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HYPERBOLIC TRIGONOMETRIC VOYAGING WAVE ARRANGEMENT FOR NON-INTEGER ADJUSTING TERM OF ECKHAUS CONDITION AND KLEIN GORDON CONDITION

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Abstract: The new wave solution of mathematical equations used in physics, engineering, and many applied sciences was found in this research using an alternative technique. For nonlinear partial differential equations without the term integer, our goal is to arrive at the analytical solution without the need for a new transformation to make the balancing term integer. To find the exact solutions to the Eckhaus equation and the cubic nonlinear Klein Gordon equation, as well as new type of complex hyperbolic trigonometric travelling wave solutions. In order to display the graphs showing the stationary wave, the parameters in these solutions are given specified values. Furthermore, few discussions about new complex solutions are presented. It is described by supplying the constants in traveling wave solutions, which are important both physically and mathematically, Finally, three-dimensional simulation is used to support these discussions.

Key words: partial differential equation; hyperbolic trigonometric travelling wave solutions; non-integer balancing term.

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1. INTRODUCTION

The proposed method has been shown to be a powerful mathematical tool for solving nonlinear waves of mathematical equations, physics, and engineering. Therefore, the discussion of exact solutions of NPDEs in nonlinear science is very important [1–4]. In recent years, many researchers have extensively used this beneficial method, for instance, a Jacobi elliptic expansion rule [5], the tanh rule [6–8], an inverse scattering rule [9], the first integral rule [10,11], extended to tanh-function rule [12], the Hirota's direct method [13], the auxiliary equation rule [14], upgraded Bernoulli sub-equation function rule [15], expansion method [16].

The Eckhaus equation is as below [16]:

$$iv_m + v_{nn} + 2(|v|^2)_n v + |v|^4 v = 0, \qquad v: R \to C \qquad (1)$$

where $v = v(n, m)$ be a complex valued function of real variables n and m. This equation is a nonlinear form of Schrödinger and recognizes a linearization of the free-time dependent linear Schrödinger equation. The Eckhaus equation was discovered as a multiscale asymptotic contraction of a certain proportion of a nonlinear partial differential equation [17]. The Eckhaus equation was linearized by the variation of the dependent variable. Many scientists have worked with this equation, for instance, the first integral method [18], weakly nonlinear effects [19], the Laplace transform [20], The Eckhaus equation was linearized (exactly) with appropriate corrections for the dependent variable [21]. Many properties of the Eckhaus equation have been studied [22], the Eckhaus equation can be integrated by the change of dependent variable [23]. The purpose of this task is to identify exact solutions by expansion method for the Eckhaus equation [24].

In this work, we use the extension rule to reduce some new cases with exact solutions to some nonlinear partial differential equations such as the cubic nonlinear Klein- Gordon equation [25]. Which is mathematical physics is so important that many researchers paid attention to balanced numbers rather than integers. In this work, we examine the third-order nonlinear Klein-Gordon equation [26] as follows:

$$v_{mm} - \omega^2 v_{nn} + \alpha v - \beta v^3 + \gamma v^5 = 0, \tag{2}$$

where ω, α, β , and γ are arbitrary constant. These equations play an important role in many scientific applications , like the quantum field theory [27], the solid state physics [28], the nonlinear Klein - Gordon's equation also found many types of exact traveling wave solutions, including the soliton solution, compact solutions, solitary patterns solutions and periodic solutions using the tanh-function rule [29], generalized Kudryashov method [30], Homotopy Perturbation Method [31].

2. STRUCTURES OF $(1/G')$ - EXPANSION METHOD

interior this portion, common substances of the $(1/G')$ - development strategy [46–48] is shown. To begin with, the common sort of nonlinear PDE subordinate on m and n factors.

$$F(v, v_m, v_n, v_{nn}, \dots), \tag{3}$$

here is $v = v(n, m)$ a function that depends on n also on m, $\xi = k(n - 2\alpha m)$ at the form of $v(n, m) = Ve^{i(\alpha n + \beta m)}$ Where k, α and β are constants that are not zero. PDE discussed to as (3)

$$\xi = k(n - 2\alpha m), \qquad v(n, m) = Ve^{i(\alpha n + \beta m)}, \tag{4}$$

utilizing this transformation

$$Q(V, V', V'', \dots) = 0, \tag{5}$$

the ODE has a form. Otherwise, the linear second order ODE solution is provided below.

$G = G(\xi)$.

$$G'' + \lambda G' + \mu = 0. \tag{6}$$

The nonlinear ODE solution given by (5) could be written as follows.

$$v(\xi) = \sum_{i=1}^{m} a_i \left(\frac{1}{G'}\right)^i, \tag{7}$$

a $a_1, a_2, \dots, \lambda, \mu$ are constants, and m is the balancing term. The term balance is a fixed number obtained in any non - linear ODE between the highest order nonlinear term and the highest order linear term. This number (7) is written in place, and the derivatives required for the solution are obtained.

In the case of such derivatives, $G'' = -\lambda G' - \mu$, taken as $(1/G')$ is a homogeneous and a polynomial condition. Here is $(1/G')^i$, $(i = 0,1,2, \dots)$. By setting the coefficients of the terms to zero, we obtain an algebraic equation system. This algebraic equation system is disassembled manually or with the help of a computer package program. Such solutions can be found in the solution function (7) and are written as (3) non-linear PDE walking wave solutions.

The common arrangement is given by the (6).

$$G = G(\xi) = -\frac{\mu\xi}{\lambda} - C_1 e^{-\lambda\xi} + C_2, \tag{8}$$

We'll take it. C_1 and C_2 are fixed in this place. (8) after determining the derivative of the solution derivative given by the ξ variable and making the necessary arrangements.

$$\frac{1}{G'} = \frac{1}{-\frac{\mu}{\lambda} + C_1 e^{-\lambda\xi}}, \tag{9}$$

if the algebraic expression (9) is transformed into a trigonometric function $C_1 = A$ to be

$$\frac{1}{G'} = \frac{\lambda}{-\mu + \lambda A\left(Cosh\left(\xi\lambda\right) - Sinh\left(\xi\lambda\right)\right)}, \tag{10}$$

it is conceivable to type in.

3. APPLICATIONS

Application 1. In this segment, the arrangement is given by (1) with $(1/G')$ - expansion method will be gotten. We will select the taking after the change of the wave for (1):

$$\xi = k(n - \frac{2\alpha m^{\tau})}{\tau}, \; v(n,m) = Ve^{i\left(\alpha n + \frac{\beta m^{\tau}}{\tau}\right)} \tag{11}$$

below the (11) wave transformation, the (1) is transformed to ODE as follows:

$$k^2v'' - (\beta + \alpha^2)v + 4kv'v^2 + v^5 = 0 \tag{12}$$

In the (12), the balancing constant between the linear term of the highest order v'' and nonlinear term of the highest order v^5 is $m = \frac{1}{2}$ that m is not integer. The arrangement of the ODE equation gotten in (12) can be given within the taking after way considering (7).

$$v(\xi) = a_1\left(\frac{1}{G'}\right)^{\frac{1}{2}}, \tag{13}$$

we take the 1st and 2nd derivatives of the (13) and put it in the (12), we get a polynomial with $(1/G')^{\frac{i}{2}}$, $i \in \mathbb{N}$ a variable. $(1/G')^{\frac{i}{2}}$ the polynomial term coefficients equal to zero.

$$\begin{aligned}
&\left(\frac{1}{G'}\right)^{\frac{1}{2}}: \quad -\alpha^2 a_1 - \beta a_1 + \frac{1}{4}k^2\lambda^2 a_1 = 0,\\
&\left(\frac{1}{G'}\right)^{\frac{3}{2}}: \quad k^2\lambda\mu a_1 + 2k\lambda a_1^3 = 0,\\
&\left(\frac{1}{G'}\right)^{\frac{5}{2}}: \quad \frac{3}{4}k^2\mu^2 a_1 + 2k\mu a_1^3 + a_1^5 = 0.
\end{aligned} \tag{14}$$

The framework of (14) with help from the computer bundle program. The arrangements gotten here are put into the (13). At last, the transformation is switched and modern complex hyperbolic trigonometric voyaging wave arrangements to the (1) are obtain.

The outcomes are as follows:

Case 1:

$$a_1 = \mp\frac{\mathrm{i}\sqrt{k}\sqrt{\mu}}{\sqrt{2}}, \quad \lambda = \mp\frac{2\sqrt{\alpha^2+\beta}}{k},$$

$$v_1(n,m) = -\frac{\mathrm{i}e^{\mathrm{i}\left(n\alpha + \frac{m^\tau \beta}{\tau}\right)}\sqrt{k}\sqrt{\mu}}{\sqrt{2}\left(\frac{k\mu}{2\sqrt{\alpha^2+\beta}} + A\cosh\left[2\left(n - \frac{2m^\tau\alpha}{\tau}\right)\sqrt{\alpha^2+\beta}\right] + A\sinh\left[2\left(n - \frac{2m^\tau\alpha}{\tau}\right)\sqrt{\alpha^2+\beta}\right]\right)^{\frac{1}{2}}}, \tag{15}$$

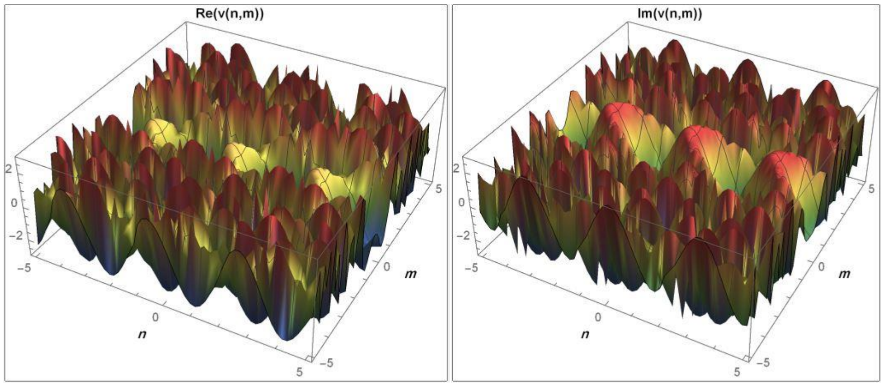

Figure 1. In terms of the (15), for the constants $A=-8,\ \alpha=1,\ \beta=-2, \tau=0.6,\ k=-1,\ \mu=-1$ the new complex hyperbolic trigonometric travelling wave solution of the (1).

Case 2:

$$\lambda=\frac{2\sqrt{\alpha^2+\beta}}{k}; \quad \mu=-\frac{2a_1^2}{k};$$

$$\mathrm{v}_2(\mathrm{n,m})=\frac{e^{i\left(n\alpha+\frac{m^\tau\beta}{\tau}\right)}a_1}{\left(A\cosh\left[2\sqrt{\alpha^2+\beta}\left(n-\frac{2m^\tau\alpha}{\tau}\right)\right]-A\sinh\left[2\sqrt{\alpha^2+\beta}\left(n-\frac{2m^\tau\alpha}{\tau}\right)+\frac{a_1^2}{\sqrt{\alpha^2+\beta}}\right]\right)^{\frac{1}{2}}}. \tag{16}$$

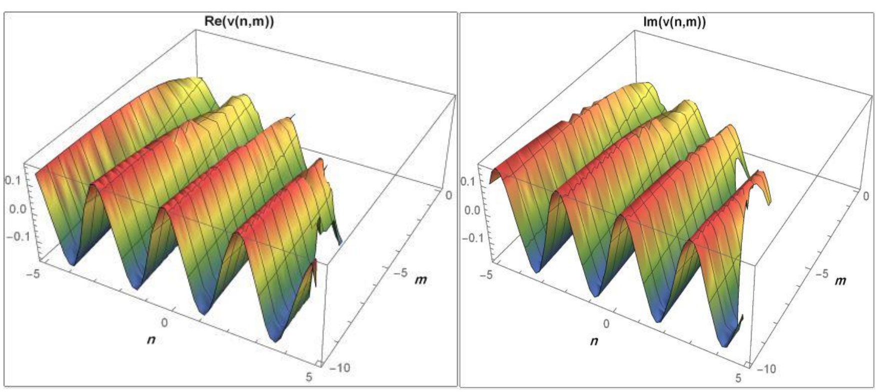

Figure 2. In terms of the (16), for the constants $A=1,\ \alpha=2.5,\ \beta=-2, \tau=0.7,\ a_1=2$ the new complex hyperbolic trigonometric travelling wave solution of the (1)

Application 2. The proposed method is used in this section to prove the cubic nonlinear Klein-Gordon equation. For (2), we can use the following wave transformation:

$$v(n,m) = v(\xi)\,, \qquad \xi = n - vm\ , \tag{17}$$

where v is constant. The voyaging wave variable (17) permits us to change over (2) to the following ODE for $v = v(\xi)$,

$$\left(v^2-\omega^2\right)v'' + \alpha v - \beta v^3 + \gamma v^5 = 0, \tag{18}$$

The balancing constant between the linear term of the highest order u'' and the nonlinear term of the highest v^5 in (18) is $m = \frac{1}{2}$ that m is not an integer. The arrangement of the ODE equation gotten in (18) can be given within the taking-after way considering (7).

$$v(\xi) = a_1\left(\frac{1}{G'}\right)^{\frac{1}{2}}, \tag{19}$$

We get a polynomial with $\left(\frac{1}{G'}\right)^{\frac{i}{2}}$, $i \in \mathbb{N}$ a variable. when we take the 2nd derivative of (18) and plug it into (19). $\left(\frac{1}{G'}\right)^{\frac{i}{2}}$ the polynomial term coefficients equal to zero.

$$\begin{aligned}
&\left(\frac{1}{G'}\right)^{\frac{1}{2}}: \quad \alpha a_1 - \frac{1}{4}\omega^2\lambda^2 a_1 + \frac{1}{4}v^2\lambda^2 a_1 = 0,\\
&\left(\frac{1}{G'}\right)^{\frac{3}{2}}: \quad -\omega^2\lambda\mu a_1 + v^2\lambda\mu a_1 - \beta a_1^3 = 0,\\
&\left(\frac{1}{G'}\right)^{\frac{5}{2}}: \quad -\frac{3}{4}\omega^2\mu^2 a_1 + \frac{3}{4}v^2\mu^2 a_1 + \gamma a_1^5 = 0,
\end{aligned} \tag{20}$$

The framework of (20) with help from the computer bundle program. The arrangements gotten here are put into the (18). At long last, the change is turned around and new complex hyperbolic trigonometric voyaging wave arrangements of the (2) are obtain.

The outcomes are as follows:

Case 1:

$$a_1 = \mp \frac{2\mathrm{i}\sqrt{\alpha}\sqrt{\mu}}{\sqrt{\beta}\sqrt{\lambda}}, \quad \gamma = \frac{3\beta^2}{16\alpha}, \quad v = \mp \frac{\sqrt{-4\alpha + \omega^2\lambda^2}}{\lambda},$$

$$v_1(n,m) = -\frac{2i\sqrt{\alpha}\sqrt{\mu}}{\sqrt{\beta}\sqrt{\lambda}\left(-\frac{\mu}{\lambda} + A\cosh\left[\lambda\left(n - \frac{m\sqrt{-4\alpha+\omega^2\lambda^2}}{\lambda}\right)\right] - A\sinh\left[\lambda\left(n - \frac{m\sqrt{-4\alpha+\omega^2\lambda^2}}{\lambda}\right)\right]\right)^{\frac{1}{2}}} \tag{21}$$

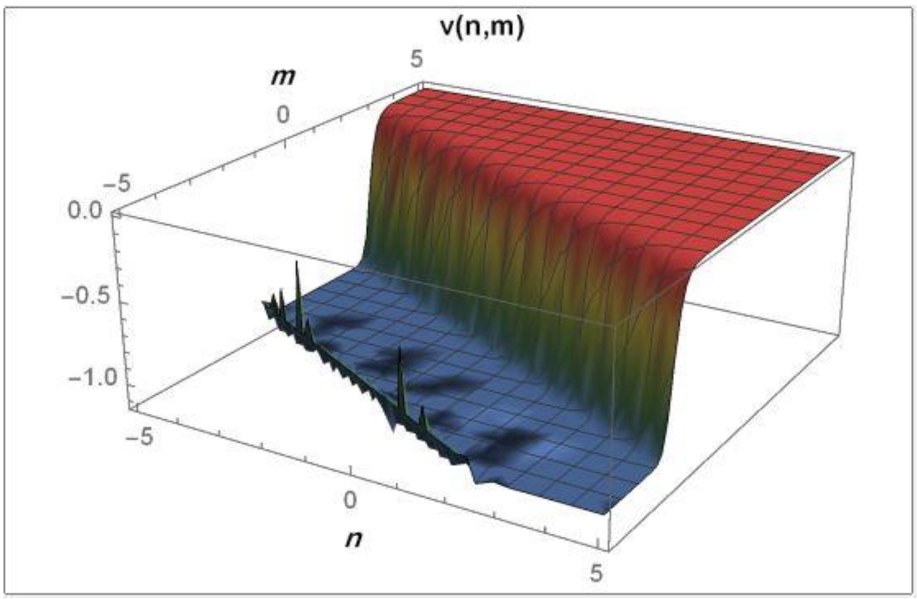

Figure 3. In terms of the (21), for the constants $A = -5.68,\ \alpha = 2.23,\ ,\beta = 2.72,\ \mu = 7.24,$ $\lambda = 1.67,\ \omega = 1.1$ the new hyperbolic trigonometric travelling wave solution of the (2).

Case 2:

$$\gamma = -\frac{3\beta\mu}{4\lambda a_1^2}; \quad \omega = \frac{\sqrt{v^2\lambda\mu - \beta a_1^2}}{\sqrt{\lambda}\sqrt{\mu}}; \quad \alpha = -\frac{\beta\lambda a_1^2}{4\mu};$$

$$v_2(n,m) = \sqrt{\frac{1}{-\frac{\mu}{\lambda} + A\cosh\left[\left(-mv+n\right)\lambda\right] - A\sinh\left[\left(-mv+n\right)\lambda\right]}}\, a_1 \tag{22}$$

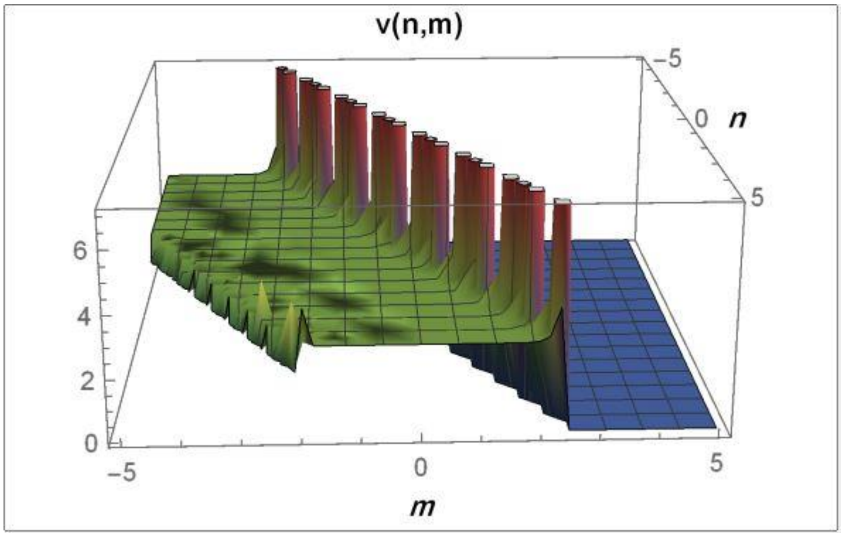

Figure 4. In terms of the (22), for the constants $A = 5,\ a_1 = -3,\ \mu = -5,\ \lambda = 1,\ v = -10$ the new hyperbolic trigonometric travelling wave solution of the (2).

4. Results and Discussions

In this paper, we have presented a modern strategy for finding complex hyperbolic trigonometric voyaging wave arrangements to partial differential conditions with non-integer balancing terms. We have actualized the application here for the balancing term m=1/2. Within the following studies, both m<0 and m≠1/2 can be utilized for balancing terms that m is not an integer. For the (1), the complex hyperbolic trigonometric travelling wave solution presented in the form of (15) and for the (2), the complex hyperbolic trigonometric voyaging wave arrangements presented in the form of (21) are new and the solutions afford the Eq. (1) and (2). In this paper, the term balancing has been effectively adapted for both partial and complex non-integer partial differential equations. Furthermore, both partial differential equations and fractional differential applications have been effectively completed. Stationary wave graphs are presented by assigning special values to constants in analytically produced solutions. In the future, the term adjusting may be attempted in non-integer negatives. In complex operations, numerous handle complexes have been experienced until the arrangement of the framework of conditions. These troubles have been overcome with computer innovation.

5. CONCLUSIONS

In this paper, a new application $(1/G')$ - expansion rule, The exact solutions of the Eckhaus and cubic nonlinear Klein-Gordon equations were obtained and discussed, in addition, for the Eckhaus and cubic nonlinear Klein Gordon equations, we obtained new types of complex hyperbolic trigonometric travelling wave solutions. Numerous researchers were less interested when the adjusting term utilized within the development strategies was non-integer or negative. In arrange to extend this intrigue, this strategy can be used to balance non-integer terms of partial differential equations. Within the arrangements displayed, extraordinary values are given to the constants to get the design to speak to the stationary wave. As these constants alter, the position, speed, adequacy, and wavelength of the wave can alter. Wolfram Mathematica 11.3 was utilized in troublesome, complex operations and realistic drawings. The arrangements given will be an imperative demonstration for analysts considering asymptotic behavior. This inquiry displayed more extensive pertinence utilizing the $(1/G')$ - expansion strategy to handle nonlinear developmental conditions. The outcome shows that this strategy was feasible. The unused wave arrangement gotten in this paper can show distinctive points of view on future research.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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