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MATHEMATICAL MODELING OF RADON CONCENTRATIONS IN SOIL

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Abstract: A mathematical model of radon concentrations in soil has been derived in its general form. Solutions so obtained are quiet new and have been derived and applied for different layers. The model has shown to match the experimental works for layers at one feet depth from the surface.

Keywords: Mathematical modelling, Radon emission, Biophysics Radiation, Soil.

2000 AMS Subject Classification: 97M60

1. Introduction

Radon is an inert gas with chemical symbol Radon-222 (^{222}Rn), and atomic number 86. It is a radioactive, colorless, odorless, tasteless noble gas, occurring naturally as a decay product of uranium [1, 2]. Radon is a gas, which is produced by the natural radioactive decay of the element radium. Spontaneous process is that an atom of one element decays or breaks down to produce another element by losing atomic particles (protons, neutrons, or electrons). Radon is formed as a part of the normal radioactive decay chain of uranium. Radon is also fairly soluble in water and organic solvents. Although reaction with other compounds is comparatively rare, it is not completely inert and forms stable molecules with highly electronegative materials. Radon is considered a noble gas that occurs in several isotopic forms, only two forms are found in significant concentrations in the human environment: Radon-222, and Radon-220.

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Radon-222 is a member of the radioactive decay chain of Uranium-238, while Radon-220 is formed in the decay chain of Thorium-232. Radon-222 is most readily occurred in the environment [3, 4]. Radon is a cancer-causing radioactive gas and it tends to have a distinctive smell, it is known to cause lungs health problems that could lead to cancer. Recent research has shown that radon is the second leading cause of lung cancer in the United States today [5]. The combination of cigarette smoke and Radon measures the probability of risk to developing lung cancer. Radon is also responsible for the public exposure to ionizing radiation. It is often the single largest contributor to an individual's background radiation dose, and it is the most variable from location to another. The primary routes of potential human exposure to radon are inhalation and ingestion. Radon in the soil, groundwater, or building materials is emitted and diffused in the working and living species and then disintegrates into its decay products. Although high concentrations of radon are in groundwater which might contribute to radon exposure through ingestion, the inhalation of radon released from water is the focus of this study. The radon concentration in soil air greatly influences the radon migration into buildings and almost all types of soil contain some radon. Radon exhalation rate in the water samples varied between 20.2 and 470.0 mBq/m^2 h [6, 7]. All rocks contain some uranium, between 1 to 3 parts per million (ppm) of uranium. In another word, the uranium in soil is about the same as in rock from which the soil was derived. Radon can get in the place through; cracks in solid floors, construction joints, cracks in walls and concrete slabs gaps in suspended floors, gaps around service pipes, and cavities inside walls and wells water. The higher Radon level can be most found in place where water has high levels of indoor radon [8, 9]. But some places have lots of uranium in the soil and low levels of indoor radon; other places on uranium-poor soils have high levels of indoor radon. This is due to other factors that affect the diffusion of radon to the atmosphere such as the location of radium atom in the mineral grain, the direction of the recoil of the radon atom, the ease and efficiency with which radon moves in the pore space or fracture, and the amount of water present in the pore space. Radon gas has much greater mobility than uranium and radium which are fixed in the solid matter, rocks and soils. Radon is usually released from fractures and openings in rocks and into the pore spaces among grains of the soil due to the existence of radium and its level depends on the radium amount within the soil.

2. Theoretical Background of a Radon Concentration in Soils

The one dimensional diffusion equation for proposed mathematical model can be written as:

$$\frac{\partial u}{\partial t} = D_e \frac{\partial^2 u}{\partial z^2} - \lambda u + S \tag{1}$$

where $\frac{\partial u}{\partial t}$ is the rate change of radon concentration,

$\lambda = 2.1 \times 10^{-6} s^{-1}$, which refers to the decay constant of radon,

D_e is a constant and for our convenience it is denoted as *D* throughout this research paper,

S is the creation rate of radon per unit bulk volume, which is measured by Bq/m^{3}s,

Equation (1) can be written with following boundary conditions:

$$\begin{gathered} \frac{\partial u}{\partial t} = D_e \frac{\partial^2 u}{\partial z^2} - \lambda u + S \\ u(0,t) = u_0 \quad , \qquad u(\infty,t) = u_{\max} \end{gathered} \tag{2}$$

<u>Case I steady state:</u> $\left(\frac{\partial u}{\partial t} = 0 \right)$

Neglecting the advection transport of radon, Equation (2) can be re-written as:

$$\frac{d^2 u}{dz^2} - \frac{\lambda}{D} u = -\frac{S}{D} \tag{3}$$

Equation (3) is a 2nd order linear non-homogenous ODE with constant coefficients and its general solution is

$$u_c = A_1 e^{\sqrt{\frac{\lambda}{D}} z} + A_2 e^{-\sqrt{\frac{\lambda}{D}} z} \tag{4}$$

The particular solution of u is:-

$$u_p = \frac{S}{\lambda}$$

The solution satisfies the regularity condition $u(\infty) = u_{\max}$

$$u = A_2 e^{-\sqrt{\frac{\lambda}{D}}z} + \frac{S}{\lambda} \,, \qquad u(\infty) = u_{\max} = \frac{S}{\lambda} \tag{5}$$

$$u(t) = A_2 e^{-\sqrt{\frac{\lambda}{D}}z} + \frac{S}{\lambda} \,, \;\; u_{\max} = \frac{S}{\lambda} \tag{6}$$

$$u(0) = u_0 = A_2 + \frac{S}{\lambda} \;\;, \;\; A_2 = u_0 - \frac{S}{\lambda} \tag{7}$$

$$u = \left(u_0 - \frac{S}{\lambda}\right)e^{-\frac{z}{l}} + \frac{S}{\lambda} \;\;, \;\; l = \sqrt{\frac{D}{\lambda}}$$

$$u(z) = \frac{S}{\lambda}\left(1 - e^{-\frac{z}{l}}\right) + u_0 \; e^{-\frac{z}{l}} \tag{8}$$

$$u(t) = u_{\max}\left(1 - e^{-\frac{z}{l}}\right) + u_0 \; e^{-\frac{z}{l}} \tag{9}$$

<u>Case II non-steady state:</u> $u(z,t), \;\; \frac{\partial u}{\partial t} \neq 0$

i.e. to solve equation 2, which is 2^{nd} order linear non-homogenous PDE.

We assume

$$u(z,t) = w(z,t) + v(z) \tag{10}$$

where $w(z,t)$ is the solution of homogenous PDE subject to boundary conditions.

i.e. $w(0,t) = 0, \quad w(\infty,t) = 0$

Substituting (11) into (2), we get

$$\frac{\partial w}{\partial t} = D\frac{\partial^2 w}{\partial z^2} + Dv''(z) - \lambda w - \lambda v + S \tag{11}$$

a) $\frac{\partial w}{\partial t} = D\frac{\partial^2 w}{\partial z^2} - \lambda w$, which is a homogeneous PDE (12a)

b) $Dv''(z) - \lambda v + S = 0$, which is 2nd order ODE (12b)

Equation (13a) can be written as:

$$\frac{1}{D}\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial z^2} - \frac{\lambda}{D}w \ , \quad w(0,t) = 0 \ , \ w(\infty,t) = 0$$

(13)

Let $w = Z(z)T(t)$

(14)

Equation (14), will take the form

$$T'Z = Z''T - \frac{\lambda}{D}ZT \tag{15}$$

$$\frac{1}{D}\left(\frac{T'}{T} + \lambda\right) = \frac{Z''}{Z} = -\gamma^2, \ Z(0) = 0 \tag{16}$$

where γ is a constant.

Let $Z(z) = \sin(\gamma z)$ (17)

Equation (16), has the solution

$$w(z,t) = e^{-\alpha^2 t} \int_0^{\infty} \beta(\gamma) e^{-\gamma^2 t} \sin(\gamma z)\, d\gamma \tag{18}$$

While, equation (12b) takes the form

$$Dv''(z) - \lambda v + S = 0, \quad v(0) = u_0\,, \quad v(\infty) = u_{\max} \tag{19}$$

Solution of equation (19), will take the form

$$v(z) = u_{\max}(1 - e^{-\alpha z}) + u_0 e^{-\alpha z} \tag{20}$$

Thus, the general solution of equation (10) can be furnished as:

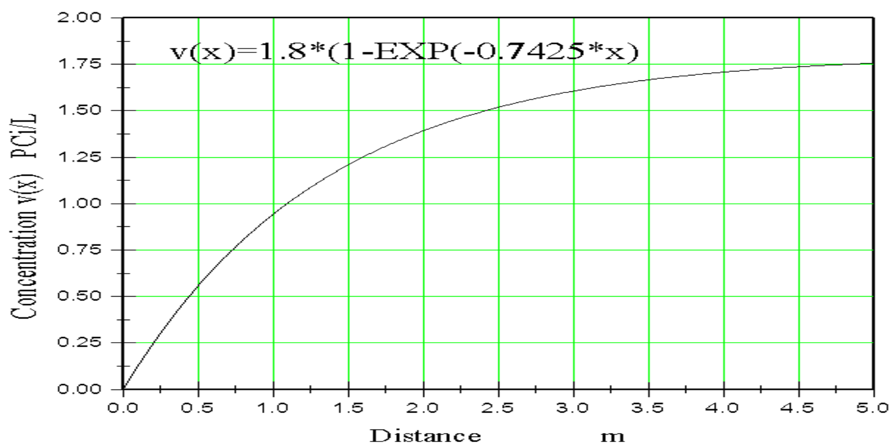
$$u(z,t) = u_{\max}(1 - e^{-\alpha z}) + u_0 e^{-\alpha z} + e^{-\alpha t} \int_0^{\infty} \beta(\gamma)\, e^{-\gamma^2 t} \sin(\gamma z)\; d\gamma \tag{21}$$

3. Physical and Sensitive Analysis

A mathematical model of radon emanation has been tackled and solved assuming uniform distribution of radium in a solid. The model deals with radon behaviour from its birthplace until its departure out of the surface. Bossus, 1984, has shown a typical model of radon according to its maximum possible value of radon emanation coefficient which is 0.25 for radon near the edge of the grain. While Nazaroff, 1992 states only the possible value of radon emanation is 0.005. This means radon emanation depends on the distribution in the grain.

An equation that describes the radon activity concentration with respect to the depth will be shown in Fig. below. However, before one does that, there are boundary conditions that need to be stated. Assumes that the soil is uniform and the problem can be approximated as one dimensional. The boundary conditions can be stated as $u \to u_{\max}$ as $z \to \infty$ and

$$u \to u_0 \; as \;\; z \to 0$$



Figure(1), shows radon concentration with distance for one feet depth from the surface

A variation of radon concentration with different layers can be shown in the figure 2 below.

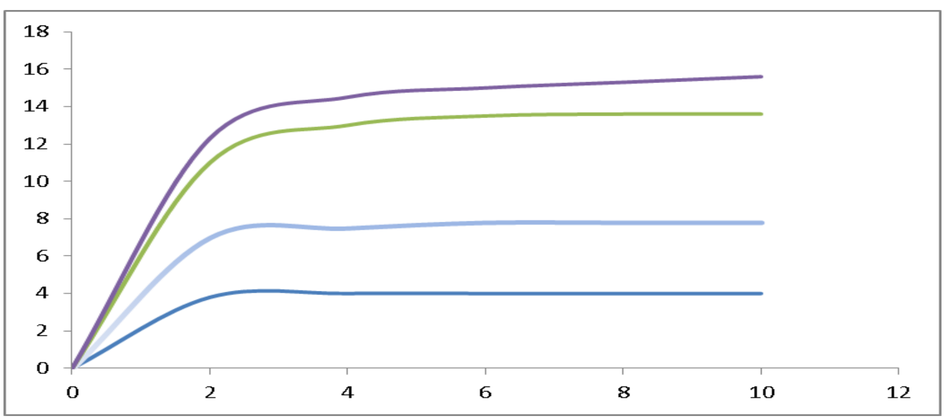


Figure (2), shows the variation of radon concentration (y-axis) with a distance (x-axis) into a soil.

The experimental works of samples were taken one feet depth from the surface and each sample had a weight of 700 grams. The sample had been placed into a system that consists of a sealed container of volume (64000 cm3) which was equipped with thermometer, humidity gauge, and radon monitor. The system had been interfaced to a computer. The data was collected for three days (72 h) and the factors considered at the beginning and end of this measurement were represented using the humidity (RH) and the temperature (T) inside the sealed container. The temperature and the humidity in the beginning of measurement were 24oC and 78%, respectively, and at the end of measurements were 26oC and 83%, respectively.

Summing up a mathematical model of a radon concentration in soil has been derived in its general form. Solutions so obtained are quiet new and have been derived and applied for different layers. While experimental works have been done and shown to match the layer of one feet depth from the surface. The experimental works will be considered in forthcoming paper.

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