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MINIMIZING TOTAL COMPLETION TIME IN A TWO-MACHINE FLOW-SHOP SCHEDULING PROBLEMS WITH A SINGLE SERVER

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Abstract. We consider the problem of two-machine flow-shop scheduling with a single server and equal processing times, we show that this problem isNP -hard in the strong sense and present a busy schedule for it with worst-case bound 7/6

Keywords:flow-shop scheduling problem, total completion time worst-case, single server.

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1. Introduction

We consider the two-machine flow-shop scheduling problem, which is described as follows.We are given n jobs $J_1, J_2, \cdots, J_n$,and two machines M_1 and M_2.Each job $J_j$consists of a chain $(O_{1,j}, O_{2,j}, \cdots, O_{n,j})$ of operations, and $O_{i,j}$ is to be processed on machine M_i for $p_{i,j}$ time units. Each machine can only process one operation at a time, and each job can be processed on at most one machine at a time. No preemption is allowed, i.e., once start, any operation can not be interrupted before it is completed. Immediately before processing an operation $O_{i,j}$ the corresponding machine,which takes a setup time of $s_{i,j}$ time units.

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During such a setup the machine is also occupied for $s_{i,j}$ time united, i.e., No other job can be processed on it.The setup times are assumed to be separable from the processing times, i.e., a setup on a subsequent machine may be performed while the job is still processed on the preceding machine. All setups have to be done by a single server M_S, which can perform at most one setup at a time. The problem we consider is to find a schedule S which minimizes the total completion times, that is $\sum_{i=1}^{n} C_j$.Following the three-field notation schedule introduced by Lentra et al [1], we denote this problem as $F2, S1||\sum_{i=1}^{n} C_j$. If all processing are equal to p, that is $p_{i,j} = p(i = 1, 2; j = 1, 2, \cdots, n)$,we have the $F2$, $S1|p_{i,j} = p|\sum_{i=1}^{n} C_j$ problem.

Complexity results for flow-shop problems obtained by Garey, et al [2], who studied two-machine flow-shop problem with minimizing total completion times, that is $F2||\sum_{i=1}^{n} C_j$. J.A.Hoogereen, et al [3] studied some special cases for two-machine flow-shop problems with minimizing total completion times, and proved that the problem with equal processing on first machine, that is $F2$, $S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$,is NP -hard in the strong sense, and present an $O(n\log(n))$ approximation algorithm for it with worst-case bound 4/3.Complexity results for flow-shop problems with a single server was obtained by Brucher, et al [4]. The complexity of parallel dedicated machine with a single server was obtained by Glass, et al [5].

In this paper , we derive some new complexity results for special cases of two-machine problem with a single server. The remainder of the paper is organized as follows. In section 2 we show that flow-shop problem with a single server ,equal processing times ,and minimizing total completion times is NP -hard in the strong sense. In section 3 we introduce a improved algorithm, and prove that its worst case is 7/6, the bound is tight.

2. Complexity of the $F2$,$S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$ problem

Let $C_{i,j}$ denote the completion times of job J_j on machine M_i . If there are no idle times on machine and machine , we have

$C_{1,1} = s_{1,1} + p_{1,1}$,$C_{2,1} = s_{1,1} + p_{1,1} + s_{2,1} + p_{2,1}$,

$C_{1,j} = C_{1,j-1} + s_{1,j} + p_{1,j}$,

$C_{2,j} = max(C_{2,j-1}, C_{1,j}) + s_{2,j} + p_{2,j})$, for $j = 2, 3, \cdots, n$

Theorem 2. *The problem of* $F2,S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$ *is* NP*-hard in the strong sense.*

Proof. Our proof is based upon a reduction from the problem Numerical Matching with Target Sums or, in short, Target Sum, which is known to be NP -hard in the strong sense[6].

Target Sum. Given two multisets $X = x_1, x_2, \cdots, x_n$ and $Y = y_1, y_2, \cdots, y_n$ of positive integers and an target vector$z_1, z_2, \cdots, z_n$,where $\sum_{i=1}^{n}(x_j + y_j)$=$\sum_{i=1}^{n} z_j$,is there a position of the set$X \bigcup Y$ into n disjoint set $Z_1, Z_2, \cdots, Z_n$,each containing exactly one element from each of X and Y ,such that the sum of the numbers in $Z_j$equal z_j,for $i = 1, 2, \cdots, n$?

(1)P-jobs: $s_{1,i} = b, p_{1,i} = b, s_{2,i} = b + x_i p_{2,i} = b(i = 1, 2, \cdots, n)$

(2)Q-jobs:$s_{1,i} = 0, p_{1,i} = b, s_{2,i} = b + y_i, p_{2,i} = b(i = 1, 2, \cdots, n)$

(3)R-jobs:$s_{1,i} = 0, p_{1,i} = b, s_{2,i} = b - z_i, p_{2,i} = b(i = 1, 2, \cdots, n)$

(4)U-jobs:$s_{1,i} = 0, p_{1,i} = b, s_{2,i} = 0, p_{2,i} = b(i = 1, 2, \cdots, n)$

(5)L-jobs:$s_{1,i} = 4b, p_{1,i} = b, s_{2,i} = b, p_{2,i} = b(i = 1, 2, \cdots, n)$

Observe that all processing times are equal to y.To prove the theorem we show that in this constructed if the $F2,S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$ problem a schedule S_0 satisfying $\sum_{i=1}^{n} C_j(S_0) \leq y$ =$\sum_{i=1}^{n} x_j + \sum_{i=1}^{n}(x_j + y_j)$ +$(77n^2 - 13n - 4)b/2$ exists if and only if Target Num has a solution.

Suppose that Target Num has a solution. The desired schedule S_0 exists and can be described as follows. No machine has intermediate idle time. Machine M_i process the jobs in order of the sequence σ , i.e., in the sequence

$\sigma = (\sigma_{P_{1,1}}, \sigma_{Q_{1,1}}, \sigma_{R_{1,1}}, \sigma_{U_{1,1}}, \sigma_{V_{1,1}}, \sigma_{W_{1,1}}, \sigma_{L_{1,1}}, \cdots, \sigma_{P_{1,n}}, \sigma_{Q_{1,n}}, \sigma_{R_{1,n}}, \sigma_{U_{1,n}}, \sigma_{V_{1,n}}, \sigma_{W_{1,n}}, \sigma_{L_{1,n}})$

While machine M_2 process the jobs in the sequence

$\tau = (\tau_{P_{2,1}}, \tau_{Q_{2,1}}, \tau_{R_{2,1}}, \tau_{U_{2,1}}, \tau_{V_{2,1}}, \tau_{W_{2,1}}, \tau_{L_{2,1}}, \cdots, \tau_{P_{2,n}}, \tau_{Q_{2,n}}, \tau_{R_{2,n}}, \tau_{U_{2,n}}, \tau_{V_{2,n}}, \tau_{W_{2,n}}, \tau_{L_{2,n}})$

as indicated in Figure 1.

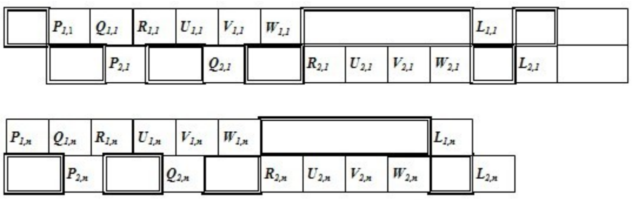

Fig.1 Gant chart for the $F2, S1|p_{i,j} = p|\sum C_j$ problem

Then we define the sequence and shown in Figure 1. Obviously, these sequence σ and τ fulfills $C(S) = C(\sigma, \tau) \leq y$. Conversely, assume that the flow-shop scheduling problem has a solution σ and τ with$C(S) \leq y$.

Considering the path composed of machine M_1 operations of jobs $(P_{1,1}, Q_{1,1}, R_{1,1}, U_{1,1}, V_{1,1}, W_{1,1})$. Machine M_2 operations of jobs $(R_{1,1}, U_{2,1}, V_{2,1}, W_{2,1}, L_{2,1}, \cdots, R_{2,n}, U_{2,n}, V_{2,n}, W_{2,1}, L_{2,n})$, we obtain that

$C(S) \geq 3b + x_1 + 5b + x_1 + y_1 + 7b + x_1 + y_1 - z_1 + 8b + 9b + 10b + \cdots (3 + (n-1)11)b + x_n + (5 + (n-1)11)b + x_n = y_n + (7 + (n-1)11)b + \cdots + (11n+1)b = \sum_{i=1}^{n} x_j + \sum_{i=1}^{n}(x_j + y_j)$ $+(77n^2 - 13n - 4)b/2 = y$, So we have$C(S) = y$.

(a) If S has a partition μ,then there is a schedule with finish times y. One such schedule is shown in Figure 1.

(b) If S has no partition, then all schedule must have a finish times$> y$. Since S has no partition, then $x_i + y_i \neq z_i (i = 1, 2, \cdots, n)$. Let $\xi_i = x_i + y_i - z_i$, we have

$\sum_{i=1}^{n} C_j(S) = \sum_{i=1}^{n} x_j + \sum_{i=1}^{n}(x_j + y_j) + (77n^2 - 13n - 4)b/2 + 5\sum_{i=1}^{n} \xi_i + 10\sum_{i=1}^{n-1} \xi_i + \cdots + 5n\xi_1 > y$.

3. Worst-case for the $F2$,$S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$ problem

In examining "worst" schedule, we restrict ourselves to busy schedule. A busy schedule is a schedule in which at all times from start to finish at least one server is processing a task.

Theorem 3. *The problem of $F2$,$S1|p_{1,j} = p|\sum_{i=1}^{n} C_j$ problem, let S_0 be a busy schedule for this problem, S^* be the optimal solution for the $F2$,$S1|p_{1,j} = 1|\sum_{i=1}^{n} C_j$ problem ,then $\sum_{i=1}^{n} C_j(S_0) / \sum_{i=1}^{n} C_j(S^*) \leq 7/6$. The bound is tight.*

Proof. *For a schedule S, let $I_{i,j}(S) (i = 1, 2, j = 1, 2, \cdots, n)$ denote the total idle times of job J_j on machine M_i.*

Considering the path composed of machine M_1 operations of jobs $1, 2, \cdots, l$, machine M_2 operation of job j, we obtain that $C_j = \sum_{i=1}^{j}(s_{1,i} + p_{1,i}) + I_{1,j} + s_{2,j} + p_{2,j}$ (1)

Considering the path composed of machine M_1 operations of job 1 , machine M_2 operation of jobs $1, 2, \cdots, j$, we obtain that $C_j = s_{1,1} + p_{1,1} + \sum_{i=1}^{j}(s_{2,i} + p_{2,i}) + I_{2,j}$ (2)

Considering the path composed of machine M_1 operations of jobs $1, 2, \cdots, l$, machine M_2 operation of jobs $l, l+1, \cdots, j$, we obtain that $C_j = \sum_{i=1}^{l}(s_{1,i} + p_{1,i}) + I_{1,j} + \sum_{i=l}^{j}(s_{2,i} + p_{2,i}) + I_{2,j}$ (3)

So we have

$6\sum_{j=1}^{n} C_j(S_0) = 2(\sum_{i=1}^{j}((s_{1,i}+p_{1,i})+I_{1,j}+s_{2,j}+p_{2,j}) + 2(s_{1,1}+p_{1,1}+\sum_{i=1}^{j}(s_{2,i}+p_{2,i})+I_{2,j})$
$+2(\sum_{i=1}^{l}(s_{1,i}+p_{1,i}) + I_{1,j} + \sum_{i=l}^{j}(s_{2,j}+p_{2,j}) + I_{2,j}) \leq 7\sum_{j=1}^{n} C_j(S^*)$

$\sum_{i=1}^{n} C_j(S_0)/\sum_{i=1}^{n} C_j(S^*) \leq 7/6.$

To prove the bound is tight, introduce the following example as show in Fig.2 and Fig. 3.

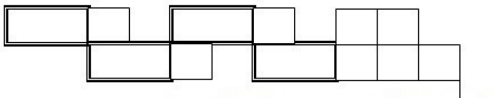

Fig.2 $\sum C_j(S_0)$

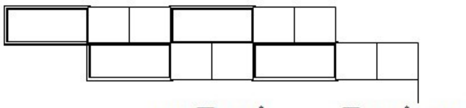

Fig.3 $\sum C_j(S^*)$

(1)$s_{1,i} = 2b, p_{1,i} = b, s_{2,i} = 2b, p_{2,i} = b(i = 1, 2)$

(2)$s_{1,i} = 0, p_{1,i} = b, s_{2,i} = 0, p_{2,i} = b(i = 3, 4)$

So we have $\sum_{i=1}^{n} C_j(S_0)/\sum_{i=1}^{n} C_j(S^*) = 35b/30b = 7/6$, *the bound is tight.*

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