Available online at http://scik.org
J. Math. Comput. Sci. 2 (2012), No. 3, 584-592
ISSN: 1927-5307

AN APPROACH FOR RANKING DISCRETE FUZZY SETS

HADI BASIRZADEH*, MADINEH FARNAM, AND ELHAM HAKIMI

Department of Mathematics, Faculty of Mathematical Sciences and Computer,
Shahid Chamran University, Ahvaz, Iran

Abstract. In this paper we present a new method for ranking discrete fuzzy sets. Comparing between two or more fuzzy numbers and ranking them is an important subject in fuzzy theory. Our method is a simple and effective parametric method to compare fuzzy numbers and discrete fuzzy sets. The proposed method can be utilized for all types of fuzzy sets whether normal or abnormal. In the end we present an example to illustrate the method.

Keywords: Discrete fuzzy sets, Ranking of fuzzy numbers, α-cuts.

2000 AMS Subject Classification: O3B52-62FO7

1. Introduction

Ranking of fuzzy numbers and fuzzy sets has been studied by many authors and several methods were introduced for ranking fuzzy numbers and sets. In many applications, ranking of fuzzy numbers and sets is an important component of decision process. This subject has been studied by many researchers since presented by Jain [6] and Dubois and Prade [5]. Many ranking methods can be found in fuzzy literature. Baas and Kwakernaak [3] are among the pioneers in this area. According to Lee and Li [7], there are two

*Corresponding author

E-mail addresses: basirzad@scu.ac.ir (H. Basirzadeh), farnam60madineh@yahoo.com (M. Farnam), hakimi_el@yahoo.com (E. Hakimi)

Received December 19, 2011

approaches to the ranking methods, namely defining a ranking function or obtaining a fuzzy set of optimal alternatives. In a more recent review, wang and Kerre [9], [10] proposed several axioms as reasonable properties to determine the rationality of a fuzzy ranking methods. Two factor play significant roles in fuzzy decision systems:

1. Contribution of the decision maker in the decision making process.
2. Simplicity of calculation.

Basirzadeh and Abbasi introduced an approach for ranking fuzzy numbers [4]. Now we want to extend this method for ranking of discrete type. In the process, each fuzzy set will be given a parametric value in terms α, which is dependent on the related α-cuts.

The remainder of this paper is organized as follows: In section 2, some definition of fuzzy set theory reviewed. In section 3, an approach for ranking fuzzy number is presented and in section 4 we express our approach for ranking discrete fuzzy sets. Finally, an illustrative example is given to demonstrate our proposed method.

2. Preliminaries

In this section, we review some fundamentals of basic fuzzy set theory, which will be used the rest of this paper.

Definition 2.1. *Let X denotes a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function*

$$\mu_A : X \to \lfloor 0, 1 \rfloor$$

Which assigns each element $x \in X$ to a real number $\mu_{\tilde{A}}(x)$ in the interval $\lfloor 0, 1 \rfloor$, where the value of $\mu_{\tilde{A}}(x)$ represents the membership function of x in $\tilde{A}$ [8].

Definition 2.2. *Let $\tilde{A}$ is a discrete fuzzy set and can be represented as follows:*

$$\tilde{A} = \{\frac{\mu_{\tilde{A}}(x_1)}{x_1}, \frac{\mu_{\tilde{A}}(x_2)}{x_2}, \cdots, \frac{\mu_{\tilde{A}}(x_n)}{x_n}\}$$

Where $\mu_{\tilde{A}}(x_j)$ is membership grade of x_j, $j = 1, 2, \cdots, n$ and $0 \leq \mu_{\tilde{A}}(x_j) \leq 1$ [8].

Definition 2.3. *A fuzzy set $\tilde{A}$ is convex if and only if*

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$

for every $x_1, x_2 \in X$ and $\lambda \in \lfloor 0, 1 \rfloor$ [8].

Definition 2.4. *The α-level set of a fuzzy set $\tilde{A}$ is defined as a crisp set A_α for which the degree of this membership function exceeds the level α* [8].

$$A_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\}, \qquad \alpha \in [0, 1]$$

It is clear that the following property holds for the α-level sets:

$$\alpha_1 \leq \alpha_2 \iff A_{\alpha_1} \supseteq A_{\alpha_2}.$$

Using the concept of α-level sets, the relationship between crisp sets and fuzzy sets can be featured by the following decomposition theorem.

Theorem 2.1.(Decomposition Theorem) *A fuzzy set can be represented by*

$$\tilde{A} = \bigcup_{\alpha \in [0,1]} \alpha A_\alpha,$$

where αA_α denotes the algebraic product of a scalar α with the α-level set A_α [8].

We know that a fuzzy number has been defined in various forms, but in this paper we appropriately employ the following definition of a fuzzy number [1], [2].

Definition 2.5. *We present an arbitrary fuzzy number $\tilde{A_\omega}$ by an ordered pair of functions $(\underline{A}(r), \overline{A}(r))$, where $0 \leq r \leq \omega$ and ω is an arbitrary constant between zero and one $(0 \leq r \leq \omega)$, in a parametric form which satisfies the following requirements:*

1. *$\underline{A}(r)$ is a bounded left continuous non-decreasing function over $\lfloor 0, \omega \rfloor$.*
2. *$\overline{A}$ is a bounded left continuous non-increasing function over $\lfloor 0, \omega \rfloor$.*
3. *$\underline{A}(r) \leq \overline{A}(r), 0 \leq r \leq \omega$.*

A crisp number k is simply represented by

$$\underline{A}(r) = \overline{A}(r) = k, \qquad 0 \leq r \leq \omega.$$

By appropriate definitions, the fuzzy number space $\{\underline{A}(r), \overline{A}(r)\}$ becomes a convex cone E^1 which is embedded isomorphically and isometrically in a Banach space. If $\tilde{A}$ be an arbitrary fuzzy number then α-cut of $\tilde{A}$ is

$$\lfloor \tilde{A} \rfloor_\alpha = \lfloor \underline{A}(\alpha), \overline{A}(\alpha) \rfloor, \qquad 0 \leq \alpha \leq \omega.$$

If $\omega = 1$, then the above defined number is called a normal fuzzy number. Fig.1 represents an arbitrary fuzzy number.

Here $\tilde{A}_\omega$ represents a fuzzy number in which ω is the maximum membership functions value that a fuzzy number takes on. Whenever a normal fuzzy number is meant, the fuzzy number is shown by $\tilde{A}$, for convenience. We can extend and apply this definition for discrete fuzzy sets.

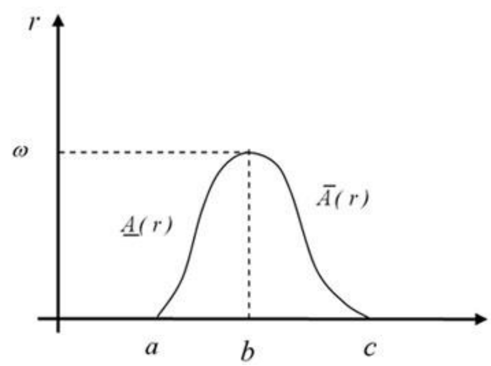

Figure 1

3. An approach for ranking fuzzy numbers

According to the definition 2.5, let $\tilde{A}_\omega\left(\underline{A}(r), \overline{A}(r)\right)$, $(0 \leq r \leq \omega)$ be a fuzzy number, then the value $Q_\alpha(\tilde{A}_\omega)$, is assigned to $\tilde{A}_\omega$ for a decision level higher than α which is calculated as follows:

$$Q(\tilde{A}_\omega) = \int_\alpha^\omega (\underline{A}(r) + \overline{A}(r))\mathrm{d}r \quad \text{where} \quad 0 \leq \alpha < 1$$

This quantity will be used as a basis for comparing fuzzy numbers in decision level higher than α.

It is clear that if $\alpha \geq \omega$, then $Q(\tilde{A}_\omega) = 0$. In order to clarify the concept of the above-mentioned quantity, consider the following fuzzy number:

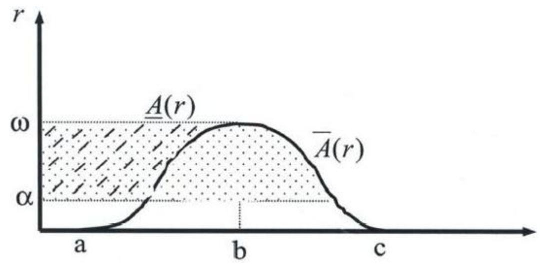

Figure 2

As shown in fig.2, the presented quantity is the summation of the dotted area and the cross-hatched area [4].

$$Q_\alpha(\tilde{A}_\omega) = \int_\alpha^\omega (\underline{A}(r) + \overline{A}(r))\mathrm{d}r + \int_\alpha^\omega \overline{A}(r)\mathrm{d}r$$

Definition 3.1. *If $\tilde{A}_\omega$ and $\tilde{B}_{\omega'}$ be two arbitrary fuzzy numbers and $\omega, \omega' \in \lfloor 0, 1 \rfloor$ then we have:*

1. $\tilde{A}_\omega \leq \tilde{B}_{\omega'} \iff \forall\alpha \in [0,1] Q_\alpha(\tilde{A}_\omega) \leq Q_\alpha(\tilde{B}_{\omega'})$.
2. $\tilde{A}_\omega = \tilde{B}_{\omega'} \iff \forall\alpha \in [0,1] Q_\alpha(\tilde{A}_\omega) = Q_\alpha(\tilde{B}_{\omega'})$.
3. $\tilde{A}_\omega \geq \tilde{B}_{\omega'} \iff \forall\alpha \in [0,1] Q_\alpha(\tilde{A}_\omega) \geq Q_\alpha(\tilde{B}_{\omega'})$ [4].

Definition 3.2. *If we compare two arbitrary fuzzy numbers including $\tilde{A}_\omega$ and $\tilde{B}_{\omega'}$ at decision levels higher than α and $\alpha, \omega, \omega' \in \lfloor 0, 1 \rfloor$, then we have:*

1. $\tilde{A}_\omega \leq \alpha\tilde{B}_{\omega'} \iff Q_\alpha(\tilde{A}_\omega) \leq Q_\alpha(\tilde{B}_{\omega'})$,
2. $\tilde{A}_\omega = \alpha\tilde{B}_{\omega'} \iff Q_\alpha(\tilde{A}_\omega) = Q_\alpha(\tilde{B}_{\omega'})$,
3. $\tilde{A}_\omega \geq \alpha\tilde{B}_{\omega'} \iff Q_\alpha(\tilde{A}_\omega) \geq Q_\alpha(\tilde{B}_{\omega'})$,

where $\tilde{A}_\omega \leq \alpha\tilde{A}'_\omega$, i.e., at decision levels higher than α, $\tilde{B}_{\omega'}$ is greater than or equal to $\tilde{A}_\omega$ [4].

4. Our approach for ranking discrete fuzzy sets

Now we want to extend the above method for discrete fuzzy sets.

Consider the discrete fuzzy set $\tilde{A} = \{\frac{\mu_{\tilde{A}}(x_1)}{x_1}, \frac{\mu_{\tilde{A}}(x_2)}{x_2}, \frac{\mu_{\tilde{A}}(x_n)}{x_n}\}$, such that $0 \leq \mu_{\tilde{A}}(x_i) \leq \omega$, for $i = 1, 2, \cdots, n$, $0 \leq \omega \leq 1$, we can rank discrete fuzzy sets as follows:

1. If all the value of membership functions in $\tilde{A}$ be increasing i.e., $\mu_{\tilde{A}}(x_1) \leq \mu_{\tilde{A}}(x_2) \leq \cdots \leq \mu_{\tilde{A}}(x_n)$ and $0 \leq \alpha \leq \mu_{\tilde{A}}(x_n)$, then corresponding to $\tilde{A}$ we define the value $Q_\alpha(\tilde{A})$ as follows:

$$Q_\alpha(\tilde{A}) = x_1(\mu(x_1) - \alpha) + \sum_{i=2}^{n} x_i(\mu(x_i) - \mu(x_{i-1}))$$

See fig.3.

2. If all the value of membership functions in $\tilde{A}$ be decreasing i.e, $\mu_{\tilde{A}}(x_1) \geq \mu_{\tilde{A}}(x_2) \geq \cdots \geq \mu_{\tilde{A}}(x_n)$ and $0 \leq \alpha \leq \mu_{\tilde{A}}(x_n)$, then corresponding to $\tilde{A}$ we define the value $Q_\alpha(\tilde{A})$ as follows:

$$Q_\alpha(\tilde{A}) = x_n(\mu(x_n) - \alpha) + \sum_{i=1}^{n-1} x_i(\mu(x_i) - \mu(x_{i+1}))$$

See fig. 4.

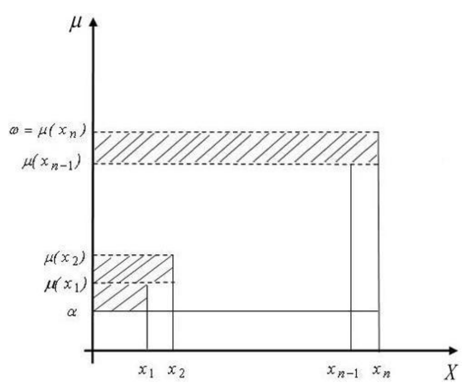

Figure 3

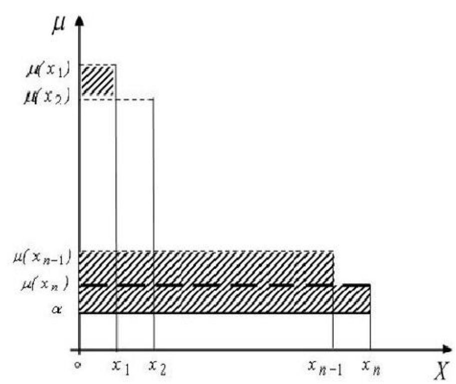

Figure 4

3. Combination of (1) and (2), i.e. when the value of membership functions in not increasing and $\tilde{A}$ not decreasing then we can formulate the problem similarly (See fig. 5).

As an example suppose that $\tilde{A}$ is a fuzzy set and there exists a unique element x_k in $\tilde{A}$ such that,

$$\mu_{\tilde{A}}(x_1) \leq \mu_{\tilde{A}}(x_2) \leq \cdots \leq \mu_{\tilde{A}}(x_k)$$

and

$$\mu_{\tilde{A}}(x_k) \geq \mu_{\tilde{A}}(x_{k+1}) \geq \cdots \geq \mu_{\tilde{A}}(x_n)$$

For $0 \leq \alpha \leq \min\{\mu_{\tilde{A}}(x_1), \cdots, \mu(x_n)\}$ we define:

$$Q_\alpha(\tilde{A}) = x_1(\mu(x_1) - \alpha) + \sum_{i=2}^{n} x_i(\mu(x_i) - \mu(x_{i-1})) + x_n(\mu(x_n) - \alpha) + \sum_{i=k}^{n-1} x_i(\mu(x_i) - \mu(x_{i+1}))$$

so we can assign to each fuzzy set $\tilde{A}$ a crisp value $Q_\alpha(\tilde{A})$.

Note: In fig.5 $\min\{\mu_{\tilde{A}}(x_1), \cdots, \mu_{\tilde{A}}(x_n)\} = \mu_{\tilde{A}}(x_1)$.

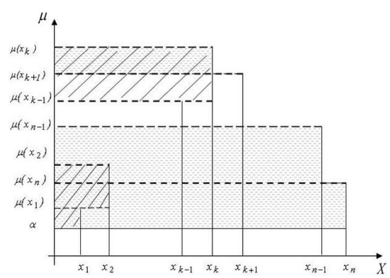

Figure 5

To clarify the concept we consider the following example.

Example:

Ranking the following discrete fuzzy sets. (Let $\alpha = 0$)

$$\tilde{A} = \left\{\frac{0.5}{1}, \frac{0.2}{2}, \frac{0.1}{3}\right\}, \tilde{B} = \left\{\frac{0.6}{3}, \frac{1}{4}\right\}, \tilde{C} = \left\{\frac{0.4}{2}, \frac{1}{4}, \frac{0.3}{5}\right\}$$
$$\tilde{D} = \left\{\frac{0.2}{1}, \frac{0.1}{2}\right\}, \tilde{E} = \left\{\frac{1}{2}, \frac{0.5}{5}, \frac{0.4}{6}\right\}, \tilde{F} = \left\{\frac{0.5}{1}, \frac{0.7}{2}, \frac{0.2}{3}\right\}$$

Solve: At first we calculate Q_0 for every set.

$Q_0(\tilde{A}) = (3 \times 0.1) + (2 \times 0.1) + (1 \times 0.3) = 0.8$ (see fig. 6a)

$Q_0(\tilde{B}) = (3 \times 0.6) + (4 \times 0.4) = 3.4$ (see fig. 6b)

$Q_0(\tilde{C}) = (2 \times 0.4) + (4 \times 0.6) + (5 \times 0.3) + (4 \times 0.7) = 7.5$ (see fig. 6c)

$Q_0(\tilde{D}) = (2 \times 0.1) + (1 \times 0.1) = 0.3$ (see fig. 6d)

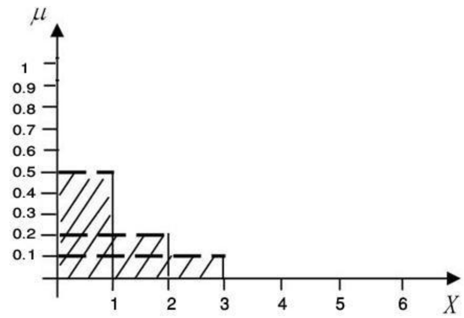

Figure 6a

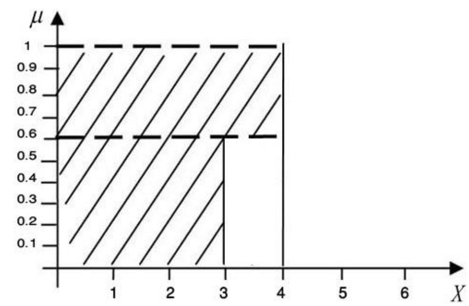

Figure 6b

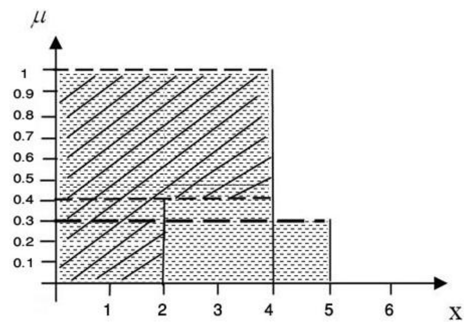

Figure 6c

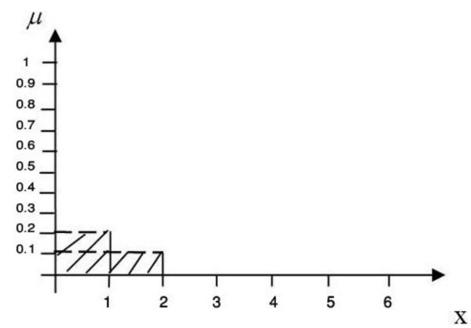

Figure 6d

$Q_0(\tilde{E}) = (6 \times 0.4) + (5 \times 0.1) + (2 \times 0.5) = 3.9$ (see fig. 6e)

$Q_0(\tilde{F}) = (1 \times 0.5) + (2 \times 0.2) + (3 \times 0.2) + (2 \times 0.5) = 2.5$ (see fig. 6f)

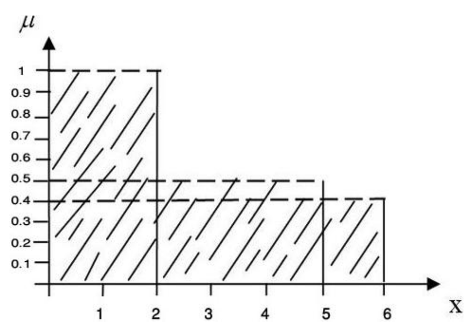

Figure 6e

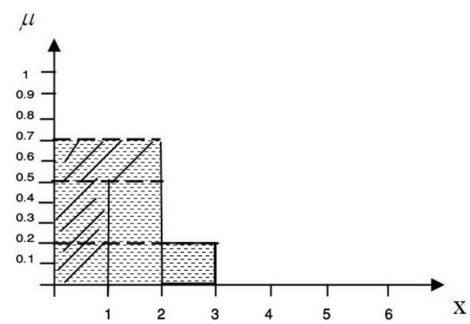

Figure 6f

By obtaining the above values we have

$$Q_0(\tilde{D}) \leq Q_0(\tilde{A}) \leq Q_0(\tilde{F}) \leq Q_0(\tilde{B}) \leq Q_0(\tilde{E}) \leq Q_0(\tilde{C})$$

so

$$\tilde{D} \leq \tilde{A} \leq \tilde{F} \leq \tilde{B} \leq \tilde{E} \leq \tilde{C}$$

5. Conclusions

In this paper, a simple and effective method was introduced for comparison between fuzzy sets. This method can be used for all kinds of fuzzy sets, whether normal or abnormal. Due to the importance of ranking fuzzy sets, especially in the theory of fuzzy decision making, it seems that the access to a simple parametric method of the comparison between fuzzy sets gives the way for better study of phenomena in fuzzy environments.

References

[1] S.Abbasbandy, B. Asady, The nearest trapezoidal fuzzy number to a fuzzy quantity, Applied mathematics and computation, 159 (2004) 381-386.

[2] T.Allahviranloo, T. Hajari, Numerical methods for approximation of fuzzy data, Applied mathematics and computation, 169 (2005) 16-33.

[3] S.M. Baas, H. Kwakernaak, Rating and ranking of multiple aspect alternative using fuzzy sets, Automatica, 13 (1977) 47-58.

[4] H. Basirzadeh, R. Abbasi, A new approach for ranking fuzzy numbers based on α- cuts, Applied mathematics and information, 26 (2008) 767-778.

[5] D. Dubios, H. Prade, Operations on fuzzy numbers, International journal of systems science, 9 (1978) 626-631.

[6] R. Jain, A procedure for multi-aspect decision making using fuzzy sets, International journal of systems science, 8 (1978) 1-7.

[7] E.S. Lee, R.J. Li, Comparison of fuzzy numbers based on the probability method of fuzzy events, computers and mathematics with applications, (1988) 887-889.

[8] M. Sakawa, Fundamentals of fuzzy set theory in: fuzzy sets and interactive multiobjective optimization, Plenum Press, New York (1993).

[9] X. Wang, E.E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy sets and systems, 118 (2001) 375-385.

[10] X. Wang, E.E. Kerre, Reasonable properties for the ordering of fuzzy quantities (II), Fuzzy sets and systems, 118 (2001) 387-405.