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# MIXED $\gamma-$FUZZY SETS: TOPOLOGICAL STRUCTURES AND PROPERTIES

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**Abstract.** In the field of fuzzy mathematics, mixed $\gamma-$fuzzy sets, which include both conventional fuzzy sets and intuitionistic fuzzy sets, have become an effective tool. The extensive uses of mixed $\gamma-$fuzzy sets in the field of mixed fuzzy topological spaces are explored in this study. The study begins by providing a comprehensive introduction to mixed $\gamma-$fuzzy sets, elucidating their essential characteristics and operations. Key concepts such as mixed $\gamma-$fuzzy neighborhoods, mixed $\gamma-$fuzzy closure, and mixed $\gamma-$fuzzy interior are introduced and analyzed, providing a powerful tool for handling uncertainty in topological spaces.

**Keywords:** topological spaces; mixed $\gamma$ fuzzy; fuzzy $\gamma$-open set.

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## 1. INTRODUCTION

The investigation of fuzzy sets theory has developed over the past ten years, spurring the development of innovative applications in mathematics, computer science, engineering, and decision-making processes, among others. The introduction of mixed$\gamma$-fuzzy sets, that extends fundamental concept of fuzzy sets through introducing graded membership and non-membership values, is a significant development in artificial intelligence, is a notable extension of this theory. These mixed $\gamma$-fuzzy sets provide a more flexible and expressive framework for

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modelling uncertainty and imprecision in realistic situations. Concurrently, the study of topological spaces has been enriched by the introduction of fuzzy topological spaces, which permit a more nuanced and adaptable exploration of spatial relationships. The combination of mixed -fuzzy sets and fuzzy topological spaces yields a potent paradigm known as "Mixed Fuzzy Topological Spaces" (MFTS), which provides an efficient method for addressing problems involving vagueness, ambiguity, and ambiguous spatial relationships.

This study investigates the fascinating domain of mixed fuzzy sets within the context of mixed fuzzy topological spaces. It investigates both the theoretical foundations and, more importantly, the practical implementations of this hybrid framework in a variety of domains. By combining the benefits of mixed -fuzzy sets with the adaptability of fuzzy topological spaces, it is possible to devise novel solutions to complex problems where traditional set theory and topology may fail to capture the inherent uncertainties and imprecisions of the real world.

Mixed $\gamma-$fuzzy sets represent a significant extension of traditional fuzzy sets, integrating both graded and tentative membership.It then proceeds to establish the foundations of mixed fuzzy topological spaces, emphasizing the integration of mixed $\gamma-$fuzzy sets into this framework. Several illustrative examples are presented, showcasing how mixed $\gamma-$fuzzy sets can effectively model and solve real-world problems.

## 2. LITERATURE REVIEW

N. R. Das and P. C. Baishya [3] combined two provided topologies on a set X to form a fuzzy topology known as a mixed fuzzy topology, and then examined its various properties. They did this by employing the closure of one topology's neighbours with respect to the other topology.Benchalli and Jenifer [4] defined fuzzy -open set and fuzzy -continuous transformations.

GC Ray et. al. (2022) [9] studied that the fuzzy topological space $(X, \tau 1(\tau 2))$ investigate a few properties of countability utilising the principles of quasi-coincidence in mixed fuzzy topological spaces and construct a countable basis for $(\tau 1(\tau 2)$, from a measurable basis for $\tau 2$

M Singh et. al. (2022) [10] defined that Second countability, countably compactness, as well as Lindel's defence of bipolar fuzzy topological spaces. Additionally defined is a division of a specific bipolar fuzzy topological space cover. Examine some of its features after defining the ultimate bipolar fuzzy topological space in a family of bipolar fuzzy topological spaces.

S Sivashanmugaraja et. al. (2021) [11] studied that there are several restrictions or limitations on the classes of topological spaces that can be addressed in topology and its allied fields of mathematics. The separation axioms specify a number of these constraints and restrictions. Additionally, investigate and assess the pre-separation axioms in fuzzy topological spaces. And also introduced pre-homeomorphism and pre-homeomorphism in ambiguous topological spaces. In addition, demonstrate few of these separation axioms' fundamental properties and theorems in fuzzy contexts.

MB Khan et. al. (2021) [12] defined that Convex and non-convex fuzzy models are generally known to be crucial to the study of fuzzy optimisation. Due to the way it is defined, convexity is essential to the study of inequality. Symmetry and convexity are notions that go hand in hand.

D Jardón et. al. (2020) [13] studied that Consider a fundamental issue in hyperspace theory: the connection between a particular dynamical system $f:(X,d)\to(X,d)$ and the expansion of it to the universe, specifically the weakly mixing, transitive properties, and point-transitivity. This problem is posed by a metric space $(X,d)$.

YKM Altalkany et. al. (2020) [14] defined that Through the characteristics and properties of the ideal, it is also possible to provide a novel definition for the neighbourhood of a particular point; however, these neighbourhoods do not necessarily contain the point. In addition to introducing a new definition of the local function by employing both the proximity relation and the concept of the specified neighbourhoods, the authors presented the most significant results and their properties.

## 3. PRELIMINARIES

Fuzzy topological spaces (fts) are represented throughout by $(X,\tau X),(Y,\tau Y)$, and $(Z,\tau Z)$ (or $X,Y$, and $Z$, respectively). By $0\,\&\,1$, we represent the constant fuzzy sets that assume the values $0\,\&\,1$ on $X$, respectively. We will now review the paper's definitions and findings.

**Definition 3.1.** Consider $\mu$ to be an imprecise set in fts $X$. Then the pre-$\gamma$-interior of $\mu$ is defined as $pint_\gamma(\mu)=\vee\{\lambda:\lambda\le\mu,\lambda\in FP_\gamma O(X)\}$ and the pre-$\gamma$-closure of $\mu$ is defined as $pcl_\gamma(\mu)=\wedge\{\lambda:\lambda\ge\mu,\lambda\in FP_\gamma C(X)\}$.

**Definition 3.2.** *A mapping $\theta:(X,\tau X))\to(Y,\tau Y)$ is called*

*(1) fuzzy continuous, if the inverse of an open fuzzy set is also fuzzy $(Y,\tau Y)$ is an open fuzzy set in $(X,\tau X)$.*

(2) *fuzzy pre- $\gamma$ -continuous if the inverse image of an open fuzzy set lies in the continuous domain $(Y, \tau Y)$ is a pre-$\gamma$-open fuzzy set in $(X, \tau X)$).*

(3) *fuzzy pre $^* - \gamma$-continuous (or fuzzy pre-$\gamma$-irresolute), if the inverse image of a pre-$\gamma$-open fuzzy set in is a pre-$\gamma$-closed fuzzy set, then $(Y, \tau Y)$ is a pre-$\gamma$-open fuzzy set in $(X, \tau X)$.*

**Definition 3.3.** *A mapping $\theta : (X, \tau X)) \to (Y, \tau Y)$ is called*

(1) *fuzzy pre-$\gamma$-open, if $\theta(\mu)$ is a pre-$\gamma$-open fuzzy set in $(Y, \tau Y)$, for every open fuzzy set $\mu$ in $(X, \tau X)$.*

(2) *fuzzy pre-$\gamma$-closed, if $\theta(\mu)$ is a pre-$\gamma$-closed fuzzy set in $(Y, \tau Y)$, for every closed fuzzy set $\mu$ in $(X, \tau X)$.*

**Definition 3.4.** *A fuzzy topological space $X$ is said to be fuzzy $T_0$ space if for every pair of fuzzy singletons $x_\alpha, x_\beta$ with various supports, there exist a fuzzy open set $\mu$ such that $x_\alpha \leq \mu \leq x^c_\beta$ or on some type of pre-$\gamma$-Separation Axioms $x_\beta \leq \mu \leq x^c_\alpha$.*

**Definition 3.5.** *A fuzzy singleton is a fuzzy set of a fuzzy topological space $X$; it has a value of zero for all points $x$ in $X$ except one. A fuzzy singleton whose value is 1 is known as a precise fuzzy singleton.*

**Definition 3.6.** *All fuzzy pre-$\tau$- open sets are fuzzy open sets if $(X, \tau X)$ is a fuzzy door and fuzzy-$\tau$ -regular space.*

## 4. FUZZY PRE-$\gamma$-HOMEOMORPHISM MAPPINGSS

This section explains pre-$\gamma$-homeomorphism and pre*-$\gamma$-homeomorphism in ambiguous contexts. Then, we examine the relationships between these homeomorphisms and demonstrate several theorems.

**Definition 4.1.** *A one-one and onto mapping $\theta : (X, \tau X)) \to (Y, \tau Y)$ is known as fuzzy pre-$\gamma$-homeomorphism, if $\theta$ and $\theta - 1$ are fuzzy pre-$\gamma$-continuous.*

**Definition 4.2.** *A one-one and onto mapping $\theta : (X, \tau X)) \to (Y, \tau Y)$ should be a fuzzy pre* -$\gamma$-homeomorphism if $\theta$ and $\theta - 1$ are fuzzy pre* -$\gamma$-continuous.*

**Theorem 4.1.** Let $(X,\tau X)$ and $(Y,\tau Y)$ are fuzzy door as well as fuzzy $\gamma$-regular spaces. A bijective mapping $\theta : (X,\tau X)) \to (Y,\tau Y)$ is fuzzy pre* -$\gamma$-homeomorphism, when and only if $\theta$ is fuzzy pre-$\gamma$-homeomorphism.

**Proof:** Let $\theta$ be a fuzzy pre* -$\gamma$-homeomorphism. We prove that $\theta$ is pre-$\gamma$ continuous. Since $\theta$ is fuzzy pre* -$\gamma$-homeomorphism, we obtain $\theta$ and $\theta - 1$ are fuzzy pre* -$\gamma$- continuous. Let $\mu$ be a fuzzy open set in $Y$. By hypothesis $Y$ is fuzzy door space as well as fuzzy $\gamma$-regular, we obtain $\mu$ is fuzzy pre-$\gamma$-open set in $Y$. Since $\theta$ is fuzzy pre* -$\gamma$-continuous, $\theta - 1(\mu)$ is fuzzy pre-$\gamma$-open set in $X$. Therefore, $\theta$ is pre-$\gamma$-continuous. Now we prove $\theta - 1$ is pre-$\gamma$-continuous. Let $(\theta - 1)(\mu)$ is fuzzy pre-$\gamma$-open set in $X$. Since $X$ is fuzzy door space and fuzzy $\gamma$-regular, we obtain $(\theta - 1)(\mu)$ is fuzzy open set in $X$. Since $\theta - 1$ is fuzzy pre* -$\gamma$-continuous, $\theta(\theta - 1(\mu)) = \mu$ is fuzzy pre-$\gamma$-open set in $Y$. Thus $\theta - 1$ is pre-$\gamma$-continuous. Hence $\theta$ is fuzzy pre-$\gamma$-homeomorphism. Converse is evident.

**Remark 4.1.** In fuzzy contexts, pre-$\gamma$-homeomorphism and pre* -$\gamma$ - homeomorphism are concepts. As demonstrated by the following example, homeomorphism is independent.

**Example 4.1.** Let

$$X = Y = \{a,b\} \ \&\ \lambda,\mu \in I^X$$

explained as

$$\lambda(a) = 1, \lambda(b) = 0; \mu(a) = 0, \mu(b) = 1.$$

Let

$$\tau X = \{1,0,\lambda\}$$

as well as

$$\tau Y = \{1,0,\mu\}$$

. Then $(X,\tau X)$ and $(Y,\tau Y)$ are fts.

Illustrate a fuzzy operation

$$\gamma : \tau X \to I^X$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\lambda) = \lambda$$

as well as also define a fuzzy operation

$$\gamma : \tau Y \to I^Y$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\mu) = \mu.$$

A mapping

$$\theta : (X, \tau X)) \to (Y, \tau Y)$$

defined by

$$\theta(a) = b, \theta(b) = a.$$

$\theta$ is Fuzzy pre-$\gamma$-homeomorphism but not fuzzy pre* -$\gamma$-homeomorphism. Since $\theta - 1 : Y \to X$ is not fuzzy pre* -$\gamma$-continuous.

**Example 4.2.** Let

$$X = Y = \{a, b\} \ \& \ \mu_1, \mu_2\mu_3, \mu_4\mu_5, \mu_6 \in I^X$$

stated as

$$\mu_1(a) = 0.4, \mu_1(b) = 0.5; \mu_2(a) = 0.1, \mu_2(b) = 0.4, \mu_3(a) = 0.5, \mu_3(b) = 0.6,$$

$$\mu_4(a) = 0.5, \mu_4(b) = 0.4, \mu_5(a) = 0.4, \mu_5(b) = 0.1, \mu_6(a) = 0.6, \mu_6(b) = 0.5.$$

Let

$$\tau X = \{1, 0, \mu_1, \mu_2, \mu_3\}$$

and

$$\tau Y = \{1, 0, \mu_4, \mu_5, \mu_6\}.$$

Then $(X, \tau X)$ as well as $(Y, \tau Y)$ are fts. Establish a fuzzy operation

$$\gamma : \tau X \to I^X$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\mu_1) = cl(\mu_1), \gamma(\mu_2) = \mu_2, \gamma(\mu_3) = \mu_3$$

also define a fuzzy operation

$$\gamma : \tau Y \to I^Y$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\mu_4) = cl(\mu_4), \gamma(\mu_5) = \mu_5, \gamma(\mu_6) = \mu_6.$$

Mapping

$$\theta : (X, \tau X) \to (Y, \tau Y)$$

defined by

$$\theta(a) = b, \theta(b) = a.$$

Then $\theta$is fuzzy pre* -$\gamma$-homeomorphism but not fuzzy pre-$\gamma$-homeomorphism. Since $\theta$ as well as $\theta - 1$ are not fuzzy pre-$\gamma$-continuous.

## 5. Fuzzy Pre-$\gamma$-Separation Axioms

**Definition 5.1.** *A fts $(X, \tau X)$ is referred to as fuzzy pre-$\gamma - T_0$ if there exists a pair of fuzzy singletons $x_\alpha$ and $x_\beta$ along different supports in $X$, there exists a fuzzy pre-$\gamma$-open set $\lambda$ such that either* $x_\alpha \leq \lambda \leq x^c_\beta$ or $x_\beta \leq \lambda \leq x^c_\alpha$.

**Example 5.1.** Let the $X = \{a, b\}$ as well as $\mu_1, \mu_2, \mu_3 \in I^X$ which are stated as

$$\mu_1(a) = 0, \mu_1(b) = 1; \mu_2(a) = 0.4, \mu_2(b) = 0; \mu_3(a) = 0.4, \mu_3(b) = 1;$$

Let the

$$\tau X = \{1, 0, \mu_1, \mu_2, \mu_3\}.$$

Then $(X, \tau X)$ is a fts. Define

$$\gamma : \tau X \to I^X$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\mu_1) = cl(\mu_1), \gamma(\mu_2) = \mu_2, \gamma(\mu_3) = \mu_3.$$

Then the fuzzy topological space $(X, \tau X)$ is fuzzy pre $-\gamma - T_0$.

**Theorem 5.1.** Fuzzy topological space $(X, \tau X)$ is fuzzy pre-$\gamma - T_0$ if it is fuzzy pre-$\gamma$ to $T_0$. The closures of any two crisp fuzzy singletons with differing supports are distinct.

**Proof:**Let $X$ be fuzzy pre $-\gamma - T_0$ and $x_\alpha, x_\beta$ be two crisp fuzzy singletons with two different supports $y_\alpha$ and $y_\beta$ respectively. By hypothesis $X$ is fuzzy pre$-\gamma - T_0$, there exists a pre-$\gamma$-open fuzzy set $\lambda$ such that $x_\alpha \leq \lambda \leq x^c_\beta$. Therefore $x_\beta \leq \lambda_c$. But $x_\beta \leq pcl_\gamma(x_\beta) \leq \lambda_c$, where $\lambda_c$ is pre-$\gamma$-closed fuzzy set. Now $x_\beta \leq \lambda_c$, this implies $x_\alpha \leq pcl_\gamma(x_\beta)$. But $x_\alpha \leq pcl_\gamma(x_\alpha)$. Thus $pcl_\gamma(x_\alpha) \neq pcl_\gamma(x_\beta)$.

Conversely, consider $x_\alpha, x_\beta$ are any two fuzzy singletons with different supports $y_\alpha, y_\beta$ respectively. Let $z_\alpha, z_\beta$ are two crisp fuzzy singletons such that $z_\alpha(y_\alpha) = 1$ and $z_\beta(y_\beta) = 1$. Since

$z_\alpha \le pcl_\gamma(z_\alpha)$, we obtain $(pcl_\gamma(z_\alpha))^c \le z^c_\alpha \le x^c_\alpha$. Since each crisp fuzzy singleton is pre-$\gamma$-closed fuzzy set, $(pcl_\gamma(z_\alpha))c$ is pre-$\gamma$-open fuzzy set and $x_\beta \le (pcl_\gamma(z_\alpha))^c \le x^c_\alpha$. Thus $X$ is fuzzy pre-$\gamma$-$T_0$.

**Definition 5.2.** *A fts $(X, \tau X)$ is known as fuzzy pre-$\gamma$-$T_1$ if for every pair of fuzzy singletons $x_\alpha$,$x_\beta$ with various supports $y_\alpha, y_\beta$, there exists fuzzy pre-$\gamma$-open sets $\lambda, \mu$ like $x_\alpha \le \lambda \le x^c_\beta$ and $x_\beta \le \mu \le x^c_\alpha$ .*

**Theorem 5.2.** Fuzzy topological space $X$ is fuzzy pre-$\gamma$-$T_1$ if and only if each crisp fuzzy singleton is a pre-processed-$\gamma$-Closed fuzzy set.

**Proof:**Let $X$ be fuzzy pre-$\gamma$-$T_1$ and $x_\alpha$ be a precise fuzzy singleton supporting $y$. For any supportable $y_\alpha$ fuzzy singleton be crisp fuzzy singleton $y$ fuzzy pre-$\lambda$-open sets $\lambda$ and $\mu$ satisfying the following conditions: $x_\alpha \le \lambda \le x^c_\beta$ and $x_\beta \le \mu \le x^c_\alpha$. Therefore, $x^c_\alpha = x_\beta$.Hence $x^c_\alpha$ is pre-$\gamma$-open fuzzy set. Thus, $x_\alpha$ is a pre-$\gamma$-closed fuzzy set.

Conversely, let $x_\alpha$ and $x_\beta$ are any pair of fuzzy singletons with different supports $y_\alpha$,$y_\beta$ respectively. Let $z_\alpha$,$z_\beta$ be two crisp fuzzy singletons with different supports $y_\alpha, y_\beta$ such that $z_\alpha(y_\alpha) = 1$ and $z_\beta(y_\beta) = 1$. Since each crisp fuzzy singleton is fuzzy pre-$\gamma$-closed set in $X$, $z^c_\alpha$ and $z^c_\beta$ are fuzzy pre-$\gamma$-open sets such that $x_\alpha \le z^c_\alpha \le x^c_\beta$ and $x_\beta \le z^c_\beta \le x^c_\alpha$. Thus $X$ is fuzzy pre-$\gamma$-$T1$.

**Example 5.2.** Let the $X = \{a,b\}$ and $\lambda, \mu, \eta \in I^X$ illustrated as

$$\lambda(a) = 0.3, \lambda(b) = 0;$$

$$\mu(a) = 0, \mu(b) = 1;$$

$$\eta(a) = 0.3, \eta(b) = 1;$$

Let

$$\tau X = \{1, 0, \lambda, \mu, \eta\}.$$

The Space $(X, \tau X)$is a fts. Define

$$\gamma : \tau X \to I^X$$

by

$$\gamma(1) = 1, \gamma(0) = 0, \gamma(\lambda) = \lambda, \gamma(\mu) = cl(\mu), \gamma(\eta) = \eta.$$

Then $(X, \tau X)$ is fuzzy pre-$\gamma$-$T_0$ but not fuzzy pre-$\gamma$-$T_1$.

**Definition 5.3.** *Fts $(X,\tau X)$ is often referred to as hazy pre-$\gamma$-strong the fuzzy set $T_1$ ifand only if fuzzy singleton is pre-$\gamma$-closed.*

**Remark 5.1.** Each fuzzy pre-$\gamma$-strong $T_1$ space is fuzzy pre-$\gamma$-$T_1$.

**Example 5.3.** Let the

$$X=\{a,b\},\lambda,\mu,\eta,\nu\in I^X$$

stated as

$$\lambda(a)=0.4,\lambda(b)=0;\mu(a)=0.2,\mu(b)=0.8;$$

$$\eta(a)=0.2,\eta(b)=0;\nu(a)=0.4,\nu(b)=0.8;$$

Let

$$\tau X=\{1,0,\lambda,\mu,\eta,\nu\}.$$

Then $(X,\tau X)$ is a fts. Define

$$\gamma:\tau X\to I^X$$

by

$$\gamma(1)=1,\gamma(0)=0,\gamma(\lambda)=\lambda,\gamma(\mu)=\mu,\gamma(\eta)=cl(\eta),\gamma(\nu)=\nu.$$

The Space $(X,\tau X)$ is fuzzy pre-$\gamma-T_1$ but not fuzzy pre-$\gamma$-strong $T_1$. Because fuzzy singletons $\lambda$ and $\eta$ aren't pre-closed fuzzy sets.

**Definition 5.4.** *A fuzzy topological space $(X,\tau X)$ is referred to as fuzzy pre-$\gamma$-Hausdorffor fuzzy pre -$\gamma$-$T_2$ if it consists of a pair of fuzzy singletons $x_\alpha,x_\beta$ with different supports, there exits fuzzy pre-$\gamma$-open sets $\lambda,\mu$ like $x_\alpha\leq\lambda\leq x^c_\beta,x_\beta\leq\mu\leq x^c_\alpha$ and $\lambda\leq\mu^c$.*

**Theorem 5.3.** A fuzzy topological space $(X,\tau X)$ is fuzzy pre-$\gamma$-$T_2$ if a pair of fuzzy singletons with different supports, there exist a pre-$\gamma$-open fuzzy set $\lambda$ such that $x_\alpha\leq\lambda\leq pcl_\gamma(\lambda)\leq x^c_\beta$.

**Proof:**Let the $X$ be fuzzy pre-$\gamma$-$T_2$ and let $x_\alpha,x_\beta$ be two fuzzy singletons with opposing supports $y_\alpha,y_\beta$ respectively. Let $\lambda,\mu$ be fuzzy pre-$\gamma$-open sets. Then $x_\alpha\leq\lambda\leq x^c_\beta,x_\beta\leq\mu\leq x^c_\alpha$ and $\lambda\leq\mu^c$. By Definition 2.2, $pcl_\gamma(\lambda)=\wedge\{\mu_c:\lambda\leq\mu_c,\mu_c\in FP^C_\gamma(X)\}$. Also $\lambda\leq pcl_\gamma(\lambda)$. Hence $x_\alpha\leq\lambda\leq pcl\gamma(\lambda)\leq\mu c\leq xc\beta$, that implies $x\alpha\leq\lambda\leq pcl_\gamma(\lambda)\leq x^c_\beta$.

In contrast, for any pre-$\gamma$-open fuzzy set $\lambda$ and any pair of fuzzy singletons $x_\alpha,x_\beta$ with distinct supports, this is not the case, let $x_\alpha\leq\lambda\leq pcl_\gamma(\lambda)\leq x^c_\beta$. That implies $x_\alpha\leq\lambda\leq x^c_\beta$. Also since

$x_\alpha \leq pcl_\gamma(\lambda) \leq x^c_\beta$, we find $x_\beta \leq pcl_\gamma(\lambda) \leq x_\alpha$. Thus $(pcl_\gamma(\lambda))c$ is fuzzy pre-$\gamma$-open set. Also $pcl_\gamma(\lambda) \leq (pcl\gamma(\mu))^c$. Hence $X$ is fuzzy pre-$\gamma$-$T_2$.

**Definition 5.5.** *A fuzzy pre-$\gamma$-regular fts $(X, \tau X)$ is defined if and only if a fuzzy singleton $x_\alpha$ and a fuzzy closed set are provided $\eta$, there exists two fuzzy pre-$\gamma$-open sets $\gamma$, that is $\eta \leq \lambda$ and $x_\alpha \leq \mu$ and $\lambda \leq \mu^c$.*

**Theorem 5.4.** A fuzzy topological space $(X, \tau X)$ is fuzzy pre-$\gamma$-regular if and only if, given a fuzzy Singleton $x_\alpha$ and a fuzzy open set $\lambda$, there exists an element such that $x_\alpha \leq \lambda$, there exits a fuzzy pre-$\gamma$-open set $\mu$ such that $x_\alpha \leq \mu \leq pcl_\gamma(\mu) \leq \lambda$.

**Theorem 5.5.** For every fuzzy set $\eta$ in a fuzzy pre-$\gamma$-regular spaces also a fuzzy singleton $x_\alpha$ like $x_\alpha \leq \eta c$ ,there exits fuzzy pre-$\gamma$-open sets $\lambda, \mu$ such that $x_\alpha \leq \lambda, \eta \leq \mu$ and $pcl_\gamma(\lambda) \leq (pcl_\gamma(\mu))^c$

**Proof:**Let the $x_\alpha$ be a fuzzy singleton and $\eta$ be a fuzzy closed set like $x_\alpha \leq \eta^c$. Since the fts $X$ is fuzzy pre-$\gamma$-regular, there exits fuzzy pre-$\gamma$-open set $\lambda$ such that $x_\alpha \leq \lambda \leq pcl_\gamma(\lambda) \leq \eta^c$. Let $\mu = (pcl_\gamma(\lambda))^c$ be a fuzzy pre-$\gamma$-open set similarly $\eta \leq (pcl_\gamma(\lambda))^c$. Now $\mu \leq pcl_\gamma(\mu)$. Hence $pcl_\gamma(\lambda) \leq (pcl_\gamma(\mu))^c$.

## Conflict of Interests

The author declares that there is no conflict of interests.

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