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MEAN TIME TO SYSTEM FAILURE ANALYSIS OF A LINEAR CONSECUTIVE 3-OUT-OF-5 WARM STANDBY SYSTEM IN THE PRESENCE OF COMMON CAUSE FAILURE

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Abstract: Studies on reliability characteristics of a redundant repairable warm standby system involving common cause failure are numerous. Little attention is paid on the effect of common cause failure, individual unit failure and repair rates on mean time to system failure and which among the common cause failure and individual unit failure will reduce the life span of the system than the other. In the present paper, we developed the explicit the expression for mean time to system failure (MTSF) for 3-out-of-5 warm standby system using kolmogorov's forward equations method and perform graphical analysis to see the behavior of common cause failure, individual unit failure and repair rates on mean time to system failure. The results have indicated that common cause failure decreases the life span (MTSF) earlier than the individual unit failure rates. The developed model helps in determining the maintenance policy, which will ensure the maximum overall mean time to system failure (MTSF) of the system.

Keywords: Mean Time to System Failure (MTSF), Redundancy, Common cause failure, warm Standby System

2000 AMS Subject Classification: 90B25

1. Introduction

One of the forms of redundancy is the k-out-of-n system which has wide application in industrial setting. Moreover, the k-out-of-n works if and only if at least k of the n components

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work. There are systems of three/four units in which two/three units are sufficient to perform the entire function of the system. Examples of such systems are 2-out-of-3,2-out-of-4,or3-out-of-4 redundant systems. These systems have wide application in the real world especially in industries. Many research results have been reported on reliability of 2-out-of-3,2-out-of-4, 3-out-of-4 redundant systems. For example, Chander and Bhardwaj [2] present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik [1] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Yusuf and Hussaini [8] have analyzed reliability characteristics of 2-out-of-3 system under perfect repair option. Yusuf [9] analyzed the availability and profit of 3-out-of-4 system with preventive maintenance. Haggag [5] deals with the cost analysis of two unit cold standby system involving common cause failure and preventive maintenance.

In this paper, a 3-out-of-5 warm standby system is constructed and its corresponding mathematical model is derived. The objectives of our analysis are primarily to capture the effect of both common cause failure, individual unit failure and repair rates on mean time to system failure (MTSF).

The organization of the paper is as follows. In Section 2, we give the notations and assumptions, and states of the system in the study. Expression for mean time to system failure (MTSF) is derived in section 3. Section 4 deals with material and method of the study. The results of our numerical simulations are presented in Section 5 and discussed in Section 6. Finally, we make a concluding remark in Section 7.

2. Notations and Assumptions

2.1 Notations

β_1, α_1: Failure and repair rate of unit 1

β_2, α_2: Failure and repair rate of unit 2

β_3, α_3: Failure and repair rate of unit 3

β_4, α_4: Failure and repair rate of unit 4

λ_{C1}, δ_1: Common cause failure and repair rates in state S_0

λ_{C2}, δ_2: Common cause failure and repair rates in states S_1 and S_2

λ_{C3}, δ_3: Common cause failure and repair rates in state S_4

2.2 Assumptions

1. The system consist of five units where three consecutive units are required for operating the rest are in warm standby

2. The system is attended by one repairman

3. The system failed when the middle unit or more than two units failed

4. Failure and repair time assumed exponential

5. Failure rates and repair rates are constant

6. Both common cause failure and individual unit failures are repairable

Table 1: Transition rates table

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
S_0		β_1	β_2	β_3					λ_{C1}
S_1	α_1				β_2	β_3			λ_{C2}
S_2	α_2						β_3	β_4	λ_{C2}
S_3	α_3								
S_4		α_2							λ_{C3}
S_5		α_3							
S_6			α_3						
S_7			α_4						
S_8	δ_1	δ_2	δ_2		δ_3				

2.3 States of the System

State S_0: Units 1,2 and 3 are working, units 4 and 5 are in standby. The system is working.

State S_1: Unit 1 failed and is under repair, units 2,3 and 4 are working, unit 5 is in standby. The system is working.

State S_2: Unit 2 failed and is under repair, units 3,4 and 5 are working, unit 1 is at rest. The system is working.

State S_3: Unit 3 failed and is under repair, units 1 and 2 are at rest, units 4 and 5 are in standby. The system is down.

State S_4: Unit 1 failed and is under repair, unit 2 failed and waiting for repair, units 3,4 and 5 are working. The system is working.

State S_5: Unit 1 failed and is under repair, unit 3 failed and waiting for repair, units 2 and 4 are at rest, unit 5 is in standby. The system is down.

State S_6: Unit 2 failed and is under repair, unit 3 failed and waiting for repair, units 1,4,and 5 are at rest. The system is down.

State S_7 : Unit 2 failed and is under repair, unit 4 failed and waiting for repair, units 1,3,and 5 are at rest. The system is down.

State S_8 : Both units failed. The system is down

3. Mean time to system failure analysis

From table 1 let $P_i(t)$ to be the probability that the System at time $t \geq 0$ is in state S_i . Also let $P(t)$ be the probability row vector at time t , we have the following initial condition.

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0)]$$

$$= \left[1,0,0,0,0,0,0,0,0\right]$$

We obtain the following differential equations using Fig. 1:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2 + \beta_3 + \lambda_{C1})P_0(t) + \alpha_1 P_1(t) + \alpha_2 P_2(t) + \alpha_3 P_3(t) + \delta_1 P_8(t)$$

$$\frac{dP_1(t)}{dt} = -(\alpha_1 + \beta_2 + \beta_3 + \lambda_{C2})P_1(t) + \beta_1 P_0(t) + \alpha_2 P_4(t) + \alpha_3 P_5(t) + \delta_2 P_8(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha_2 + \beta_3 + \beta_4 + \lambda_{C2})P_2(t) + \beta_2 P_0(t) + \alpha_3 P_6(t) + \alpha_4 P_7(t) + \delta_2 P_8(t)$$

$$\frac{dP_3(t)}{dt} = -\alpha_3 P_3(t) + \beta_3 P_0(t)$$

$$\frac{dP_4(t)}{dt} = -(\alpha_2 + \lambda_{C3})P_4(t) + \beta_2 P_1(t) + \delta_3 P_8(t)$$

$$\frac{dP_5(t)}{dt} = -\alpha_3 P_5(t) + \beta_3 P_1(t)$$

$$\frac{dP_6(t)}{dt} = -\alpha_3 P_6(t) + \beta_3 P_2(t)$$

$$\frac{dP_7(t)}{dt} = -\alpha_4 P_7(t) + \beta_4 P_2(t)$$

$$\frac{dP_8(t)}{dt} = -(\delta_1 + 2\delta_2 + \delta_3)P_8(t) + \lambda_{C1} P_0(t) + \lambda_{C2} P_1(t) + \lambda_{C2} P_2(t) + \lambda_{C3} P_4(t) \qquad (1)$$

Which is in matrix form as :

$$\dot{P} = A_1 P \qquad (2)$$

$$A=\begin{bmatrix} -(\beta_1+\beta_2+\beta_3+\lambda_{C1}) & \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & 0 & 0 & \delta_1 \\ \beta_1 & -(\alpha_1+\beta_2+\beta_3+\lambda_{C2}) & 0 & 0 & \alpha_2 & \alpha_3 & 0 & 0 & \delta_2 \\ \beta_2 & 0 & -(\alpha_2+\beta_3+\beta_4+\lambda_{C2}) & 0 & 0 & 0 & \alpha_3 & \alpha_4 & \delta_2 \\ \beta_3 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & -(\alpha_2+\lambda_{C3}) & 0 & 0 & 0 & \delta_3 \\ 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & \beta_4 & 0 & 0 & 0 & 0 & -\alpha_4 & 0 \\ \lambda_{C1} & \lambda_{C2} & \lambda_{C2} & 0 & \lambda_{C3} & 0 & 0 & 0 & -(\delta_1+2\delta_2+\delta_3) \end{bmatrix}$$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q_1 (see El said [3, 4], Haggag [5,6], Wang et al [7]) .

The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\int_0^\infty e^{Qt}dt \tag{3}$$

and

$$\int_0^\infty e^{Qt}dt=Q^{-1}\text{, since } Q^{-1}<0 \tag{4}$$

For system 1, explicit expression for the $MTSF$ is given by

$$E\left[T_{P(0)\to P(absorbing)}\right]=MTSF=P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\frac{N}{D} \tag{5}$$

where

$$N=(\alpha_2\beta_4\lambda_{C2}+2\beta_3\lambda_{C2}\lambda_{C3}+\alpha_1\alpha_3\beta_4+\alpha_1\beta_3\lambda_{C3}+\alpha_2\beta_3\beta_4+\alpha_1\alpha_2\lambda_{C3}+\alpha_1\alpha_2\beta_3+\alpha_1\beta_4\lambda_{C3}+\alpha_1\alpha_2\lambda_{C2}+\alpha_1\lambda_{C2}\lambda_{C3}+$$
$$\alpha_2\beta_2\lambda_{C3}+\alpha_1\alpha_2^2+\alpha_2^2\beta_3+\alpha_2\beta_3^2+\beta_3^2\lambda_{C3}+\alpha_2^2\lambda_{C2}+\alpha_2\lambda_{C2}^2+\lambda_{C2}^2\lambda_{C3}+\beta_2\beta_3\lambda_{C3}+\beta_2\beta_4\lambda_{C3}+\beta_2\lambda_{C2}\lambda_{C3}+\alpha_2\beta_3\lambda_{C3}+$$
$$\beta_3\beta_4\lambda_{C3}+2\alpha_2\beta_3\lambda_{C2}+\alpha_2\lambda_{C2}\lambda_{C3}+\beta_4\lambda_{C2}\lambda_{C3})+\beta_1(\alpha_2+\beta_3+\beta_4+\lambda_{C2})(\alpha_2+\lambda_{C3})+\beta_2(\alpha_1\alpha_2+\alpha_1\lambda_{C3}+\beta_2\lambda_{C3}+\alpha_2\beta_3+$$
$$\beta_3\lambda_{C3}+\alpha_2\lambda_{C2}+\lambda_{C2}\lambda_{C3})+\beta_1\beta_2(\alpha_2+\beta_3+\beta_4+\lambda_{C2})$$

$$D=\beta_3^2\lambda_{C1}\lambda_{C3}+\alpha_1\alpha_2^2\lambda_{C1}+\alpha_2\beta_2\lambda_{C2}^2+\alpha_2\beta_3^2\lambda_{C1}+\beta_3\lambda_{C2}^2\lambda_{C3}+\alpha_2^2\beta_1\lambda_{C2}+\alpha_2\beta_1\beta_4\lambda_{C2}+2\beta_1\beta_3\lambda_{C2}\lambda_{C3}+\alpha_2\beta_1\beta_3\beta_4+$$
$$\alpha_2\beta_1\beta_2\lambda_{C3}+\beta_1\beta_2\beta_3\lambda_{C3}+\beta_1\beta_2\beta_4\lambda_{C3}+\beta_1\beta_2\lambda_{C2}\lambda_{C3}+\alpha_2\beta_1\beta_3\lambda_{C3}+\beta_1\beta_3\beta_4\lambda_{C3}+2\alpha_2\beta_1\beta_3\lambda_{C2}+\alpha_2\beta_1\lambda_{C2}\lambda_{C3}+\beta_1\lambda_{C2}^2\lambda_{C3}+$$
$$\alpha_1\alpha_2^2\beta_3+\beta_2^2\beta_3\lambda_{C3}+\beta_2^2\beta_4\lambda_{C3}+\beta_2^2\lambda_{C2}\lambda_{C3}+2\beta_3^2\lambda_{C2}\lambda_{C3}+\alpha_1\beta_3^2\lambda_{C3}+\alpha_2\beta_3^2\beta_4+\alpha_1\alpha_2\beta_3^2+\alpha_2\beta_3^2\lambda_{C3}+\beta_3^2\beta_4\lambda_{C3}+$$

$$2\alpha_2\beta_3^2\lambda_{C2} + \alpha_2^2\beta_1\beta_3 + \alpha_2\beta_1\beta_3^2 + \beta_1\beta_3^2\lambda_{C3} + \alpha_2\beta_1\lambda_{C2}^2 + \alpha_2^2\beta_3^2 + \alpha_2\beta_3^3 + \beta_3^3\lambda_{C3} + \beta_1\beta_4\lambda_{C2}\lambda_{C3} + \alpha_2\beta_2\beta_4\lambda_{C2} + 3\beta_2\beta_3\lambda_{C2}\lambda_{C3} +$$
$$\alpha_1\alpha_2\beta_2\beta_4 + \alpha_1\beta_2\beta_3\lambda_{C3} + \alpha_2\beta_2\beta_3\beta_4 + \alpha_1\alpha_2\beta_2\beta_3 + \alpha_1\beta_2\beta_4\lambda_{C3} + \alpha_1\alpha_2\beta_2\lambda_{C2} + \alpha_1\beta_2\lambda_{C2}\lambda_{C3} + \alpha_2\beta_2\beta_3\lambda_{C3} + 2\beta_2\beta_3\beta_4\lambda_{C3} +$$
$$2\alpha_2\beta_2\beta_3\lambda_{C2} + \beta_2\beta_4\lambda_{C2}\lambda_{C3} + \alpha_2\beta_3\beta_4\lambda_{C2} + \alpha_1\alpha_2\beta_3\beta_4 + \alpha_1\alpha_2\beta_3\lambda_{C3} + \alpha_1\beta_3\beta_4\lambda_{C3} + \alpha_1\alpha_2\beta_3\lambda_{C2} + \alpha_1\beta_3\lambda_{C2}\lambda_{C3} + \alpha_2\beta_3\lambda_{C2}\lambda_{C3} +$$
$$\beta_3\beta_4\lambda_{C2}\lambda_{C3} + \alpha_2\beta_4\lambda_{C1}\lambda_{C2} + 2\beta_3\lambda_{C1}\lambda_{C2}\lambda_{C3} + \alpha_1\alpha_2\beta_4\lambda_{C1} + \alpha_1\beta_3\lambda_{C1}\lambda_{C3} + \alpha_2\beta_3\beta_4\lambda_{C1} + \alpha_1\alpha_2\lambda_{C1}\lambda_{C3} + \alpha_1\alpha_2\beta_3\lambda_{C1} +$$
$$\alpha_1\beta_4\lambda_{C1}\lambda_{C3} + \alpha_1\alpha_2\lambda_{C1}\lambda_{C2} + \alpha_1\lambda_{C1}\lambda_{C2}\lambda_{C3} + \alpha_2\beta_2\lambda_{C1}\lambda_{C3} + \beta_2\beta_3\lambda_{C1}\lambda_{C3} + \beta_2\beta_4\lambda_{C1}\lambda_{C3} + \beta_2\lambda_{C1}\lambda_{C2}\lambda_{C3} + \alpha_2\beta_3\lambda_{C1}\lambda_{C3} +$$
$$\beta_3\beta_4\lambda_{C1}\lambda_{C3} + 2\alpha_2\beta_3\lambda_{C1}\lambda_{C2} + \alpha_2\lambda_{C1}\lambda_{C2}\lambda_{C3} + \beta_4\lambda_{C1}\lambda_{C2}\lambda_{C3} + \alpha_2\beta_2\beta_3^2 + 2\beta_2\beta_3^2\lambda_{C3} + \beta_2\lambda_{C3}^2\lambda_{C2} + \alpha_2^2\beta_3\lambda_{C2} + \alpha_2\beta_3\lambda_{C2}^2 +$$
$$\alpha_2^2\beta_3\lambda_{C1} + \alpha_2^2\lambda_{C1}\lambda_{C2} + \alpha_2\lambda_{C1}\lambda_{C2}^2 + \lambda_{C1}\lambda_{C2}^2\lambda_{C3}$$

and

$$Q = \begin{bmatrix} -(\beta_1+\beta_2+\beta_3+\lambda_{C1}) & \beta_1 & \beta_2 & 0 \\ \alpha_1 & -(\alpha_1+\beta_2+\beta_3+\lambda_{C2}) & 0 & \beta_2 \\ \alpha_2 & 0 & -(\alpha_2+\beta_3+\beta_4+\lambda_{C2}) & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_2+\lambda_{C3}) \end{bmatrix}$$

4. Material and Method

In this study, the system is analyzed by using Kolmogorov's equations method. Explicit expression for mean time to system failure has been obtained.

5. Results

In this section, we numerically obtained the results for MTSF for the developed models. For analysis of the model, the following set of parameters values are fixed throughout the simulations for consistency:

Fig. 2: shows relation between α_1 and MTSF of the system

Fig. 3: shows the relation between β_1 and MTSF of the system

Fig. 4: shows the relation between λ_{C1} and MTSF of the system

For the numerical simulations of the system, the following set of parameters values are used for consistency:

(i) In Fig. 2, we fixed $\beta_1 = 0.8$, $\beta_2 = 0.8$, $\beta_3 = 0.0007$, $\beta_4 = 0.00009$, $\alpha_2 = 0.00009$, $\lambda_{C1} = 0.59$, $\lambda_{C2} = 0.09$, $\lambda_{C3} = 0.9$ and vary α_1.

(iii) In Fig. 3, we fixed $\beta_2 = 0.09$, $\beta_3 = 0.0007$, $\beta_4 = 0.009$, $\alpha_1 = 0.08$, $\alpha_2 = 0.009$, $\lambda_{C1} = 0.009$, $\lambda_{C2} = 0.009$, $\lambda_{C3} = 0.9$ and vary β_1.

(iv) In Fig.4, we fixed $\beta_1 = 0.8$, $\beta_2 = 0.9$, $\beta_3 = 0.0007$, $\beta_4 = 0.0009$, $\alpha_1 = 0.00009$, $\alpha_2 = 0.8$, $\lambda_{C2} = 0.009$, $\lambda_{C3} = 0.9$ and vary λ_{C1}.

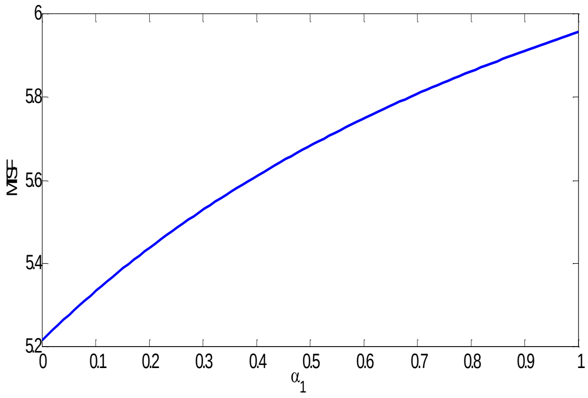

Fig. 2 effect of α_1 on MTSF

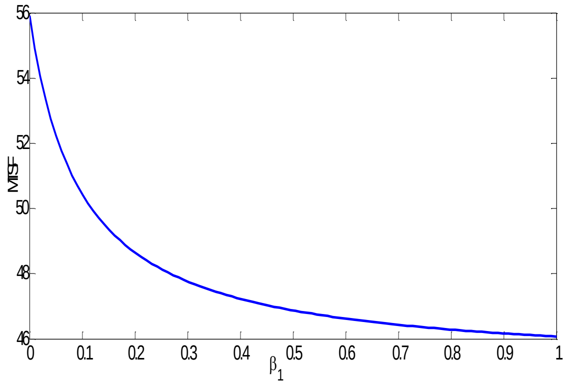

Fig. 3 effect of β_1 on MTSF

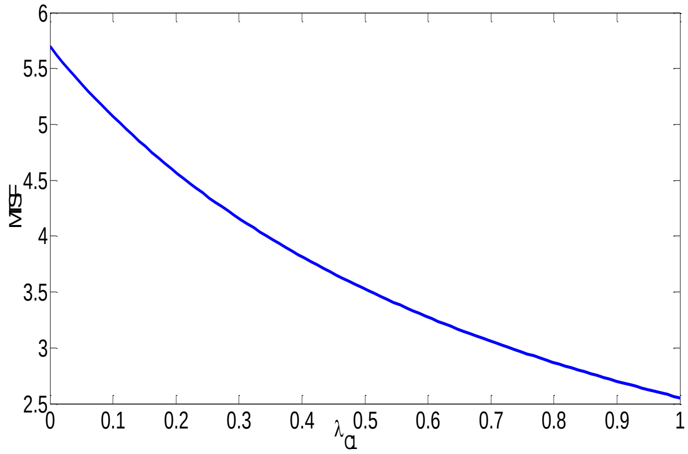

Fig. 4 effect of λ_{C1} on MTSF

6. Discussion

Using numerical solution with Matlab, we obtained the results depicted in Fig. 2 to 4. Figure 2 reveal the effect of repair rates of first unit 1 on mean time to system failure. In Fig. 2, as repair rate of unit 1 increases from 0 to 1, the mean time to system failure increases. Figure 3 reveal the effect of failure rates of unit 1 on the mean time to system failure. It is observed that for some known values of failure and repair rates of other units, as failure rate of unit 1 increases from 0 to 1, the mean time to system failure decreases. Figure 4 reveal the effect of common cause failure of both units on mean time to system failure. It is observed that for some known values of failure / repair rates of other units, as common cause common cause failure rate of all unit increases from 0 to 1 the mean time to system failure decreases. From figures 3 and 4, the mean time to system failure decreases with respect to the parameter in question (β_1 and λ_{C1}). It is evident from figure 4 that common cause failure (λ_{C1}) decrease the mean time to system failure than β_1 .

7. Conclusion

In this paper we constructed a redundant 3-out-of-5 linear consecutive warm standby system. The system is attended by one repair man. Explicit expression for mean time to system failure (MTSF) is developed in the paper. Numerical simulations obtained provide description on the effect of common cause failure rate λ_{C1} , unit 1 failure rate (β_1) and unit 1 repair rate α_1 on mean time to system failure (MTSF). From the simulations, MTST decreases as β_1 and λ_{C1} increases, and increases as α_1 increase. Thus, the life span of the system (MTSF) is shortening by λ_{C1} than β_1 . The developed model helps in determining the maintenance policy, which will ensure the maximum overall mean time to system failure (MTSF) of the system.

Conflict of Interests

The author declares that there is no conflict of interests.

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