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J. Math. Comput. Sci. 3 (2013), No. 2, 505-512

ISSN: 1927-5307

SOME QUESTIONS OF A CLASS OF THE TIME SERIES MODEL BASE ON HAAR WAVELET

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Abstract: Time series model is a sort of important stochastic system, and is a useful stochastic process in practices. In this paper, we study a time series model based on Haar wavelet and wavelet transform. We investigate its some properties and wavelet expansion.

Keywords: time series model; wavelet; Haar; expansion.

2000 AMS Subject Classification: 60H15, 60H30, 60H35

1. Introduction

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from extragalactic astronomy to molecular spectroscopy to medical imaging to computer vision, one must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data. Wavelets have contributed to this already intensely developed and rapidly advancing field.

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This paper was funded by Hunan Provinial Natural Science Foundation of China(12JJ9004) and Scientific Research Found of Hunan Provincial Education Department (12A033).

Received January 22, 2013

Wavelet analysis consists of a versatile collection of tools for the analysis and manipulation of signals such as sound and images as well as more general digital data sets, such as speech, electrocardiograms, image. Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering .The basic idea is always to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks themselves come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes.

There are two important aspects to wavelets, which we shall call “mathematical” and “algorithmic”. Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates. As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties. Recently some persons have studied wavelet problems of stochastic process or stochastic system ([1]-[19]).

2. Basic definitions

Definition 1. Let

$$x(t) = a_0 y(t) + a_1 y\ (t-1) \tag{1}$$

We call (1) as MA (1) model; for more details, see [20] and the reference therein.

We have

$$\begin{aligned} R(s,t) &= EX(t)X(s) \\ &= E(a_0 y(t) + a_1 y(t-1))(a_0 y(s) + a_1 y(s-1)) \\ &= E(a^2{}_0 y_t y_s + a_0 a_1 y_t y_{s-1} + a_0 a_1 y_{t-1} y_s + a_1{}^2 y_{t-1} y_{s-1}) \qquad (2) \\ &= \begin{cases} a_0{}^2 + a^2{}_1, s=t \\ a_0 a_{1,} t=s-1 \\ 0, other \end{cases} \end{aligned}$$

$$here, let Dy_t = \sigma^2 = 1$$

Definition 2. Let {x(t) , t∈R} is a stochastic processes on probability space (Ω, g, P). We call

$$W(s, x) = \frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt \quad (3)$$

is wavelet transform of x(t), where,ψis mather wavelet; see[11] and the reference therein.

Then, we have

$$w(s, x+\tau) = \frac{1}{s}\int_R x(t)\psi(\frac{x+\tau-t}{s})dt \quad (4)$$

Definition 3. Let mather waveletψ(x) be the function:

$$\psi(x)=\begin{cases} 1, 0 \le x < \frac{1}{2} \\ -1, \frac{1}{2} \le x < 1 \\ 0, other \end{cases} \quad (5)$$

we call ψ(x) is the Haar wavelet.

Then, we have

$$\psi(\frac{x-t}{s}) = \begin{cases} 1, x-\frac{s}{2} \le t < x \\ -1, x-s \le t < x-\frac{s}{2} \end{cases} \quad (6)$$

$$\psi(\frac{x+\tau-t}{s}) = \begin{cases} 1, x+\tau-\frac{s}{2} \le t < x+\tau \\ -1, x+\tau-s \le t < x+\tau-\frac{s}{2} \end{cases} \quad (7)$$

3. Some results about density degree

We have

$$R(\tau) = E[w(s,y)w(s,y+\tau)]$$
$$= E[\frac{1}{s}\int_R x(t)\psi(\frac{y-t}{s})dt][\frac{1}{s}\int_R x(t_1)\psi(\frac{y+\tau-t_1}{s})dt_1]$$
$$= \frac{1}{s^2}E[\iint_{R^2} x(t)x(t_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1]$$
$$= \frac{1}{s^2}\iint E[x(t)x(t_1)]\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t_1}{s})dtdt_1$$

$$=\frac{1}{s^2}\iint_{R^2}(a_0{}^2+a^2{}_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t}{s})dtdt$$

$$+\frac{1}{s^2}\iint_{R^2}(a_0a_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t}{s})dtdt$$

=I.

Then, we have

$$\begin{aligned} I &= \frac{1}{s^2}\iint_{R^2}\ (a_0{}^2+a_1{}^2+a_0a_1)\psi(\frac{y-t}{s})\psi(\frac{y+\tau-t}{s})dtdt\ ] \\ &= \frac{1}{s^2}[\int_{y-s/2}^{y}dt\int_{y+\tau-s/2}^{y+\tau}dt\ -\int_{y-s/2}^{y}dt\int_{y+\tau-s}^{y+\tau-s/2}dt \\ &\quad -\int_{y-s}^{y-s/2}dt\int_{y+\tau-s/2}^{y-s/2}dt\ +\int_{y-s}^{y-s/2}dt\int_{y+\tau-s}^{y+\tau-s/2}dt\ ] \\ &= \frac{s(s+\tau)(a_0{}^2+a_1{}^2+a_0a_1)}{2} \\ &\textit{then} \\ &R''(\tau)=0, R''(0)=0 \end{aligned} \tag{8}$$

Then, the zero density degree of W(s , y) is

$$\sqrt{\left|\frac{R''(0)}{\pi^2 R(0)}\right|}=0\,.$$

4. Wavelet expansion

We consider wavelet expansions of stochastic processes and show that for certain wavelets, the coefficients of the expansion have negligible correlation for different scales. We can introduce a modification of the wavelets. Certain non-stationary processes the wavelets may be chose to give uncorrelated coefficients.

In order to use the idea of multiresolution, we will start by defining the scaling function and then define the wavelet in terms of it.

Let real function φ be standard orthogonal element of multiresolution analysis $\{V_j\}\ j\in Z$ (see [4]), then there exists $h_k \in l^2$, have

$$\varphi(t) = \sqrt{2}\sum_k \varphi(2t-k).$$

Let $\psi(t) = \sqrt{2}\sum_k (-1)^k h_{1-k}\varphi(2t-k)$.

Then wavelet express of $y(t)$ in mean square is

$$y(t) = 2^{-\frac{J}{2}}\sum_K C_n^J \varphi(2^{-J}t-n) + \sum_{j\le J} 2^{-\frac{j}{2}}\sum_{n\in Z} d_n^j \psi(2^{-j}t-n),$$

where $C_n^j = 2^{-\frac{j}{2}}\int_R x(t)\varphi(2^{-j}t-n)dt$, and

$$d_n^j = 2^{-\frac{j}{2}}\int_R x(t)\psi(2^{-j}t-n)dt.$$

We, therefore, find that

$$E\left[C_n^j C_m^k\right] = 2^{-\frac{j+k}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\varphi(2^{-j}t-n)\varphi(2^{-k}s-m)dsdt,$$

$$E\left[d_n^j d_m^k\right] = 2^{-\frac{j+k}{2}}\iint_{R^2} E\left[x(t)x(s)\right]\psi(2^{-j}t-n)\psi(2^{-k}s-m)dsdt,$$

where

$$\psi(2^j t-m) = \begin{cases} 1, m2^{-j} \le t < (1/2+m)2^j \\ -1, (1/2+m)2^{-j} \le t < (1+m)2^{-j} \end{cases}$$

$$\psi(2^k s-n) = \begin{cases} 1, n2^{-k} \le s < (1/2+n)2^{-k} \\ -1, (1/2+n)2^{-k} \le s < (1+n)2^{-k} \end{cases} \tag{9}$$

In view of (8) and (10), we can obtain the value of $E\left[d_n^j d_m^k\right]$.

If we let normalized scaling function to have compact support over [0,1], then a solution is a scaling function that is a simple rectangle function

$$\varphi(t) = \begin{cases} 1, 0 \le t \le 1 \\ 0, otherwise \end{cases} \tag{10}$$

Now we consider the function $\psi(t)$ that exists a compact support set on $[-k_1, k_2], k_1, k_2 \ge 0$, and exists a enough large number M such that $\int_R t^m \psi(t)dt = 0, 0 \le m \le M-1$, then φ exists a compact support set on $[-k_3, k_4]$ satisfing $k_1 + k_2 = k_3 + k_4, k_3, k_4 \ge 0$.

Let $b(j,k) =< y(t), \psi_{jk} >$, $a(j,k) =< y(t), \phi_{jk} >$

Let J be a constant. We find that $\left\{2^{\frac{J}{2}}\varphi(2^J x-k), k\in Z\right\}\cup\left\{2^{\frac{j}{2}}\psi(2^j t-k), k\in Z\right\}_{j\geq J}$ is a standard orthonormal basis of the space $L^2(R)$. It follows that

$$y(t)=2^{\frac{J}{2}}\sum_{K\in Z}a(J,K)\varphi(2^J t-K)+\sum_{j\geq J}\sum_{K\in Z}2^{\frac{j}{2}}b(j,K)\psi(2^j t-K). \tag{11}$$

Therefore, the self-correlation function of $b(j,m)$

$$\begin{aligned}R_b(j,K;m,n)&=E\left[b(j,m)b(k,n)\right]\\&=2^{-\frac{j+K}{2}}\iint_{R^2}E\left[x(t)x(s)\right]\psi(2^j t-m)\psi(2^K s-n)dtds\end{aligned} \tag{12}$$

And have also the self-correlation function of $a(j,m)$

$$\begin{aligned}R_a(j,K;m,n)&=E\left[a(j,m)a(k,n)\right]\\&=2^{-\frac{j+K}{2}}\iint_{R^2}E\left[x(t)x(s)\right]\varphi(2^j t-m)\varphi(2^K s-n)dtds.\end{aligned} \tag{13}$$

It follows from (8) and (9) that

$$R_b(j,k;m,n)=2^{-\frac{j+k}{2}}\iint_{R^2}E[x(t)x(s)]\psi(2^j t-m)\psi(2^k s-n)dtds.$$

$$R_a(j,k;m,n)=2^{-\frac{j+k}{2}}\iint_{R^2}E[x(t)x(s)]\varphi(2^j t-m)\varphi(2^k s-n)dtds.$$

On the other hand, we have

$$\begin{aligned}&R_b(j,k;m,n)=\\&-2^{-\frac{j+k}{2}}\Big[\int_{m2^{-j}}^{(1/2+m)2^{-j}}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}}dt+\int_{m2^{-j}}^{(1/2+m)2^{-j}}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}dt+\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}dt\int_{n2^{-k}}^{(1/2+n)2^{-k}}dt\\&-\int_{(1/2+m)2^{-j}}^{(1+m)2^{-j}}dt\int_{(1/2+n)2^{-k}}^{(1+n)2^{-k}}dt\Big](a_0{}^2+a^2{}_1+a_0a_1)\\&=\frac{2^{j+1}+2^{k+1}-1}{2^{j+k+2}}(a_0{}^2+a^2{}_1+a_0a_1)2^{-\frac{j+k}{2}}\\&=\frac{2^{j+1}+2^{k+1}-1}{2^{3(j+k)/2+2}}(a_0{}^2+a^2{}_1+a_0a_1)\end{aligned}$$

It follows that

$$\varphi(2^j t-n)=\begin{cases}1, n2^j \le t \le (n+1)2^j \\ 0, other\end{cases}$$

$$\varphi(2^k s-m)=\begin{cases}1, m2^k \le s \le (m+1)2^k \\ 0, other\end{cases}$$

Then we have

$$R_a(j,k;m,n)=$$

$$-2^{-\frac{j+k}{2}}\iint_{R^2}(a^2{}_0+a^2{}_1+a_0a_1)\varphi(2^j s-n)\varphi(2^k s-m)dtds\ .$$

REFERENCES

[1] S. Cambancs, Wavelet approximation of deterministic and random signal, IEEE Trans.Information Theory, 40 (1994) 1013-1029

[2] P. Malliavin, Stochastic analysis, Springer-Verlag, 2003

[3] P. Flandrin, Wavelet analysis and synthesis of fractional Brownian motion,IEEE Trans.Information Theory,38 (1992), 910-916

[4] Y. Meyer, In Ondelettes et operatears, Hermann, 1990

[5] A. Bobrowski, Functional analysis for probability and stochastic processes, Cambridge University Press, 2007

[6] Anju Bala, Devendra Kumar, Balbir Singh, Convergence of wavelet series and applications to computerized tomography, J. Math. Comput. Sci. 2 (2012), No. 3, 673-683

[7] Devendra Kumar, Stability of wavelet Frames and Riesz basis, with respect to translations in Lp(Rn), J. Math. Comput. Sci. 2 (2012), No. 3, 593-603

[8] H. Krim, Multire solution analysis of a class non-stationary processes, IEEE Tran.on Information Theory, 41 (1995) 1010-1019

[9] H. Ren, Wavelet estimation for jumps on a heterosedastic regression model, Acta Math. Sci. 22 (2002) 269-277

[10] X. Xia, Wavelet analysis of the stochastic system with coular stationary noise, Eng. Sci.3 (2005) 43-46

[11] X. Xia, Wavelet density degree of continuous parameter AR model, Int. J. Nonlinear Sci. 7 (2008) 237-242

[12] X. Xia, Wavelet analysis of Browain motion, World J. Model. Simulation, 3 (2007)106-112

[13] X. Xia, T. Dai, Wavelet density degree of a class of Wiener processes, Int. J. Nonlinear Sci. 7 (2009) 327-332

[14] A. Hendi, New exact travelling wave solutions for some nonlinear evolution equations, Int. J. Nonlinear Sci. 7 (2009) 259-267

[15] X. Xia, The morlet wavelet density degree of the linear regress processes with random coefficient, Int. J. Nonlinear Sci. 9 (2010) 349-351

[16] X. Xia, The study of a class of the fractional Brownian motion based on wavelet, Inter.J. Nonlinear Sci.11 (2011) 290-294

[17] X. Xia, D. Ting, The energy of convolution of 2-dimension exponential random variables base on Haar wavelet, Progress Appl. Math.2012,3(2)

[18] X. Xia, The study of Wiener processes with linear-trend base on wavelet, Stud. Math. Sci.2012,4(2)

[19] X. Xia, The energy of stochastic vibration system of a class of protein base on wavelet, Adv. Nature Sci. 2012,5(2)

[20] S.M. Pandit, S.M. Wu, Time series analysis and applications, John wiley, 1983