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VECTOR BASIS S-CORDIAL LABELING OF GRAPHS

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Abstract. Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $< \varphi(u), \varphi(v) >$. Then φ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. In this paper, we investigate the vector basis S-cordial labeling behavior of certain standard graphs like path, cycle, complete graph and star graph.

Keywords: path; cycle; complete graph; star graph.

2020 AMS Subject Classification: 05C38, 05C78.

1. INTRODUCTION

This study considers a finite, simple and undirected connected graph. The first research paper on graph theory was published by Leonhard Euler. However, he did not use the word 'graph' in his work. In the early stages of the development of the subject, the vertices of a graph were specified as $v_1, v_2, \ldots$ and the edges were denoted by $e_1, e_2, \ldots$. In the recent times, several researchers have attempted to provide different types of labeling to the vertices and edges of a

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graph by identifying the relevant mathematical properties. The present paper provides a novel method of labeling by employing the basis of an inner product space and so it may form a foundation for future research work. Graph labeling is one of the important branches of graph theory. Graph labeling is used in network analysis, cryptography, radar, circuit design, etc. The concept of graph labeling was introduced by Rosa [12]. For a detailed survey on graph labeling, we refer the book of Gallian [7]. Super root cube of cube difference labeling of some special graphs has been studied [16]. Labeling of some pan graphs were examined by Sarang Sadawartea, Sweta Srivastava [13]. The notion of cordial labeling was first introduced by Cahit [3]. Abdel-Aal, et al. [1] was discussed a study on the varieties of equivalent cordial labeling graphs. A novel problem for solving permuted cordial labeling of graphs has been investigated [6]. Prime cordial labeling for some path, cycle and wheel related graphs have been investigated by Barasara and Prajapati [3]. Product cordial labelling for some bicyclic graphs have been studied by Meena and Usharani [10]. Prajapati and Patel [11] proved that the pentagonal snake and double pentagonal snake are edge product cordial. Results on parity combination of cordial labeling have been brought out by Seoud and Aboshady [15]. Barasara et al.[4] proved that the degree splitting of path, cycle, shell graph, comb and crown are divisor cordial graphs. Edge sum divisor cordial labeling of some graphs with python implementation has studied in [2]. Sudha Rani and Sindu Devi [15] have proved that the path, cycle graph, star graph, jelly fish and wheel graphs are dihedral group divisor cordial. For the terminologies and different notations of graph theory, we refer the book of Harary [8] and for algebra, we refer the book of Herstein [9]. The comb $P_n \odot K_1$ [7] is a connected graph obtained by joining a pendent edge to each vertex of the path P_n. It has $2n$ vertices and $2n-1$ edges. The vector space V over a field F [9] is said to be an inner product space if there is defined for any two vectors $u, v \in V$ an element $< u, v >$ in F such that

(1) $< u, v > = \overline{< (v, u) >}$
(2) $< u, u > \geq 0$ and $< u, u > = 0$ if and only if $u = 0$
(3) $< \alpha u + \beta v, w > = \alpha < u, w > + \beta < v, w >$ for any $u, v, w \in V$ and $\alpha, \beta \in F$.

A subset S of a vector space V is called a basis [9] of V if S consists of linearly independent elements and $V = L(S)$. In this paper, we introduce a new graph labeling method using a basis of an inner product space and investigate some standard graphs like path, cycle, star, complete graph and comb for this labeling technique.

2. VECTOR BASIS S-CORDIAL LABELING

Definition 2.0.1. *Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $< \phi(u), \phi(v) >$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph.*

In this paper, we consider the inner product space R^n and the standard inner product $< x,y >= x_1y_1 + x_2y_2 + \cdots + x_ny_n$ where $x = (x_1,x_2,\ldots,x_n), y = (y_1,y_2,\ldots,y_n)$, $x_i,y_i \in R$.

3. MAIN RESULTS

3.1. Vector Basis $\{(1,0),(0,1)\}$-Cordial Labeling.

Theorem 3.1.1. *A graph G is vector basis $\{(1,0),(0,1)\}$-cordial if and only if G is a cordial graph.*

Proof. Consider the basis $S = \{(1,0),(0,1)\}$ of R^2. The following table (1) illustrates the inner product space due to $(1,0)$ and $(0,1)$.

<>	(1,0)	(0,1)
(1,0)	1	0
(0,1)	0	1

TABLE 1.

Vertex labels appear in the first row and the first column of table 1 and the interior values in the table are edge labels. Suppose ϕ is a vector basis S-cordial labeling of G. Replacing the label $(1,0)$ by 1 and $(0,1)$ by 0, we get a cordial labeling by table 1. Similarly if f is a cordial labeling of G, then it is also a vector basis S-cordial labeling of G. □

3.2. Vector Basis $\{(1,0,0),(0,1,0),(0,0,1)\}$-Cordial Labeling.

Theorem 3.2.1. *The path P_n is vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$-cordial if and only if $n \neq 3$.*

Proof. Let P_n be the path $u_1, u_2, \ldots, u_n$.

Case (i): $n = 1$.

In this case, a vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling of P_1 is given in figure (1).

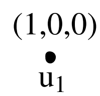

Fig. 1: Vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling of P_1.

Case (ii):$n = 2$.

A vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling in this case is provided in figure (2).

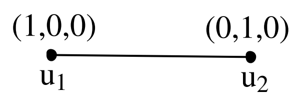

Fig. 2: Vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling of P_2.

Case (iii): $n = 3$

In this case, $\phi_{(1,0,0)} = \phi_{(0,1,0)} = \phi_{(0,0,1)} = 1$.This implies $\gamma_1 = 0$, $\gamma_2 = 2$, a contradiction.

Case (iv): $n = 4$

Next, assign the vector $(1,0,0)$ to the vertices u_1, u_2. Assign the vector $(0,1,0)$ to the vertex u_3. Finally, assign the vector $(0,0,1)$ to the vertex u_4.

Case (v): $n = 5$

Now, assign the vector (1,0,0) to the vertices u_1, u_2. Then, assign the vector (0,1,0) to the vertices u_3, u_4. Finally, assign the vector (0,0,1) to the vertex u_5.

Case (vi):$n \equiv 0 \pmod 6$

First, we assign the vector (1,0,0)to the vertices u_1, u_2. Also, assign the vector (0,1,0) to the vertices u_3, u_4. We assign the vector (0,0,1) to the vertices u_5, u_6. Continue the above process till reach the vertex u_n.

Case (vii):$n \equiv 1 \pmod 6$

Assign the vector (1,0,0) to the vertices u_1,u_2. We assign the vector (0,1,0) to the vertices u_3,u_4. Further, assign the vector (0,0,1) to the vertices u_5,u_6. Continue the above process till the vertex u_{n-1} is reached. Assign the vector (1,0,0) to the final vertex u_n.

Case (viii): $n \equiv 2 \pmod 6$

We assign the vector (1,0,0) to the vertices u_1,u_2. Then, assign the vector (0,1,0) to the vertices u_3,u_4. Next, assign the vector (0,0,1) to the vertices u_5,u_6. Continue the above process till reaching the vertex u_{n-2}. Finally, assign the vector (1,0,0) to the vertex u_{n-1} and assign the vector (0,1,0) to the vertex u_n.

Case (ix): $n \equiv 3 \pmod 6$

In this case, assign the vector (1,0,0) to the vertices u_1,u_2. Assign the vector (0,1,0)to the vertices u_3,u_4. Next, assign the vector (0,0,1) to the vertices u_5,u_6. Continue the above process till reaching the vertex u_{n-3}. Further, assign the vector (0,0,1) to the vertex u_{n-2}. Assign the vector (0,1,0) to the vertex u_{n-1}. Assign the vector (1,0,0) to the final vertex u_n.

Case (x):$n \equiv 4 \pmod 6$

We assign the vector (1,0,0) to the vertices u_1,u_2. Assign the vector (0,1,0) to the vertices u_3,u_4. Then, assign the vector (0,0,1) to the vertices u_5,u_6. Continue the above process till the vertex u_{n-4} is reached. Thereafter, assign the vector (1,0,0) to the vertices u_{n-3},u_{n-2}. Assign the vector (0,1,0) to the vertex u_{n-1}. Assign the vector (0,0,1) to the final vertex u_n.

Case (xi): $n \equiv 5 \pmod 6$

Now, assign the vector (1,0,0) to the vertices u_1,u_2. Assign the vector (0,1,0) to the vertices u_3,u_4. Then assign the vector (0,0,1) to the vertices u_5,u_6. Continue the above process till reaching the vertex u_{n-5}.Then assign the vector (1,0,0) to the vertices u_{n-4},u_{n-3}. Assign the vector (0,1,0) to the vertices u_{n-2},u_{n-1}. Finally, assign the vector (0,0,1) to the vertex u_n.

Clearly the above labeling method provides a vector basis {(1,0,0),(0,1,0),(0,0,1)}-cordial labeling. □

Theorem 3.2.2. *The star $K_{1,n}$ is vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial if and only if $n = 1,3$.*

Proof. Consider the star graph $K_{1,n}$.

Case (i): $n = 1,3$

When $n=1$, the star graph $K_{1,1} \simeq P_2$, the path P_2 is a vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial.

When $n=3$, a vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling of the star graph $K_{1,3}$ is given in figure (3).

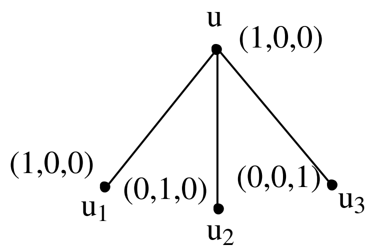

Fig.3: A vector basis $\{(1,0,0),(0,1,0),(0,0,1)\}$–cordial labeling of $K_{1,3}$.

Case (ii): $n=2$

In this case, $\phi_{(1,0,0)} = \phi_{(0,1,0)} = \phi_{(0,0,1)} = 1$. Then, $\gamma_1 = 0$, $\gamma_2 = 2$, a contradiction.

Case (iii):$n>3$

To get the edge label 1, the adjacent vertices should be receiving the same label.

Sub-case (i): $n=3k$, $k>1$

Then $\phi_{(1,0,0)} = k+1$ or $\phi_{(0,1,0)} = k+1$ or $\phi_{(0,0,1)} = k+1$. In this case, $\gamma_1 = k$, whereas $\gamma_0 = 2k$. As $k>1$, a contradiction arises.

Sub-case (ii): $n=3k+1$, $k>1$

As in subcase (i), $\gamma_1 = k$, whereas $\gamma_0 = 2k+1$. Then, $\gamma_0 - \gamma_1 = 2k+1-k = k+1 > 1$, as $k>1$, we get a contradiction.

Sub-case (iii): $n=3k+2$, $k>1$

Then $\phi_{(1,0,0)} = k+1$ or $\phi_{(0,1,0)} = k+1$ or $\phi_{(0,0,1)} = k+1$.Hence $\gamma_1 = k$, but $\gamma_0 = 2k+2$ Therefore, $\gamma_0 - \gamma_1 = 2k+2-k = k+2 > 1$, as $k>1$, we get a contradiction. Hence the result. □

Before moving to the next section, prove that the set $S = \{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$ is a basis of R^4.

Theorem 3.2.3. *The set* $S = \{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$ *is a basis for* R^4 *over* R.

Proof. Consider the relation

$$a(1,1,1,1)+b(1,1,1,0)+c(1,1,0,0)+d(1,0,0,0)=(0,0,0,0)$$

$$(a,a,a,a)+(b,b,b,0)+(c,c,0,0)+(d,0,0,0)=(0,0,0,0)$$

$$(a+b+c+d,a+b+c,a+b,a)=(0,0,0,0)$$

We get, $a+b+c+d=0$,

$a+b+c=0$,

$a+b=0$,

$a=0$.

Then, $a=0,b=0,c=0,d=0$ is the only solution. So the four vectors are linearly independent over R. Hence $S=\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$ is a basis for R^4. □

3.3. Vector Basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}–Cordial Labeling.

Theorem 3.3.1. *Any path P_n is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial.*

Proof. Let P_n be the path $u_1u_2\dots u_n$.

Case (i):$n \equiv 0 \pmod 4$

Let $n=4k$. Assign the vector (1,1,1,1) to the first k vertices $u_1,u_2,\dots,u_k$. Then, assign the vector (1,1,1,0) to the next k vertices $u_{k+1},u_{k+2},\dots,u_{2k}$. We assign the vector (1,1,0,0) to the next k vertices $u_{2k+1},u_{2k+2},\dots,u_{3k}$. Assign the vector (1,0,0,0) to the next k vertices $u_{3k+1},u_{3k+2},\dots,u_{4k}=u_n$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n=4k+1$. We assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1,u_2,\dots,u_(k+1)$. Then, assign the vector (1,1,1,0) to the next k vertices $u_{k+2},u_{k+3},\dots,u_{2k+1}$. Also assign the vector (1,1,0,0) to the next k vertices $u_{2k+2},u_{2k+3},\dots,u_{3k+1}$. Thereafter, assign the vector (1,0,0,0) to the next k vertices $u_{3k+2},u_{3k+3},\dots,u_{4k+1}=u_n$.

Case (iii): $n \equiv 2 \pmod 4$

Let $n=4k+2$. Assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1,u_2,\dots,u_{k+1}$. Assign the vector (1,1,1,0) to the next $k+1$ vertices $u_{k+2},u_{k+3},\dots,u_{2k+2}$. Assign the vector (1,1,0,0) to the next k vertices $u_{2k+3}, u_{2k+4}, \dots, u_{3k+2}$. Finally, assign the vector (1,0,0,0) to the next k vertices $u_{3k+3}, u_{3k+4}, \dots, u_{4k+2} = u_n$.

Case (iv):$n \equiv 3 \pmod 4$

Let $n = 4k+3$. Assign the vector (1,1,1,1)to the fist $k+1$ vertices $u_1, u_2, \dots, u_{k+1}$. This is followed by the assignment of the vector (1,1,1,0) to the next $k+1$ vertices $u_{k+2}, u_{k+3}, \dots, u_{2k+2}$. Thereafter assign the vector (1,1,0,0) to the next $k+1$ vertices $u_{2k+3}, u_{2k+4}, \dots, u_{3k+3}$. Further assign the vector (1,0,0,0) to the next k vertices $u_{3k+4}, u_{3k+5}, \dots, u_{4k+3} = u_n$.

Clearly the above labeling provides a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),$ $(1,0,0,0)\}$–cordial labeling of P_n. □

Theorem 3.3.2. *The cycle C_n is vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial if and only if $n \equiv 1,2,3 \pmod 4$.*

Proof. Let C_n be the cycle $u_1 u_2 \dots u_n u_1$.

Case (i):$n \equiv 0 \pmod 4$

Let $n = 4k$. To get the edge label 4 the vector (1,1,1,1) should be assigned to the consecutive vertices of the cycle. As the size of C_n is n, the maximum edges with label 4 is $k-1$, a contradiction.

Case (ii):$n \equiv 1 \pmod 4$

Let $n = 4k+1$. Assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1, u_2, \dots, u_{k+1}$. Further, assign the vector (1,1,1,0) to the next k vertices $u_{k+2}, u_{k+3}, \dots, u_{2k+1}$. Thereafter, assign the vector (1,1,0,0) to the next k vertices $u_{2k+2}, u_{2k+3}, \dots, u_{3k+1}$. Finally assign the vector (1,0,0,0) to the next k vertices $u_{3k+2}, u_{3k+3}, \dots, u_{4k+1} = u_n$.

Case (iii):$n \equiv 2 \pmod 4$

Let $n = 4k+2$. Assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1, u_2, \dots, u_{k+1}$. Assign the vector (1,1,1,0)to the next $k+1$ vertices $u_{k+2}, u_{k+3}, \dots, u_{2k+2}$. We assign the vector (1,1,0,0) to the next k vertices $u_{2k+3}, u_{2k+4}, \dots, u_{3k+2}$. Then assign the vector (1,0,0,0) to the next k vertices $u_{3k+3}, u_{3k+4}, \dots, u_{4k+2} = u_n$.

Case (iv):$n \equiv 3 \pmod 4$

Let $n = 4k+3$. Assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1, u_2, \dots, u_{k+1}$. Assign the vector (1,1,1,0) to the next $k+1$ vertices $u_{k+2}, u_{k+3}, \dots, u_{2k+2}$. Next, assign the vector

(1,1,0,0) to the next $k+1$ vertices $u_{2k+3}, u_{2k+4}, \ldots, u_{3k+3}$. Then assign the vector (1,0,0,0) to the next k vertices $u_{3k+4}, u_{3k+5}, \ldots, u_{4k+3} = u_n$. Clearly the above labeling pattern is a vector basis $\{(1,1,1,1),(1,1,1,0),$
$(1,1,0,0),(1,0,0,0)\}$–cordial labeling of C_n. □

Theorem 3.3.3. *The star $K_{1,n}$ is vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial for all natural numbers n.*

Proof. Let $K_{1,n}$ be a given star. Let $V(K_{1.n}) = \{u, u_i \mid 1 \le i \le n\}$ and $E(K_{1,n}) = \{uu_i \mid 1 \le i \le n\}$ be the vertex and edge sets respectively of $K_{1,n}$. Then $|V(K_{1,n})| = n+1$ and $|E(K_{1,n})| = n$. Assign the vector (1,1,1,1) to the vertex u. Next, assign the vectors (1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1) to the first 4 vertices u_1, u_2, u_3, u_4,respectively. Then, assign the vectors (1,0,0,0), (1,1,0,0), (1,1,1,0)(1,1,1,1) to the next 4 vertices u_5, u_6, u_7, u_8,respectively. Continuing in this way, we reach the vertex u_n. It is seen that the edge uu_n receives the label 4 or 3 or 2 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 3 \pmod 4$ or $n \equiv 2 \pmod 4$ or $n \equiv 1 \pmod 4$.Clearly the above labeling pattern bestows a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial labeling to $K_{(1,n)}$. □

Theorem 3.3.4. *The comb $P_n \odot K_1$ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial for all natural numbers n.*

Proof. Let $V(P_n \odot K_1) = \{u_i, v_i \mid 1 \le i \le n\}$ and $E(P_n \dot{K}_1) = \{u_i v_i \mid 1 \le i \le n\}$ be the vertex and edge sets of the comb $P_n \odot K_1$. Then $|V(P_n \odot K_1)| = 2n$ and $|E(P_n \odot K_1)| = 2n-1$. Let $2n = m$. There arise two cases, namely $m \equiv 0 \pmod 4$ and $m \equiv 2 \pmod 4$

Case (i): $m \equiv 0 \pmod 4$.

Let $m = 4k$. Now, we assign the vector (1,1,1,1)to the fist k vertices $u_1, u_2, \ldots, u_k$. Assign the vector (1,1,1,0) to the next k vertices $u_{k+1}, u_{k+2}, \ldots, u_n$. Assign the vector (1,1,0,0) to the first k vertices $v_1, v_2, \ldots, v_k$. Then assign the vector (1,0,0,0) to the next k vertices $v_{k+1}, v_{k+2}, \ldots, v_n$.

Case (ii): $m \equiv 2 \pmod 4$

Let $m = 4k+2$. In this case, assign the vector (1,1,1,1) to the fist $k+1$ vertices $u_1, u_2, \ldots, u_{k+1}$. Assign the vector (1,1,1,0) to the first $k+1$ vertices $v_1, v_2, \ldots, v_{k+1}$. We assign the vector (1,1,0,0) to the k vertices $u_{k+2}, u_{k+3}, \ldots, u_n$. Then assign the vector (1,0,0,0) to the next k

vertices $v_{k+2}, v_{k+3}, \ldots, v_n$.

Clearly the above labeling provides a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial labeling to the comb $P_n \odot K_1$. □

Theorem 3.3.5. *The complete graph K_n is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial if and only if $n \leq 3$.*

Proof. Consider the complete graph K_n. When $n = 1$, the complete graph $K_1 \simeq P_1$, the path P_1 is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$–cordial. When $n = 2$, $K_2 \simeq P_2$, the path P_2 is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial. When $n = 3$, the complete graph $K_3 \simeq C_2$, the cycle C_3 is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial. Let $\binom{n}{r}$ denote the number of combinations of n objects taken r at a time.

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4k$. Then $\phi_{(1,1,1,1)} = k$, $\phi_{(1,1,1,0)} = k$, $\phi_{(1,1,0,0)} = k$ and $\phi_{(1,0,0,0)} = k$. Hence we get $\gamma_1 = \binom{k}{2} + k^2 + k^2 + k^2$, $\gamma_2 = \binom{k}{2} + k^2 + k^2$, $\gamma_3 = \binom{k}{2} + k^2$, $\gamma_4 = \binom{k}{2}$. Consequently, $\gamma_1 - \gamma_3 = 2k^2 > 1$, as $k \geq 1$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4k+1$. Then any one of the following types of cases can occur:

Type I: $\phi_{(1,1,1,1)} = k+1$, $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = k$.

Then $\gamma_1 = \binom{k}{2} + k^2 + k^2 + k(k+1), \gamma_2 = \binom{k}{2} + k^2 + k(k+1), \gamma_3 = \binom{k}{2} + k(k+1), \gamma_4 = \binom{k+1}{2}$. So we obtain $\gamma_1 - \gamma_3 = 2k^2 > 1$, as $k \geq 1$.

Type II: $\phi_{(1,1,1,0)} = k+1, \phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = k$.

Then $\gamma_1 = \binom{k}{2} + k^2 + k^2 + k(k+1), \gamma_2 = \binom{k}{2} + k^2 + k(k+1), \gamma_3 = \binom{k+1}{2} + k(k+1), \gamma_4 = \binom{k}{2}$. Therefore, $\gamma_1 - \gamma_4 = 2k^2 + k(k+1) > 1$, as $k > 1$.

Type III: $\phi_{(1,1,0,0)} = k+1$, $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,0,0,0)} = k$.

Hence, $\gamma_1 = \binom{k}{2} + k^2 + k^2 + k(k+1), \gamma_2 = \binom{k+1}{2} + k(k+1) + k(k+1), \gamma_3 = \binom{k}{2} + k^2, \gamma_4 = \binom{k}{2}$. So we obtain $\gamma_1 - \gamma_4 = 2k^2 + k(k+1) > 1$, as $k > 1$.

Type IV: $\phi_{(1,0,0,0)} = k+1$, $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = k$.

Then, $\gamma_1 = \binom{k+1}{2} + k(k+1) + k(k+1) + k(k+1), \gamma_2 = \binom{k}{2} + k^2 + k^2, \gamma_3 = \binom{k}{2} + k^2, \gamma_4 = \binom{k}{2}$. We see that $\gamma_2 - \gamma_4 = 2k^2 > 1$, as $k > 1$.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4k+2$. Then any one of the following types of cases can arise.

Type I: $\phi_{(1,0,0,0)} = \phi_{(1,1,0,0)} = k+1$, $\phi_{(1,1,1,0)} = \phi_{(1,1,1,1)} = k$.
Then, $\gamma_1 = \binom{k+1}{2} + (k+1)^2 + k(k+1) + k(k+1), \gamma_2 = \binom{k+1}{2} + k(k+1) + k(k+1), \gamma_3 = \binom{k}{2} + k^2, \gamma_4 = \binom{k}{2}$. So we get $\gamma_1 - \gamma_4 = \binom{k+1}{2} + (k+1)^2 + k(k+1) + k(k+1) - \binom{k}{2} > 1$, as $k > 1$.

Type II: $\phi_{(1,0,0,0)} = \phi_{(1,1,1,0)} = k+1$, $\phi_{(1,1,0,0)} = \phi_{(1,1,1,1)} = k$.
Thus, $\gamma_1 = \binom{k+1}{2} + k(k+1) + k(k+1) + (k+1)^2, \gamma_2 = \binom{k}{2} + k(k+1) + k^2, \gamma_3 = \binom{k+1}{2} + k(k+1), \gamma_4 = \binom{k}{2}$. Hence we obtain $\gamma_2 - \gamma_4 = k(k+1) + k^2 > 1$, as $k > 1$.

Type III: $\phi_{(1,0,0,0)} = \phi_{(1,1,1,1)} = k+1$, $\phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = k$.
Hence, $\gamma_1 = \binom{k+1}{2} + k(k+1) + k(k+1) + (k+1)^2, \gamma_2 = \binom{k}{2} + k^2 + k(k+1), \gamma_3 = \binom{k}{2} + k(k+1), \gamma_4 = \binom{k+1}{2}$. Consequently, $\gamma_1 - \gamma_4 = 2k(k+1) + (k+1)^2 > 1$, as $k > 1$.

Type IV: $\phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = k+1$, $\phi_{(1,0,0,0)} = \phi_{(1,1,1,1)} = k$.
Then, $\gamma_1 = \binom{k+1}{2} + k(k+1) + k(k+1) + k^2, \gamma_2 = \binom{k+1}{2} + (k+1)^2 + k(k+1), \gamma_3 = \binom{k+1}{2} + k(k+1), \gamma_4 = \binom{k}{2}$. Therefore it is seen that $\gamma_1 - \gamma_4 = 2k(k+1) + k^2 > 1$, as $k > 1$.

Type V: $\phi_{(1,1,0,0)} = \phi_{(1,1,1,1)} = k+1$, $\phi_{(1,0,0,0)} = \phi_{(1,1,1,0)} = k$.
We have, $\gamma_1 = \binom{k}{2} + k(k+1) + k^2 + k(k+1), \gamma_2 = \binom{k+1}{2} + k(k+1) + (k+1)^2, \gamma_3 = \binom{k}{2} + k(k+1), \gamma_4 = \binom{k+1}{2}$.Consequently, $\gamma_2 - \gamma_4 = k(k+1) + (k+1)^2 > 1$, as $k > 1$.

Type VI: $\phi_{((1,0,0,0)} = \phi_{(1,1,1,1)} = k+1$, $\phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = k$.
Now, $\gamma_1 = \binom{k+1}{2} + k(k+1) + k(k+1) + (k+1)^2, \gamma_2 = \binom{k}{2} + k^2 + k(k+1), \gamma_3 = \binom{k}{2} + k(k+1), \gamma_4 = \binom{k+1}{2}$. We obtain $\gamma_1 - \gamma_4 = 2k(k+1) + (k+1)^2 > 1$, as $k > 1$.

Case (iv): $n \equiv 3 \pmod 4$

Let $n = 4k+3$. Then any one of the following types of cases will arise.

Type I: $\phi_{(1,0,0,0)} = \phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = k+1, \phi_{(1,1,1,1)} = k$.
Then, $\gamma_1 = \binom{k+1}{2} + (k+1)^2 + (k+1)^2 + k(k+1), \gamma_2 = \binom{k+1}{2} + (k+1)^2 + k(k+1), \gamma_3 = \binom{k+1}{2} + k(k+1), \gamma_4 = \binom{k}{2}$. This implies that $\gamma_1 - \gamma_3 = 2(k+1)^2 > 1$, as $k > 1$.

Type II: $\phi_{(1,0,0,0)} = \phi_{(1,1,0,0)} = \phi_{(1,1,1,1)} = k+1, \phi_{(1,1,1,0)} = k$.
We obtain $\gamma_1 = \binom{k+1}{2} + (k+1)^2 + k(k+1) + (k+1)^2, \gamma_2 = \binom{k+1}{2} + k(k+1) + (k+1)^2, \gamma_3 = \binom{k}{2} + k(k+1), \gamma_4 = \binom{k+1}{2}$. Hence $\gamma_1 - \gamma_4 = 2(k+1)^2 + k(k+1) > 1$, as $k > 1$.

Type III: $\phi_{(1,0,0,0)} = \phi_{(1,1,1,0)} = \phi_{(1,1,1,1)} = k+1, \phi_{(1,1,0,0)} = k$.

We have $\gamma_1 = \binom{k+1}{2} + k(k+1) + (k+1)^2 + (k+1)^2, \gamma_2 = \binom{k}{2} + k(k+1) + k(k+1), \gamma_3 = \binom{k+1}{2} + (k+1)^2, \gamma_4 = \binom{k+1}{2}$. Consequently $\gamma_1 - \gamma_4 = k(k+1) + 2(k+1)^2 > 1$, as $k > 1$.

Type IV: $\phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = \phi_{(1,1,1,1)} = k+1, \phi_{(1,0,0,0)} = k$.

Then, $\gamma_1 = \binom{k}{2} + k(k+1) + k(k+1) + k(k+1), \gamma_2 = \binom{k+1}{2} + (k+1)^2 + (k+1)^2, \gamma_3 = \binom{k+1}{2} + (k+1)^2, \gamma_4 = \binom{k+1}{2}$. We get $\gamma_2 - \gamma_4 = 2(k+1)^2 > 1$, as $k > 1$.

Hence the complete graph K_n is vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),$
$(1,0,0,0)\}$–cordial if and only if $n \leq 3$. □

4. CONCLUSION

In this paper, we have introduced a new labeling concept called a vector basis S-cordial labeling. Vector basis S-cordial labeling behavior of certain standard graphs like path, cycle, complete graph and star graph has been studied for some basis. Results on the non-existence of this labeling in certain cases have been brought out in this study. Investigation of several other families of graphs for the existence of vector basis S-cordial labeling is an open problem. Labeling the vertices and edges of the graph with the application of an inner product space can be expected to attract researches to undertake more work in this unexplored area.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] M.E. Abdel-Aal, S.A. Bashammakh, A study on the varieties of equivalent cordial labeling graphs, AIMS Math. 9 (2024), 34720-34733.

[2] A.A.C. Aanisha, R. Manoharan, Edge sum divisor cordial labeling of some graphs with python implementation, J.Propuls. Technol. 45 (2024), 5254-5263.

[3] C.M. Barasara, P.J. Prajapati, Prime cordial labeling for some path, cycle and wheel related graphs, Adv. Appl. Discr. Math. 30 (2018), 35-58.

[4] C.M. Barasara, Y.B. Thakkar, Divisor cordial labeling for some snakes and degree splitting related graphs, South East Asian J. Math. Math. Sci. 19 (2023), 211-224.

[5] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin. 23 (1987), 201–207.

[6] A. ELrokh, M.M.A. Al-Shamiri, M.M.A. Almazah, A.A. El-hay, A novel problem for solving permuted cordial labeling of graphs, Symmetry. 1 (2023), 825.

[7] J.A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin. 27 (2021), 1–712.

[8] F. Harary, Graph theory, Addison Wesely, New Delhi, (1972).

[9] I.N. Herstein, Topics in algebra, John Wiley and Sons, New York, (1991).

[10] S. Meena, S. Usharani, Product cordial labeling for some bicyclic graphs, Commun. Math. Appl. 13 (2022), 1329-2336.

[11] U.M. Prajapati, N.B. Patel, Edge product cordial labeling of some graphs, J. Appl. Math. Comput. Mech. 18 (2019), 69-76.

[12] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966) Gordon and Breach, N. Y. and Dunod Paris, (1967), 349–355.

[13] S. Sadawartea, S. Srivastava, Cordial labeling of some pan graphs, WCNC-2021: Workshop on Computer Networks & Communications-CEUR Workshop Proceedings, (2021), 1-5.

[14] M. Seoud, M. Aboshady, Further results on parity combination of cordial labeling, J. Egypt. Math. Soc. 28 (2020), 1-10.

[15] A.S. Rani, S.S. Devi, Dihedral group divisor cordial labeling for path, cycle graph, star graph, jelly fish and wheel graphs, AENG Int. J. Appl. Math. 54 (2024), 2148-2153.

[16] G. Vembarasi, R. Gowri, Super root cube of cube difference labeling of some special graphs, Indian J. Sci. Technol. 14 (2021), 2778-2783.