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ON OPTIMAL SEARCH TECHNIQUE FOR A RANDOMLY LOCATED TARGET

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Abstract: In order to modify the time-saving coordination search procedure for a main objective randomly located on one line, a novel search procedure is provided in this paper. From the origin of the line, the search for the hidden target begins with two searchers or robots, and in each particular section of the search, every searcher wants to identify the lost objective (target). The expected time to detect the objective is calculated. For the optimal search procedure, the required conditions are obtained. It introduces an approximation algorithm that makes the procedure easier for searchers to detect the objective. The real-life effective application is illustrated.

Keywords: time saving; search in a straight line; lost objective; expected value; optimal search plane; hidden target.

2020 AMS Subject Classification: 60K30, 90B40.

1. INTRODUCTION

Dates of search theory returns to the Second World War, search plans for located or moving targets have become solutions for complex problems [1-4]. Our study on a special technique of search plans it called coordinated search, we must illustrate the coordinated linear search in order to describe some essential models of search programs. see Abd EL-Moneim.A.M et al. [5-7]. In earlier works by Abd EL-Moneim.A.M and his collaborators [8-10], the authors illustrated the

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coordinated technique in the Plane where the target was located. Recently, due to the technological W. Afifi et al. [11,12,13] presented symmetric and asymmetric coordinated scan on one of two intersecting lines for a randomly located target. Recently, W. Afifi et al. [14,15] illustrated for one of two and n disjoint lines, a random walker target. In the discrete search problems using a monitoring system, W. Afifi et al. [16] obtained an ideal discrete search for a randomly moving COVID-19 between many cells in the region of the human nasopharynx. Also, W.Afifi et al. [17] proposed and investigated a current three-dimensional search model for numerous searchers to discover a 3-D randomly located object. More recently the coordinated search technique between n disjoint lines with multi searchers W. Afifi et al [18]. W. Afifi et al [19] have also recently discussed the coordinated search strategy in the plane.

In this paper we use a new technique that updated the technique which was used in W. Afifi [11]. We use two cooperative searchers to looking for the lost target from the origin at the same time which located inside cylinder (real line) continuously and in one direction, where one of the searchers looking for the target in the right direction of the line to $+\infty$ and the other searcher looking for the target in the left direction of the line to $-\infty$ without going back to the origin point, and using modern means of communication is supported our model. This significantly reduces the predicted time to detect the lost target, which is our primary goal.

The following is a summary of the rest of the paper:

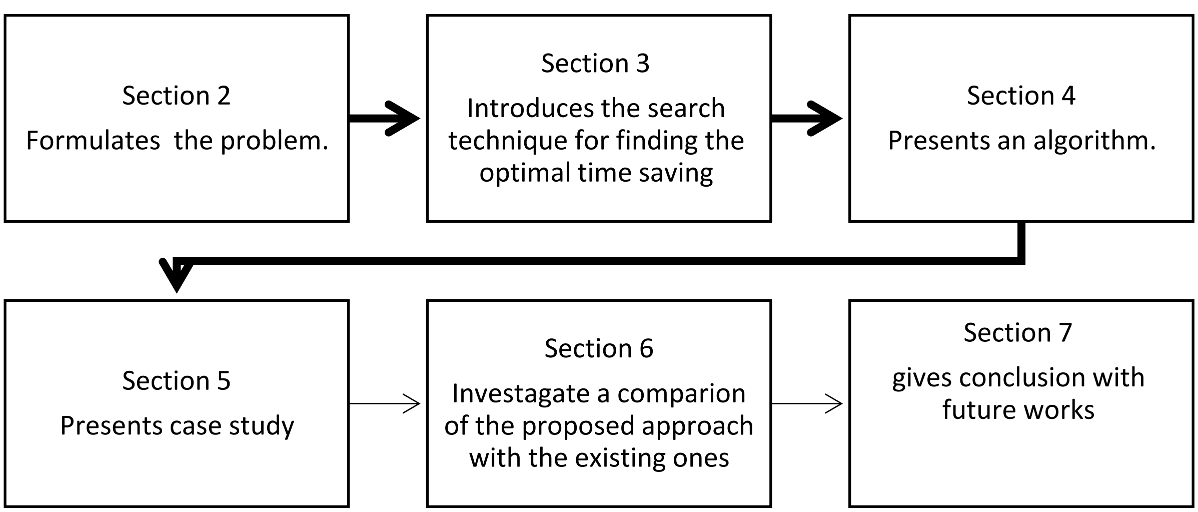

Fig. 1 Layout of Remaining Paper

2. Problem Formulation and Solution Concepts

On 27 march 2013, Internet service in Egypt and the UAE has been affected as a result of a major submarine cable cut in the Mediterranean. Whereas, the "Smw4" cable was cut off near the coastal city of Alexandria, which weakened the country's Internet service for a short period. What if the distribution of the cable cut (lost target) problem is asymmetrical, see Figure (1). Of course, the problem will be more complicated. The searchers, on the other hand, are aware of the probability distribution function of the cable severed site (sensors), which searches for it and aims to discover it in the shortest possible time. We describe a modern technique for finding on the wire that involves collaboration between two searchers on the (line) to calculate the optimal search plan which minimize the lost target detection.

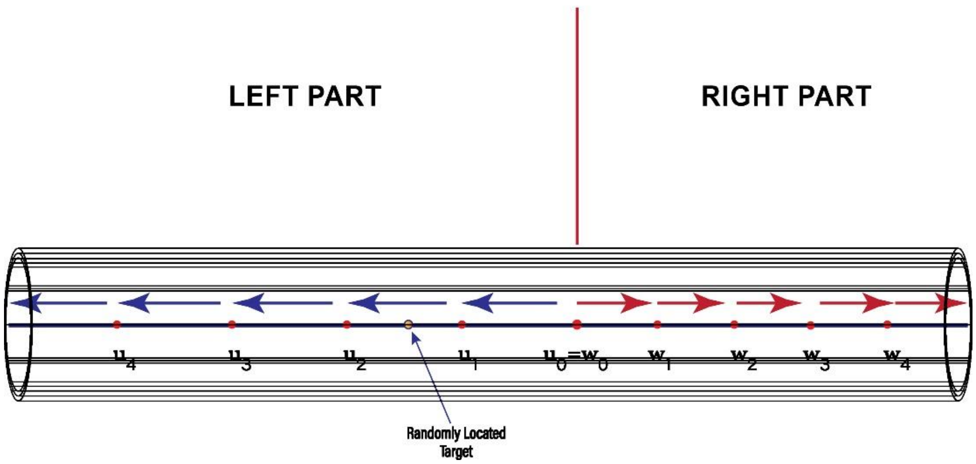

Fig. 2 Search Technique for Asymmetric Distribution for A Located Target (Cut) Inside One Cable.

2.1. The Searching Framework,

The search area is as follows: Axis of one cylinder (internet cable).

The objective: On one line (internet cable), the target is placed at random.

Methods of search: Two searches on the line were used to look for the missing target. The searchers begin their hunt for the target at the line's origins, using continuous pathways in one direction at equal speeds. Furthermore, the search spaces (internet cable) are separated by a wide range of distances.

2.2. The Searching technique.

The procedure of locating a cut internet cable in this model is based on the first sensor R_1 , it scans the area of the right part of the cable and bounded by W. Also, the second sensor R_2, it scans the area of the left part of the cable and bounded by U. At the origin point, there is a control center, it receives external signals from the sensors. There are two forms of detection:

1- Correct detection: The target has been detected in the area set for scanning on the cable, according to one of the sensors, and then emits positive signals to the control center.
2- False detection: One of the sensors announces that the target has not been detected in the area specified for scanning on the cable, and then emits negative signals to the control center.

A target is assumed to be randomly located on the cable (line) L, and let X be the position of the target (internet cable cut), which has a certain asymmetrical distribution .Therefore we assume that $|u_i| \neq |w_i|, i \geq 0$, where w_i and u_i are distances scanned by searchers by R_1 and R_2 respectively to detect the lost target as quickly as possible, where the searchers have equal speeds. Suppose that the location of the missing target has different probabilities at any point in the interval $[u, w]$, it has a certain (CDF) $F(x)$, and its corresponding (PDF) is $f(x)$.

The search process will be done according to the following procedure:

Step 1: The search for the missing target begins at the origin point $|u_0| = |w_0| = 0$ (the control center location) as an asymmetric coordination process between the two searchers R_1 and R_2 where the searcher R_1 searches for the lost target on the right (positive) part of L until he reaches the point w_1. At the same time the searcher R_2 searches for the lost target on the left (negative) part of L until he reaches the point u_1.But the searchers move at different distances. If $u_1 \geq w_1$, then the control center receive the signs of the searchers after R_2 reaches to u_1, and if $w_1 \geq u_1$, then the control center receive the signs of the searchers after R_1 reaches to w_1. At this point, they report whether or not they have detected the target; if one of them finds it, they stop searching, but if not, they continue the following step.

Step 2: the searcher R_1 completes the search for the lost target on the right (positive) part of L and he moves from w_1 until he reaches the point w_2. At the same time the searcher R_2 searches for the lost target on the left (negative) part of L and he moves from u_1 until he reaches the point u_2. If $u_2 \geq w_2$, then the control center receive the signs of the searchers after R_2 reaches to u_2, and if $w_2 \geq u_2$, then the control center receive the signs of the searchers after R_1 reaches to w_2. At this moment they inform if they detect the target or no; if one of them finds it, they stop searching, but if not, they continue the following step.

Step 3: the searcher R_1 completes the search for the lost target on the right (positive) part of L and he moves from w_2 until he reaches the point w_3.Simultaneously, the searcher looks for the misplaced target on the left (negative) side of L. and he moves from u_2 until he reaches the point u_3 .If $u_3 \geq w_3$, then the control center receive the signs of the searchers after R_2 reaches to u_3, and if $w_3 \geq u_3$, then the control center receive the signs of the searchers after R_1 reaches to w_3. At this point, they report whether they have detected the target or not; if one of them has, they cease their search, and so on until one of them detect the lost target.

Let w and u are the search paths of the two searchers R_1 and R_2, respectively, where w consists of a sequence $\{w_i, i \geq 0\}$ .Also, u consists of a sequence $\{u_i, i \geq 0\}$,where I is nonnegative integer set .Assume the search technique be $\phi = (w, u) \in \Phi$, where Φ is the set of all search technics such that:

$$\phi = \left\{ \{w_i, u_i\}_{i \geq 0} : \cdots < u_3 < u_2 < u_1 < u_0 = w_0 < w_1 < w_2 < w_3 < \cdots \right\},$$

where, $w = \lim_{i \to \infty} w_i$ and $u = \lim_{i \to \infty} u_i$. All distances w_1, w_2, w_3,… and u_1, u_2, u_3 are successfully scanned by the searchers R_1 and R_2 until the lost target is found without returning to the origin point again.

Let $\nu(u,w)=1$, where $\nu(x_1,x_2)=F(x_2)-F(x_1)$. Let B_1 be the time for R_1 to reach to any specific point of search in the right part of L, and B_2 be the time for R_2 to reach to any specific point of search in the left part of L. the $B(\phi)$ be the time for the searchers if one of them has found the target without going back to the origin point.

In the following theorem, we assume that the target is distributed asymmetrically on both sides of the origin point, so we find $B_1 > B_2$ or $B_2 > B_1$.

Theorem 1: The time that the searchers should expect to spend discovering the objective without returning to the starting place., until one of them detects the lost target is given by:

$$E\left[B(\phi)\right]=\sum_{i=1}^{\infty}(w_i-u_i)\left[\nu(w_{i-1},w_i)-\nu(w_i,w_{i+1})+\nu(u_i,u_{i-1})-\nu(u_{i+1},u_i)\right] \quad (1)$$

Proof: If $B_1 > B_2$, in this case we take B_1 as a greater time of detecting the target as the following:

If the target finds out in $]0,w_1]$, then $B_1=(w_1-w_0)$,

If the target finds out in $]w_1,w_2]$, then $B_1=(w_1-w_0)+(w_2-w_1)$,

If the target finds out in $]w_2,w_3]$, then $B_1=(w_1-w_0)+(w_2-w_1)+(w_3-w_2)$,

If the target finds out in $]w_3,w_4]$, then $B_1=(w_1-w_0)+(w_2-w_1)+(w_3-w_2)+(w_4-w_3)$

and so on.

If the target finds out in $[u_1,0[$, then $B_1=w_1-w_0$,

If the target finds out in $[u_2,u_1[$, then $B_1=(w_1-w_0)+(w_2-w_1)$,

If the target finds out in $[u_3,u_2[$, then $B_1=(w_1-w_0)+(w_2-w_1)+(w_3-w_2)$,

If the target finds out in $[u_4,u_3[$, then $B_1=(w_1-w_0)+(w_2-w_1)+(w_3-w_2)+(w_4-w_3)$,

and so on.

If $B_2 > B_1$, in this case we take B_2 as a greater time of detecting the target as the following:

If the target finds out in $]0, w_1]$, then $B_2 = -(u_1 - u_0)$,

If the target finds out in $]w_1, w_2]$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1))$,

If the target finds out in $]w_2, w_3]$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2))$,

If the target finds out in $]w_3, w_4]$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2) + (u_4 - u_3))$,

and so on.

If the target finds out in $[u_1, 0[$, then $B_2 = -(u_1 - u_0)$,

If the target finds out in $[u_2, u_1[$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1))$,

If the target finds out in $[u_3, u_2[$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2))$,

If the target finds out in $[u_4, u_3[$, then $B_2 = -((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2) + (u_4 - u_3))$,

and so on.

$$
\begin{aligned}
E\left[B(\phi)\right] = &(w_1 - w_0)\nu(0, w_1) + \left((w_1 - w_0) + (w_2 - w_1)\right)\nu(w_1, w_2) \\
&+ \left((w_1 - w_0) + (w_2 - w_1) + (w_3 - w_2)\right)\nu(w_2, w_3) + \cdots \\
&+ (w_1 - w_0)\nu(u_1, 0) + \left((w_1 - w_0) + (w_2 - w_1)\right)\nu(u_2, u_1) \\
&+ \left((w_1 - w_0) + (w_2 - w_1) + (w_3 - w_2)\right)\nu(u_3, u_2) + \cdots \\
&- (u_1 - u_0)\nu(0, w_1) - \left((u_1 - u_0) + (u_2 - u_1)\right)\nu(w_1, w_2) \\
&- \left((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2)\right)\nu(w_2, w_3) - \cdots \\
&- (u_1 - u_0)\nu(u_1, 0) - \left((u_1 - u_0) + (u_2 - u_1)\right)\nu(u_2, u_1) \\
&- \left((u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2)\right)\nu(u_3, u_2) - \cdots \\
= &(w_1 - w_0)[\nu(0, w_1) + \nu(w_1, w_2) + \cdots + \nu(u_1, 0) + \nu(u_2, u_1) + \cdots]
\end{aligned}
$$

$$+(w_2-w_1)[\nu(w_1,w_2)+\nu(w_2,w_3)+\cdots+\nu(u_2,u_1)+\nu(u_3,u_2)+\cdots]$$

$$+(w_3-w_2)[\nu(w_2,w_3)+\nu(w_3,w_4)+\cdots+\nu(u_3,u_2)+\nu(u_4,u_3)]+\cdots]+\cdots$$

$$-(u_1-u_0)[\nu(0,w_1)+\nu(w_1,w_2)+\cdots+\nu(u_1,0)+\nu(u_2,u_1)+\cdots]$$

$$-(u_2-u_1)[\nu(w_1,w_2)+\nu(w_2,w_3)+\cdots+\nu(u_2,u_1)+\nu(u_3,u_2)+\cdots]$$

$$-(u_3-u_2)[\nu(w_2,w_3)+\nu(w_3,w_4)+\cdots+\nu(u_3,u_2)+\nu(u_4,u_3)]+\cdots]+\cdots$$

$$=(w_1-w_0)+(w_2-w_1)\left[1-\nu(u_1,w_1)\right]+(w_3-w_2)\left[1-\nu(u_2,w_2)\right]+\cdots$$

$$-(u_1-u_0)-(u_2-u_1)\left[1-\nu(u_1,w_1)\right]-(u_3-u_2)\left[1-\nu(u_2,w_2)\right]-\cdots$$

$$=\sum_{i=0}^{\infty}(w_{i+1}-w_i)[1-\nu(u_i,w_i)]-(u_{i+1}-u_i)[1-\nu(u_i,w_i)].$$

In the next section, we'll go over the conditions that must be met in order to acquire the best search method.

2.3 The Optimal Search Strategy for Detecting the Target

We can get the different values of $E\left[B(\phi)\right]$ by substituting in (1) in our problem, by using different and many search strategy. Let $\phi_1=\left(w^1,u^1\right)$, where $w^1=\{w_1=m_{11},w_2=m_{12},w_3=m_{13},\ldots\}$, then we can calculate $E\left[B(\phi_1)\right]$ from (1).Also, put $\phi_2=\left(w^2,u^2\right)$, where $w^2=\{w_1=m_{21},w_2=m_{22},w_3=m_{23},\ldots\}$, then we can get $E\left[B(\phi_2)\right]$. Consider $\phi_3=\left(w^3,u^3\right)$, where $w^3=\{w_1=m_{31},w_2=m_{32},w_3=m_{33},\ldots\}$, then we can calculate $E\left[B(\phi_3)\right]$ and so on, where $m_{11},m_{12},m_{13},\ldots,m_{21},m_{22},m_{23},\ \ldots,m_{31},m_{32},m_{33},\ldots$ are constants which given by the searchers on the both different sides. Hence the optimal search expected time value is determined by $E^*\left[B\left(\phi^*\right)\right]$, where

$$E^*\left[B\left(\phi^*\right)\right]=\inf\left\{E\left[B(\phi_1)\right],E\left[B(\phi_2)\right],E\left[B(\phi_3)\right],\ldots\right\}.$$

So, we must compute all important values of the expected value of time in order to obtain the ideal expected value of time. Then, as the following theorem shows, we choose the best of them.

Theorem 2: Suppose F(x) is a differentiable function for a continuous random variable X, with a density function f(x). $\phi=(w,u)$ is an optimal search strategy if

$$w_{i+1}=\frac{\nu(u_i,w_i)-\nu(u_{i-1},w_{i-1})+(w_i-u_i)\,f(w_i)}{f(w_i)}+u_{i+1}, \tag{2}$$

and

$$u_{i+1}=-\frac{\nu(u_i,w_i)-\nu(u_{i-1},w_{i-1})+(w_i-u_i)\,f(w_i)}{f(u_i)}+w_{i+1}, \quad i\geq 1 \tag{3}$$

Proof: We can get from (1) the following

$$B[(\phi)]=(w_1-w_0)+(w_2-w_1)[1-\nu(u_1,w_1)]+(w_3-w_2)[1-\nu(u_2,w_2)]+\cdots$$
$$-(u_1-u_0)-(u_2-u_1)[1-\nu(u_1,w_1)]-(u_3-u_2)[1-\nu(u_2,w_2)]-\cdots,$$

then

$$\frac{\partial B[(\phi)]}{\partial w_1}=(w_1-w_2)\,f(w_1)+\nu(u_1,w_1)+(u_2-u_1)\,f(w_1)=0,$$

which leads to

$$w_2=\frac{\nu(u_1,w_1)+(w_1-u_1)f(w_1)}{f(w_1)}+u_2\,.$$

Also,

$$\frac{\partial B[(\phi)]}{\partial w_2}=(w_2-w_3)\,f(w_2)+\nu(u_2,w_2)-\nu(u_1,w_1)+(u_3-u_2)\,f(w_2)=0,$$

due to

$$w_3=\frac{\nu(u_2,w_2)-\nu(u_1,w_1)+(w_2-u_2)\,f(w_2)}{f(w_2)}+u_3,$$

and so on, we can get:

$$w_{i+1}=\frac{\nu(u_i,w_i)-\nu(u_{i-1},w_{i-1})+(w_i-u_i)\,f(w_i)}{f(w_i)}+u_{i+1}, i\geq 1.$$

Remark we can prove relation (3) in terms of u_{i+1} by similar way.

Theorem 3: Suppose Φ is the set of all search techniques and if $\phi\in\Phi$ represents the optimal strategy, hence

$$f(w_{i+1}) < f(w_i) \tag{4}$$

Proof: According to the optimal search strategy definition and from (1), (2) and (3) we can find the following

$$f(w_1) = \frac{\nu(u_1, w_1)}{(w_2 - w_1) - (u_2 - u_1)},$$

$$f(w_2) = \frac{\nu(u_2, w_2) - \nu(u_1, w_1)}{(w_3 - w_2) - (u_3 - u_2)},$$

and

$$f(w_3) = \frac{\nu(u_3, w_3) - \nu(u_2, w_2)}{(w_4 - w_3) - (u_4 - u_3)},$$

science

$$\nu(u_1, w_1) > \nu(u_2, w_2) - \nu(u_1, w_1) > \nu(u_3, w_3) - \nu(u_2, w_2),$$

and

$$(w_2 - w_1) - (u_2 - u_1) < (w_3 - w_2) - (u_3 - u_2) < (w_4 - w_3) - (u_4 - u_3).$$

Hence,

$$\frac{\nu(u_3, w_3) - \nu(u_2, w_2)}{(w_4 - w_3) - (u_4 - u_3)} < \frac{\nu(u_2, w_2) - \nu(u_1, w_1)}{(w_3 - w_2) - (u_3 - u_2)} < \frac{\nu(u_1, w_1)}{(w_2 - w_1) - (u_2 - u_1)},$$

and this leads to

$$f(w_3) < f(w_1), \quad \text{then} \qquad f(w_{i+1}) < f(w_i).$$

Hint: equation (4) can be written in terms of $(u_{i+1}), i \geq 1$, by using equation (3) as the following:

$$f(u_{i+1}) < f(u_i). \tag{5}$$

If we look closely, we note that w_{i+1} is a function of $(u_{i+1}), i \geq 1.$ This means that each new optimal distance depends on the previous distances on the right and left sides. Thus, we find the relationship is simplified by the existence of a function ξ_{i+1}, where $w_{i+1} = \xi_{i+1}(u_{i+1}), i \geq 1,$ that connects the new distance to the previous distances on both sides. Hence, The optimal estimated time for one of the searchers to locate the target without returning to the origin location is given by

$$E^*\left[B\left(\phi^*\right)\right]=\inf\left\{E\left(D\left(\phi(\xi_{i+1}\left(u_{i+1}\right),i\geq 1)\right)\right\}. \quad (6)$$

We can get $\xi_{i+1}\left(u_{i+1}\right), i\geq 1,$ for different and varied values of u_{i+1}. Also, we can obtain the optimal value by the following algorithm.

3. ALGORITHM

To facilitate the process of reaching the desired goal in the ideal time, without returning to the starting position, one of the searchers finds the missing target. we'll go over the algorithm for computing the optimal predicted time for locating the object. So, the optimal points generated $w_{i+1}, i\geq 1$ must be determined by the searchers. The following stages can be used to review the algorithm procedures

procedure 1: Input the following values w_1 is the first distance that will be scan by the first sensor R_1 to detect the target in the positive part of the cable L.

u_1 is the first distance that will be scan by the second sensor R_2 to locate the target in the cable's negative section

U, W are the bounds of L.

procedure 2: Generate w_{i+1}, where $w_{i+1}=\xi_{i+1}\left(u_{i+1}\right), i\geq 1,$ from (2).

procedure 3: Apply the condition to achieve the relationship $w_i < w_{i+1} < W$. Proceed to step 4 if you are satisfied. Continue to step 5 after stopping the process, if you're not already there.

procedure 4: Proceed to step 5 after calculating $E\left[B(\phi)\right]$ from (1)**.**

procedure 5: Re-entre new values for $\left(u_{i+1}\right), i\geq 1$, and repeat steps 1 through 4 until E* is computed from (6), and then proceed to step 6.

procedure 6: End (stop).

4. Case Study

Assume that the position of a cut is randomly located in the cable net under the sea follows the truncated exponential distribution, with the probability density function

$$f(x) = \lambda e^{-\lambda(x-q)}, x \geq q, q < 0,$$

and distribution function

$$F(x) = \begin{cases} 1 - e^{-\lambda(x-q)}, & x \geq q, \\ 0, & x < q, \end{cases}$$

Suppose $u_0 = w_0 = 0, \lambda = 1,\ U = -4,\ W = 6,\ w_1 = 0.3,\ u_1 = -0.5,\ u_2 = -1,\ u_3 = -1.5$ and $u_4 = -3$.

We can find the value of q from the following relation

$$\int_{-4}^{6} \left[\lambda e^{-\lambda(x-q)} \right] dx = 1$$

$$= \left[e^{4+q} - e^{-6+q} \right] = 1$$

$$= \left[e^{4} - e^{-6} \right] = e^{-q},$$

hence, $q = -10$, from (2) we can get ,

$$w_2 = \frac{v(u_1, w_1) - v(u_0, w_0) + (w_1 - u_1) f(w_1)}{f(w_1)} + u_2,$$

$$w_2 = \frac{e^{-9.5} - e^{-10.3} + (0.8) f(0.3)}{f(0.3)} - 1,$$

$$w_2 = 1.02553.$$

Also, from (2) we can generate new values of w_{i+1}, as a following

$$w_3 = \frac{e^{-9} - e^{-11.02553} + (2.02553) f\left(1.6280 * 10^{-5}\right)}{f\left(1.6280 * 10^{-5}\right)} - 1.5,$$

hence,

$$w_3 = 4.5738870.$$

Also,

$$w_4 = \frac{e^{-8.5} - e^{-14.5738870} + (2.02553)\, f\left(4.68426*10^{-7}\right)}{f\left(1.6280*10^{-5}\right)} - 3,$$

hence,

$$w_4 = 207.739808.$$

We note that the value $w_4 > W$ and it is impossible, and this leads us to search on the cut cable is bounded by $w_1, w_2, w_3, u_1, u_2, u_3, u_4 \in [U, W]$.

5. DISCUSSION

In this section, the proposed approach is compared with some existing literature to illustrate the advantages of the proposed approach. Table 1 investigates this comparison in the case of some parameters.

Table 1. Comparison of different researcher's contributions

Author's name	Paper's Title	Interactive approach	Efficient solution	Best compromise solution	Fuzzy goal programming	Environment
W.Afifi and EL-Bagoury, 2021	Optimal Multiplicative Generalized Linear Search Plane for ADiscrete Randomly Located Target	Detected the lost target with out saving time of search	√	×	×	Fuzzy set
Mohamed, A. A and W.Afifi 2019	Quasi Coordinated Search for a Randomly Moving Target	Long time search plan	×	√	√	Crisp set
Mohamed, A. A et al., 2007	On the Coordinated search problem	Long time search plan	√	×	×	Fuzzy set
Diana J. Reyniers. 1995	Coordinated two searchers for an object hidden on an interval	Usual search with long time	√	×	×	Fuzzy set
Proposed approach		√	√	√	×	Fuzzy Set

6. MAIN RESULTS AND FUTURE WORKS

In this paper, A novel study technique has been proposed to help with the optimization of a time-saving coordination search technique for a randomly situated target. From the beginning of the line, two searchers or robots begin looking for the hidden target, and each searcher wants to find the target in his or her own segment of the line. The expected time to detect the target has calculated. The required conditions for the best search technique have been met. An approximation algorithm that facilitates searching procedures for searchers was presented. Our contribution involved:

1. Existence of optimal time-saving coordination search technique for randomly located target on the line has been obtained, see Theorem 1 and 2.
2. The origin point $|u_0| = |w_0| = 0$ is the starting point of the sensor's movement.
3. We avoided wasting time for searching a goal with continuous search process, and the sensors not returning to the origin point.
4. The necessary conditions which helped us to calculate the optimal search strategy has been presented, see Theorem 3.

Future work could include extending this research to other fuzzy-like structures (e.g., interval-valued fuzzy sets), Neuromorphic set, Pythagorean fuzzy set, Spherical fuzzy set etc. with more discussion and suggestive comments. In addition, we can study the existence of optimal time-saving coordination search.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] A.B. El-Rayes, A. Mohamed, H. Fergani, On the generalized linear search problem, Delta J. 6 (1993), 1-10.

[2] A. Mohamed, H.M.A. Gabal, Generalized optimal search paths for a randomly located target, in: Annual Conference, Cairo ISSR, 35, Math. Statistics (2000), 17-19.

[3] A. Mohamed, H.M.A. Gabal, Linear search with multiple searchers for a randomly moving target, in: International Conference for Statistics, Computer Science and Its Application, Egypt (2003), 115-124.

[4] A. Mohamed, H.M.A. Gabal, Multiplicative linear search problem, Egypt. Stat. J. 48 (2004), 34-45.

[5] D.J. Reyniers, coordinated two searchers for an object hidden on an interval, J. Oper. Res. Soc. 46 (1995), 1386-1392.

[6] D.J. Reyniers, Coordinated search for an object on the line, Eur. J. Oper. Rese. 95 (1996), 663-670.

[7] A. Mohamed. M. Teamah, H.M.A. Gabal, W.A. Afifi, On the coordinated search problem, Int. J. Appl. Math. 5 (2007), 627-636

[8] A. Mohamed, H. Abou Gabal, M. El-Hadidy, Coordinated search for a randomly located target on the plane, Eur. J. Pure Appl. Math, 2 (2009), 97-111.

[9] A. Mohamed, H.A. Gabal, W. Afifi, On the coordinated search problem, Int. J. Appl. Math. 5 (2007), 627-636.

[10] A. Mohamed, H. Fergany, M. El-Hadidy, On the coordinated search problem on the plane, J. Sch. Bus. Admin., Istan. Univer. 41 (2012), 80-102.

[11] A. Mohamed, H.A. Gabal, W. Afifi, On the coordinated search problem, Int. J. Appl. Math. 5 (2007), 627-636.

[12] A.A. Mohamed, H.M. Abou-Gabal, W.A. Afifi, Double coordinate search problem, Int. J. Contemp. Math. Sci. 8 (2013), 5-8.

[13] A.A. Mohamed, H.M. Abou-Gabal, W.A. Afifi, Generalized coordinated search for a randomly located target, Delta J. 38 (2016),12-22.

[14] A.A. Mohamed, W. Afifi, Quasi coordinated search for a randomly moving target, J. Appl. Phys. 7 (2019), 1814-1826.

[15] W. Afifi, A.H. EL-Bagoury, S.N. AL-Aziz, A novel search algorithm for a multi searchers random walk, J. Applied Math. Inform. Sci. 14 (2020), 115-122.

[16] W. Afifi, A.H. EL-Bagoury, S.N. AL-Aziz, et al. Optimal discrete search for a randomly moving COVID19, J. Stat. Appl. Probab. 9 (2020), 473-481.

[17] W. Afifi, A.H. EL-Bagoury, M.A. Kassem, et al. On minimum expected search time algorithm for 3-dimensional randomly located target, J. Stat. Appl. Probab. 10 (2021), 159-166.

[18] W. Afifi, A.A.H. EL-Bagoury, Optimal multiplicative generalized linear search plane for a discrete randomly located target, Inform. Sci. Lett. 10 (2021), 153-158.

[19] S.N. Al-Aziz, A.A.H. EL-Bagoury, W. Afif, A coordinated search algorithm for a lost target on the plane, J. Computer Syst. Sci. Eng. 38 (2022), 127-137.