is attributed to be used as suitable disease-preventative measures.

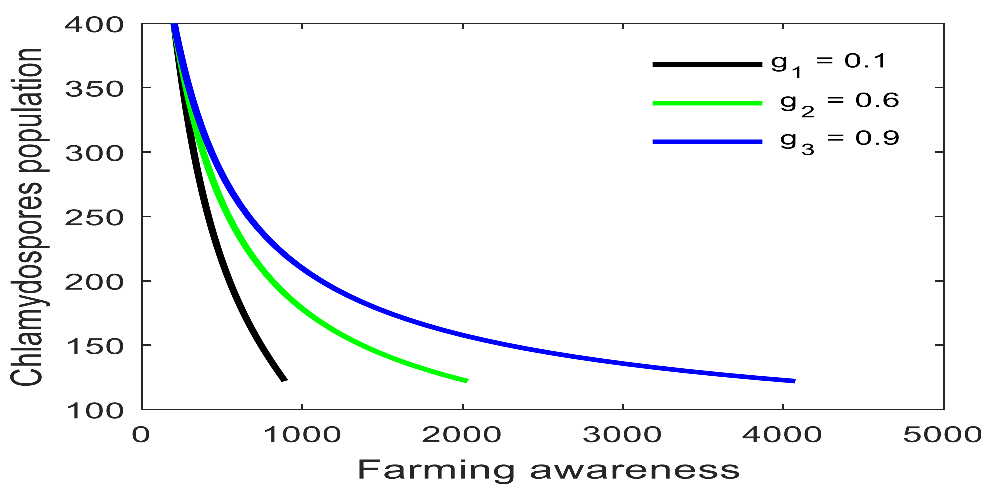

Figure 7: Effect of farming awareness on disease transmission.

An increase in farming awareness through public awareness decreases disease transmission.

5.2. Optimal Control Numerical Results

The primary goal of this part was to see how control techniques, including cutting and burning, severely infected cashew plants, fungicides, and advertising, minimize the spread of Fusarium wilt disease in cashew plants. In addition, graphical representations of alternative methods are visualized to support the analysis results and determine their influence whenever the control is applied to the system (10).

Strategy A: Combination of Cutting and Burning and Fungicides

Figure 8 showed that the number of infecting plants reduced over time when the two controls were employed. The number of Chlamydospores declined slowly and reached zero at 15 weeks. This study demonstrates that the measure should be used to best control the disease.

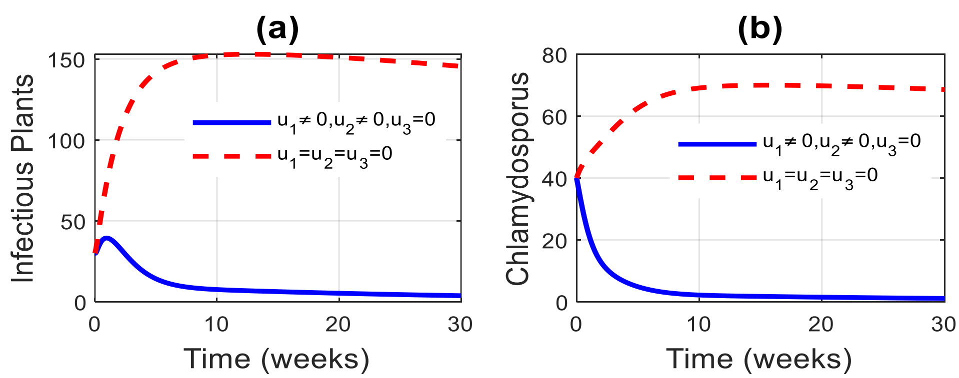

Figure 8: Impact of cutting and burning and fungicides on Fusarium wilt transmission dynamic.

Strategy B: Combination of Cutting and Burning and Advertisements

The results demonstrate that putting control measures can help reduce disease transmission. According to Figure 9 (a), careful strategy implementation for 14 weeks is sufficient to minimize the infected plants to zero. The population of Chlamydospores drops for the first 25 days before remaining constant, as shown in figure 9 (b). As a result, both the infected plant and the fungus suffer a decline.

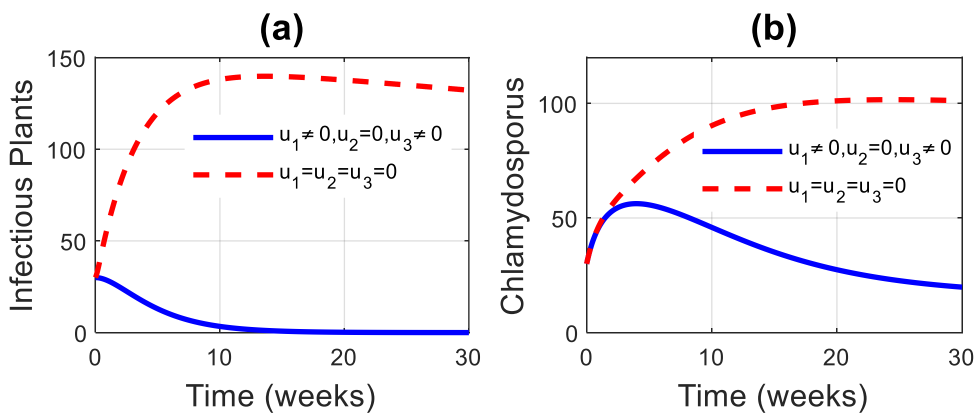

Figure 9: Impact of cutting and burning and advertisements on Fusarium wilt transmission dynamic.

Strategy C: Combination of Fungicides and Advertisements

According to the study, fungicides and advertisements can help reduce disease transmission in the field. Figures 10 (a) and (b) show that the number of infectious plants decreases but not to zero. In addition, the number of Chlamydospores decreases when control is utilized for 13 weeks, as indicated in Figure 10 (b).

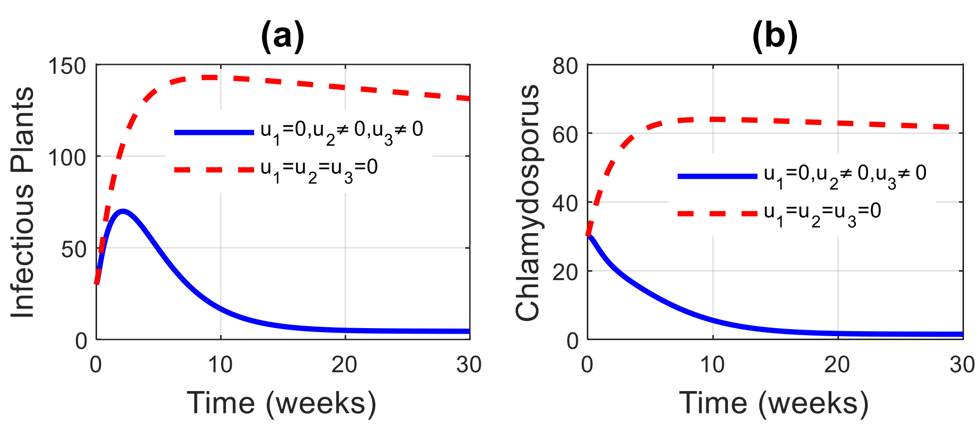

Figure 10: Impact of fungicides and Awareness on fusarium wilt transmission dynamic

Strategy D: Combination of Fungicides, Advertisements, Cutting and Burning.

The findings show that employing fungicides and advertising significantly decreased the number of infected plants and Chlamydospores population, as indicated in Figures 11 (a) and (b).

Furthermore, the disease-free status of this strategy is achieved immediately after 15 weeks. It is worth noting that a mix of fungicides and advertisements is the key to minimizing Fusarium wilt disease. Also, mixing fungicides with advertisements produces positive results in optimizing the objective function.

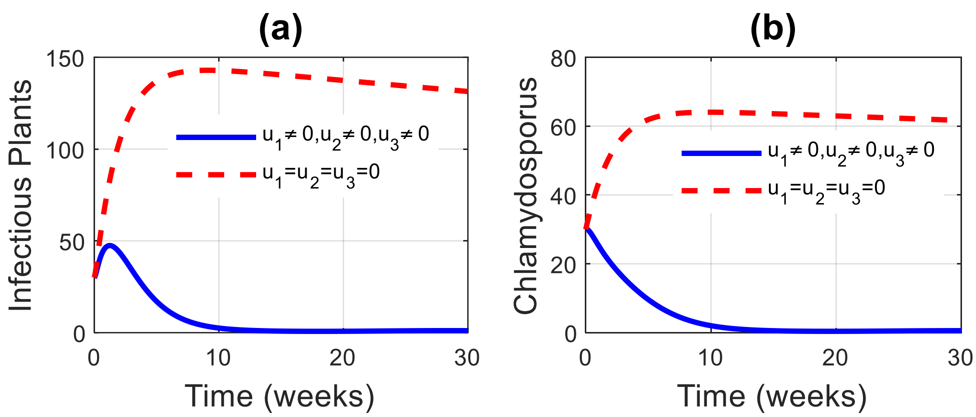

Figure 11: Impact of fungicides, advertisements, cutting and burning on Fusarium wilt transmission dynamic

6. COST-EFFECTIVE ANALYSIS

The cost-effectiveness analysis aids in calculating the financial advantage of the given control action. It is a tool for comparing various strategies' relative costs and outcomes. The analysis evaluation of Fusarium wilt disease was done to discover the best cost-effective method for adopting the approach with restricted resources. The cost-effectiveness ratio (ICRE) was used in this study to analyze the cost and health results of the given pair of strategies [43], [44]. Each intervention is weighed against the next least effective option. The difference between the total number of plants without control and those with control is used to calculate the avoided plant.

The entire cost of control is calculated as follows:

$$B(u)=\underset{u1,u2,u3}{Min}\int_{0}^{3}\left(\frac{G_1}{2}u_1^2+\frac{G_2}{2}u_2^2+\frac{G_3}{2}u_3^2\right)dt$$

Each control strategy's relative cost weight $G_1u_1^2, G_2u_2^2 \quad G_3u_3^2$ calculates the total control costs. The quantitative result for the control strategies is grouped by control combination in table 1, then organized by increasing order of effectiveness (in terms of infection avoided) in table 2.

Strategies	Infectious	Infection averted(E)	Cost $($C_T$)
Status quo	82654		
A	13868	68785	315.00
B	41937	40717	349.50
C	36810	45844	412.00
D	9205.3	73448.7	473.00

Table 1: The overall cost of each strategy and the number of infections avoided.

Strategies	E	ΔE	C_T	ΔC_T	ICER($\Delta C_T/\Delta E$)
B	40717	40717	349.50	349.50	0.008584
C	5127	-35590	412.00	62.50	-0.00176
A	22941	17814	315.00	-92.00	0.00545
D	4663.7	-18277.3	473.00	158.00	0.00864

Table 2: Various optimal control techniques' incremental cost-effectiveness ratios

First, we reorder the control strategies in the table (1) to increase effectiveness. After that, the incremental effectiveness and cost and respectively, are calculated. Finally, we get incremental effectiveness when dividing the incremental cost by incremental effectiveness (ICER). Table 2 contains ICER.

$$ICER(B) = \frac{40717}{349.50} = 0.008584$$

$$ICER(C) = \frac{-35590}{62.50} = -0.00176$$

When comparing strategies B and C, strategy C has a lower ICER than strategy B.

The ICER for the remaining alternatives is then recalculated after removing option B from the list of possibilities. Consequently, option B is more costly and ineffective than strategy C. Table 3 is the result of dropping approach B.

Strategies	E	ΔE	C_T	ΔC_T	ICER($\Delta C_T/\Delta E$)
C	45844	45844	412.00	412.00	0.008987
A	203743	157899	315.00	-97.00	-0.00061
D	208435.7	4692.7	473.00	158.00	0.033669

Table 3: Various optimal control techniques' incremental cost-effectiveness ratios.

From Table 6, we have

$$ICER(C) = \frac{45844}{412.00} = 0.008987$$

$$ICER(A) = \frac{157899}{-97.00} = -0.00061$$

According to this analysis, strategy A is less expensive than strategy C. Therefore, strategy C is ignored, and the comparison is focused on the remaining strategies.

Strategies	E	ΔE	C_T	ΔC_T	ICER($\Delta C_T/\Delta E$)
A	68785	68785	412.00	412.00	0.00599
D	208435.7	139650.7	473.00	6100	0.000437

Table 4: Various optimal control techniques' incremental cost-effectiveness ratios

$$ICER(A) = \frac{68785}{412.00} = 0.00599$$

$$ICER(D) = \frac{139650.70}{473.00} = 0.000437$$

Table 4 compares strategies A and D and reveals that approach A is more expensive and ineffective than strategy D. Strategy D has a lower ICER. The findings determine that strategy D (control with fungicides, advertising, cutting and burning) gives the lowest ICER, hence the most cost-effective strategy. However, it is less costly and more effective than strategy A. Since it utilizes limited resources, Strategy A must be eliminated from the list of options.

7. CONCLUSIONS

This paper presents a deterministic model for Fusarium wilt disease transmission with awareness, and three control options are studied. The increase in awareness shows a more significant impact in decreasing disease transmission. The maximal principle of Pontryagin was utilized to investigate and determine the optimum condition for fusarium wilt control disease using control strategies such as cutting and burning severely infected cashew plants u_1, fungicides u_2 and advertisement u_3. The number of infected cashew plants and the chlamydospores fungus drops when control is used. The numerical analysis indicates that the control strategies minimize disease transmission. According to cost-effectiveness analysis, using fungicides, cutting and burning the severely infected plants, and advertising is the most cost-effective optimal control method for controlling the fusarium wilt disease outbreak with limited resources.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interest.

REFERENCES

[1] I. Wonni, D. Sereme, A.I. Kassankagno, et al. Diseases of cashew nut plants (Anacardium Occidentale L .) in Burkina Faso, Adv. Plants Agric. Res. 6 (2017), 78–83.

[2] G. Brufau, J. Boatella, M. Rafecas, Nuts: Source of energy and macronutrients, Br. J. Nutr. 96 (2006), 24–28.

[3] H. A. Ramadhani, N. Kassim, B. Lyimo, et al. Physicochemical quality of street vended roasted cashew nuts in Tanzania, Amer. J. Res. Commun. 2 (2014), 175–184.

[4] J. Mukuddem-Petersen, W. Stonehouse Oosthuizen, J. C. Jerling, et al. Effects of a high walnut and high cashew nut diet on selected markers of the metabolic syndrome: A controlled feeding trial, Br. J. Nutr. 97 (2007), 1144–1153.

[5] M.E.R. Sijaona, R.H. Reeder, J.M. Waller, Cashew leaf and nut blight – a new disease of cashew in Tanzania caused by Cryptosporiopsis spp., Plant Pathol. 55 (2006), 576–576.

[6] J.P.N. Madeni, Genotype x environment interaction on performance of selected cashew (Anacardium occidentale L.) hybrids in Tanzania, Sokoine Univ. Agric. 18 (2016), 22280.

[7] S. A. Lilai, F. A. Kapinga, W. A. Nene, et al. Ecological factors influencing the severity of cashew fusarium wilt disease in Tanzania, Res. Plant Dis. 27 (2021), 49–60.

[8] L.C. Brown, E. Minja, A.S. Hamad, Cashew production in East Africa, Paper presented at CABI'sFirst Scientific Conference on Advancing Agricultural Production in Africa, ARUSHA, 1984.

[9] D.J. Majune, P.A. Masawe, E.R. Mbega, Status and management of cashew disease in Tanzania, Int. J. Environ. Agric. Biotechnol. 3 (2018), 1590–1597.

[10] D.D. Tibuhwa, S. Shomari, Fusarium wilt disease: an emerging threat to cashew nut crop production in Tanzania, Asian J. Plant Pathol. 10 (2016), 36-48.

[11] N.Y. Moore, S. Bentley, K.G. Pegg, et al. Musa Disease Fact Sheet No 5 Fusarium wilt of Banana, no. 33, INIBAP, Montpellier, 1995.

[12] J. H. Bowers, J. C. Locke, Effect of botanical extracts on the population density of Fusarium oxysporum in soil and control of fusarium wilt in the greenhouse, Plant Dis. 84 (2000), 300–305.

[13] R.J. Hillocks, T.H.M. Kibani, Factors affecting the distribution, incidence and spread of Fusarium wilt of cotton in Tanzania, Experiment. Agric. 38 (2002), 13-27.

[14] A. Daly, G. Walduck, Fusarium wilt of bananas (Panama disease) Agnote 151. Northern Territory Government, Australia, (2006).

[15] A.N. Cianchetta, R.M. Davis, Fusarium wilt of cotton: Management strategies, Crop Protect. 73 (2015), 40–44.

[16] M. Dita, M. Barquero, D. Heck, et al. Fusarium wilt of banana: Current knowledge on epidemiology and research needs toward sustainable disease management, Front. Plant Sci. 871 (2018), 1–21.

[17] W.V. Mbasa, W.A. Nene, F.A. Kapinga, et al. Characterization and chemical management of Cashew Fusarium Wilt Disease caused by Fusarium oxysporum in Tanzania, Crop Protect. 139 (2020), 105379.

[18] W.V. Mbasa, In vitro efficacy of botanical leaf extracts on Fusarium oxysporum causing cashew wilt disease in Tanzania, Arch. Phytopathol. Plant Protect. 54 (2021), 2397–2406.

[19] P.L.M. Vasterman, N. Ruigrok, Pandemic alarm in the Dutch media: Media coverage of the 2009 influenza A (H1N1) pandemic and the role of the expert sources, Eur. J. Commun. 28 (2013), 436–453.

[20] L. Zuo, M. Liu, Effect of awareness programs on the epidemic outbreaks with time delay, Abstr. Appl. Anal. 2014 (2014), 940841.

[21] F. Al Basir, A.M. Elaiw, S. Ray, Effect of time delay in controlling crop pest using farming awareness, Int. J. Appl. Comput. Math. 5 (2019), 110.

[22] T. Abraha, F. Al Basir, L.L. Obsu, et al. Pest control using farming awareness: Impact of time delays and optimal use of biopesticides, Chaos Solitons Fractals. 146 (2021), 110869.

[23] D. Gao, S. Ruan, An SIS patch model with variable transmission coefficients, Math. Biosci. 232 (2011), 110–115.

[24] J.M. Tchuenche, N. Dube, C.P. Bhunu, et al. The impact of media coverage on the transmission dynamics of human influenza, BMC Public Health. 11 (2011), S5.

[25] J.A. Cui, X. Tao, H. Zhu, An SIS infection model incorporating media coverage, Rocky Mt. J. Math. 38 (2008), 1323–1334.

[26] F. Al Basir, E. Venturino, P.K. Roy, Effects of awareness program for controlling mosaic disease in Jatropha curcas plantations, Math. Methods Appl. Sci. 40 (2016), 2441–2453.

[27] N. Anggriani, L.N. Putri, A.K. Supriatna, Stability analysis and optimal control of plant fungal epidemic: An explicit model with curative factor, AIP Conf. Proc. 1651 (2015), 40.

[28] X. Yang, F. Wang, L. Meng, et al. Farmer and retailer knowledge and awareness of the risks from pesticide use: A case study in the Wei River catchment, China, Sci. Total Environ. 497–498 (2014), 172–179.

[29] S. Lenhart, J.T. Workman, Optimal control applied to biological models, Chapman and Hall/CRC, 2007.

[30] U. Ledzewicz, H. Schättler, Optimal bang-bang controls for a two-compartment model in cancer chemotherapy, J. Optim. Theory Appl. 114 (2002), 609–637.

[31] P. Waltman et al., Lecture Notes in Biomathematics Martin Eisen Mathematical Models in Cell Biology, vol. 2. 1978.

[32] T. Abraha, F. Al Basir, L.L. Obsu, et al. Farming awareness based optimum interventions for crop pest control, Math. Biosci. Eng. 18 (2021), 5364–5391.

[33] J. Chowdhury, F. Al Basir, Y. Takeuchi, et al. A mathematical model for pest management in Jatropha curcas with integrated pesticides - An optimal control approach, Ecol. Complex. 37 (2019), 24–31.

[34] C.J. Silva, D.F. M. Torres, E. Venturino, Optimal spraying in biological control of pests, Math. Model. Nat. Phenom. 12 (2017), 51–64.

[35] F. Chilinga, A.K. Hugo, A.M. Wanyama, Modelling the effect of fusarium oxysporum in transmission dynamics of fusarium wilt disease of cashew plants in the context of south-eastern Tanzania, J. Math. Comput. Sci. 12

(2022), 69.

[36] B. Dutta, J. Searcy, T. Coolong, Fusarium Wilt of Watermelon, UGA Coop. Ext. Bull. 1485, 2017.

[37] F.I. Okungbowa, H. Shittu, Fusarium wilts : An overview, Environ. Res. J. 6 (2016), 83-102.

[38] N. Nyerere, Mathematical models for the infectiology and cost-effectiveness of the control strategies for brucellosis, PhD Theses, The Nelson Mandela AFrican Institution of Science and Technology, 2021.

[39] M.A. Choghadi, H.A. Talebi, The routh-hurwitz stability criterion, revisited: The case of multiple poles on imaginary axis, IEEE Trans. Automat. Contr. 58 (2013), 1866–1869.

[40] N. Shaban, Modelling the effects of public health education in the spread of hepatitis B disease, Appl. Math. Sci. 9 (2015), 3967–3981.

[41] D.L. Lukes, Differential equations: Classical to controlled, vol. 162, Academic Press, New York,1982.

[42] W.H. Fleming, R.W. Rishel, Deterministic and stochastic optimal control, Springer, vol. 139, no. 4. 1976.

[43] K.O. Okosun, R. Ouifki, N. Marcus, Optimal control analysis of a malaria disease transmission model that includes treatment and vaccination with waning immunity, BioSystems, 106 (2011), 136–145.

[44] P. Rodrigues, C.J. Silva, D.F.M. Torres, Cost-effectiveness analysis of optimal control measures for tuberculosis, Bull. Math. Biol. 76 (2014), 2627–2645.