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A STUDY IN INTUITIONISTIC Q – FUZZY IDEALS OF KU – ALGEBRAS

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Abstract. In this paper, we introduce the notion of intuitionistic Q – fuzzy KU – ideal in KU – algebra, upper and lower level cuts of Q – fuzzy sets and some properties are investigated.

Keywords: KU – algebras; intuitionistic Q – fuzzy set; intuitionistic Q – fuzzy sub algebra; intuitionistic Q – fuzzy KU – ideal; upper level cuts; lower level cuts.

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1. INTRODUCTION

The introduction of BCI – algebra and BCK – algebras by Y. Imai and K. Iseki [1, 2], BCK algebras class is a proper sub class of BCI – algebras class. J. Neggers et al [3] introduced Q – algebras as a generalization of BCK and BCI algebras. In [4], C. Prabpayak and U. Leerawat introduced KU – algebra as a new algebraic structure. They obtained the notion of KU – algebras homomorphism. L. A. Zadeh [5] in 1965 gave the concept of a fuzzy subset of a set. This notion has been applied to many mathematical branches, such as groups, rings, topology, real and so on.

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Xi [6] applied this concept to BCK – algebras, and he introduced fuzzy sub algebras notion. Mostafa and Abdel Naby [7] introduced fuzzy KU – ideals in KU – algebras. Sithar Selvam and Ramachandran [8] introduced the concept of anti Q – fuzzy KU – ideal and sub algebras of KU – algebras and investigated some related properties. Atanassov [9, 10] introduced the concept of intuitionistic fuzzy subset as generalization of fuzzy set. Mostafa et al [11] introduced intuitionistic fuzzy KU ideals and fuzzy intuitionistic image of KU – ideals in KU – algebras. On the other hand Massa'deh and Massa'deh et al used the intuitionistic fuzzy concept in more than one paper (see [12, 13, 14, 15]).

In this paper, we introduce the concept of intuitionistic Q –fuzzy KU – ideal in KU – algebra and we define upper and lower level cuts of Q – fuzzy sets and discuss some results related to this subject.

2. PRELIMINARIES

Definition 2.1 [4] An algebra system (A, *, 0) for type (2, 0) is said to be KU – algebra if the following conditions are satisfied.

1. $(a * b) *[(b *c) * (a * c)] = 0$
2. $a * 0 = 0$
3. $0 * a = a$
4. If $a * b = 0 = b * a$ then $a = b$.

For all $a, b, c \in A$.

In KU – algebra A, we get $(0 * 0) * [(0 * a) * (0* a)] = 0$. It follows that $a * a = 0$ for all $a \in A$, and if we put $b = 0$ in condition.1, we obtain $c * (a * c) = 0$ for all $a, c \in A$, A subset B of a KU – algebra A is called sub algebra of A, if $u, v \in B$ then $u * v \in B$.

Definition 2.2 [4] If S is a non empty subset of a KU – algebra A, then its said to be KU – sub algebra of A, if $a, b \in S$ then $a * b \in S$.

Definition 2.3 [4] A KU – ideal S is non empty subset of KU – algebra A if it satisfied the following axioms:

I. $0 \in S$.

II. $a *(b * c) \in S$, $b \in S$ then $a * c \in S$ for all $a, b, c \in S$.

Proposition 2.4 [4] In KU – algebra A, the following statement are holds

1. $v \geq u \Rightarrow u * z \geq v * z$

2. $z * (v * u) = v * (z * u)$

3. $v * [(v * u) * u] = 0 \ \forall u, v, z \in A$

Proof: Straightforward.

Definition 2.5 [5] Let A be a nonempty set, a fuzzy subset λ of a set A is a mapping λ: A $\rightarrow$ [0,1].

Definition 2.6 If A, Q are any two sets, a mappingλ: $A \times Q \rightarrow [0, 1]$ is called Q – fuzzy set in A.

Definition 2.7 [8] A Q – fuzzy set λ in A is said to be a Q – fuzzy KU – ideal of A if

1. $\lambda(0, q) \geq \lambda(u, q)$

2. $\lambda(u * z, q) \geq \min \{ \lambda(u * (v * z), q), \lambda(v, q) \}$

for all $u, v, z \in A$ & $q \in Q$.

Lemma 2.8 [8] Let δ be a Q – fuzzy ideal of KU – algebra A

1. If $u * v \leq z$, then $\lambda(v, q) \geq \min \{ \lambda(u, q), \lambda(z, q) \}$

2. $u \leq v$, then $\lambda(v, q) \leq \lambda(u, q)$.

Proof: Straightforward.

Definition 2.9 [8] If λ is a Q – fuzzy set on a KU – algebra A, then λ is called a Q – fuzzy KU – sub algebra of A if $\lambda(u * v, q) \geq \min \{ \lambda(u, q), \lambda(v, q) \}$ for all $u, v \in A$ & $q \in Q$.

3. INTUITIONISTIC Q – FUZZY KU – IDEAL IN KU – ALGEBRA

Definition 3.1 [8] Let A, Q are arbitrary non empty sets. An intuitionistic Q - fuzzy subset μ in a set $A \times Q$ is defined as an object of the form $\mu = \{ < (a,q); \delta_\mu (a, q), \lambda_\mu (a, q) > ; a \in A \ \& \ q \in Q\}$, where $\delta_\mu : A \times Q \rightarrow [0,1]$ and $\lambda_\mu: A \times Q \rightarrow [0,1]$ define the degree of membership and the degree of non membership of the element $(a,q) \in A \times Q$ respectively and for every $a \in A$, $q \in Q$ satisfying $0 \leq \delta_\mu (a, q) + \lambda_\mu (a, q) \leq 1$.

We shall use the symbol $\mu = (\delta_\mu , \lambda_\mu)$ for intuitionistic Q – fuzzy set $\mu = \{< (a,q); \delta_\mu (a, q), \lambda_\mu (a, q) > ; a \in A \ \& \ q \in Q\}$.

Definition 3.2 An intuitionistic Q – fuzzy set $\mu = (\delta_\mu , \lambda_\mu)$ in a KU – algebra A is said to be an intuitionistic Q – fuzzy KU – sub algebra of A. If it satisfies the following conditions.

1. $\lambda_\mu(u * v, q) \geq \min \{ \lambda_\mu(u, q), \lambda_\mu(v, q) \}$

2. $\delta_\mu(u * v, q) \leq \max \{ \delta_\mu(u, q), \delta_\mu(v, q) \}$

For all $u, v \in A$ & $q \in Q$.

Lemma 3.3 If $\mu = (\delta_\mu , \lambda_\mu)$ is an intuitionistic Q – fuzzy sub algebra of A, then . $\lambda_\mu(0, q) \geq \lambda_\mu(u, q)$ and $\delta_\mu(0, q) \leq \delta_\mu(u, q)$ for all $u \in A$ & $q \in Q$.

Proof:

$\lambda_\mu(u * u, q) \geq \min \{ \lambda_\mu(u, q), \lambda_\mu(u, q) \} = \lambda_\mu(u, q) = \lambda_\mu(0, q)$

and $\delta_\mu(0, q) = \delta_\mu(u * v, q) \leq \max \{ \delta_\mu(u, q), \delta_\mu(u, q) \} = \delta_\mu(u, q)$.

Definition 3.4 An intuitionistic Q – fuzzy set $\mu = (\delta_\mu , \lambda_\mu)$ in a KU – algebra A is said to be an intuitionistic Q – fuzzy ideal of A, if it satisfy the following conditions

1. $\lambda_\mu(0, q) \geq \lambda_\mu(u, q)$ and $\delta_\mu(0, q) \leq \delta_\mu(u, q)$
2. $\lambda_\mu(u * z, q) \geq \min \{ \lambda_\mu(u *(v * z), q), \lambda_\mu(v, q) \}$
3. $\delta_\mu(u * z, q) \leq \max \{ \delta_\mu(u *(v * z), q), \delta_\mu(v, q) \}$

For all $u, v, z \in A$ & $q \in Q$.

Theorem 3.5 Let μ be an intuitionistic Q – fuzzy KU – ideal of KU – algebra A such that $u * v \leq z$, then

1. $\lambda_\mu(v, q) \geq \min \{ \lambda_\mu(u, q), \lambda_\mu(v, q) \}$
2. $\delta_\mu(v, q) \leq \max \{ \delta_\mu(u, q), \delta_\mu(v, q) \}$

For all $u, v \in A$ & $q \in Q$.

Proof:

We know $u * v \leq z$ for all $u, v, z \in A$ thus $z *(u * v) = 0$. Now

1. $\lambda_\mu(v, q) = \lambda_\mu(0 *v, q)$

$\geq \min \{\lambda_\mu(0 *(u * v), q), \lambda_\mu(u, q) \}$

$= \min \{\lambda_\mu (u * v , q), \lambda_\mu(u, q) \}$

$\geq \min \{\min \{ \lambda_\mu(u *(z * v), q), \lambda_\mu(z, q) \}, \lambda_\mu(u, q) \}$

$= \min \{\min \{ \lambda_\mu(z *(u * v), q), \lambda_\mu(z, q) \}, \lambda_\mu(u, q) \}$

$= \min \{\min \{ \lambda_\mu(0, q), \lambda_\mu(v, q) \}, \lambda_\mu(u, q) \}$

$= \min \{\lambda_\mu(u, q), \lambda_\mu(v, q) \}$

Therefore $\lambda_\mu(v, q) \geq \min \{ \lambda_\mu(u, q), \lambda_\mu(v, q) \}$.

2. $\delta_\mu(v, q) = \delta_\mu(0 *v, q)$

$\leq \max \{\delta_\mu(0 *(u * v), q), \delta_\mu(u, q) \}$

$= \max \{\delta_\mu (u * v , q), \delta_\mu(u, q) \}$

$\leq \max \{\max \{ \delta_\mu(u *(z * v), q), \delta_\mu(z, q) \}, \delta_\mu(u, q) \}$

$= \max \{\max \{ \delta_\mu(z *(u * v), q), \delta_\mu(z, q) \}, \delta_\mu(u, q) \}$

$= \max \{\max \{ \delta_\mu(0, q), \delta_\mu(v, q) \}, \delta_\mu(u, q) \}$

$= \max \{\delta_\mu(u, q), \delta_\mu(v, q) \}$

Therefore $\delta_\mu(v, q) \leq \max \{ \delta_\mu(u, q), \delta_\mu(v, q) \}$.

Theorem 3.6 If μ is an intuitionistic Q – fuzzy KU – ideal of KU – algebra A, then for all u, v ∈ A & q ∈ Q, we have $\lambda_\mu(u *(u * v), q) \geq \lambda_\mu(v, q)$ and $\delta_\mu(u *(u * v), q) \leq \delta_\mu(v, q)$.

Proof:

Let u, v ∈ A & q ∈ Q. Then

$\lambda_\mu(u *(u * v), q) \geq \min \{ \lambda_\mu(u *(u * v), q), \lambda_\mu(v, q)\}$

$= \min \{ \lambda_\mu(u *(u *(v * v)), q), \lambda_\mu(v, q)\}$

$= \min \{ \lambda_\mu(u *(u * 0), q), \lambda_\mu(v, q)\}$

$= \min \{ \lambda_\mu(u * 0), q), \lambda_\mu(v, q)\}$

$= \min \{ \lambda_\mu(0, q), \lambda_\mu(v, q)\}$

$= \lambda_\mu(v, q)$

Hence $\lambda_\mu(u *(u * v), q) \geq \lambda_\mu(v, q)$.

On the other hand

$\delta_\mu(u *(u * v), q) \leq \max \{ \delta_\mu(u *(u * v), q), \delta_\mu(v, q)\}$

$= \max \{ \delta_\mu(u *(u *(v * v)), q), \delta_\mu(v, q)\}$

$= \max \{ \delta_\mu(u *(u * 0), q), \delta_\mu(v, q)\}$

$= \max \{ \delta_\mu(u * 0), q), \delta_\mu(v, q)\}$

$= \max \{ \delta_\mu(0, q), \delta_\mu(v, q)\}$

$= \delta_\mu(v, q)$

Hence $\delta_\mu(u *(u * v), q) \leq \delta_\mu(v, q)$.

Definition 3.7 For any α, β ∈[0, 1] and a Q – fuzzy set μ in a non empty set A, the set $\mu^\alpha = \{u \in A, q \in Q; \mu(u, q) \geq \alpha\}$ is called an upper α – level cut of μ and $\mu_\beta = \{u \in A, q \in Q; \mu(u, q) \leq \beta\}$ is called a lower β – level cut of μ.

Theorem 3.8 If μ is an intuitionistic Q – fuzzy KU – ideal of KU – algebra A, then $\mu^\alpha{}_{\lambda\mu}$, $\mu_{\beta\delta\mu}$ are a KU – ideal of A for every α, β ∈ [0, 1].

Proof:

Hence μ is an intuitionistic Q – fuzzy KU – ideal of A

1. Let $u \in \mu^{\alpha}{}_{\lambda\mu}$ this means that $\lambda_\mu(u, q) \geq \alpha$,

$\lambda_\mu(0, q) = \lambda_\mu(v * 0), q)$

$\geq \min \{ \lambda_\mu(v *(u * 0), q), \lambda_\mu(u, q)\}$

$= \min \{ \lambda_\mu((v * 0), q), \lambda_\mu(u, q)\}$

$= \min \{ \lambda_\mu(0, q), \lambda_\mu(u, q)\}$

$= \lambda_\mu(u, q)$

$\geq \alpha$

Thus $0 \in \mu^{\alpha}{}_{\lambda\mu}$.

2. Let $u *(v * z) \in \mu^{\alpha}{}_{\lambda\mu}$ and $v \in \mu^{\alpha}{}_{\lambda\mu}$ for all u, v, z $\in$ A & $q \in Q$, $u *(v * z) \in \mu^{\alpha}{}_{\lambda\mu}$ and $v \in \mu^{\alpha}{}_{\lambda\mu}$ for all u, v, z $\in$A this implies that $\lambda_\mu(u *(v * z), q) \geq \alpha$ and $\lambda_\mu(v, q) \geq \alpha$. $\lambda_\mu((v * z), q) \geq \min\{ \lambda_\mu(u *(v * z), q) , \lambda_\mu(v, q)\} \geq \min\{\alpha, \alpha\} = \alpha$, thus $v * z \in \mu^{\alpha}{}_{\lambda\mu}$ and we get $\mu^{\alpha}{}_{\lambda\mu}$ an KU – ideal of A for every $\alpha \in [0, 1]$.

On the other hand

1. Let $u \in \mu_{\beta\delta\mu}$ this means that $\delta_\mu(u, q) \leq \beta$,

$\delta_\mu (0, q) = \delta_\mu (v * 0), q)$

$\leq \max \{ \delta_\mu (v *(u * 0), q), \delta_\mu (u, q)\}$

$= \max \{ \delta_\mu ((v * 0), q), \delta_\mu (u, q)\}$

$= \max \{ \delta_\mu (0, q), \delta_\mu (u, q)\}$

$= \delta_\mu (u, q)$

$\leq \beta$

Thus $0 \in \mu_{\beta\delta\mu}$.

2. Let $u *(v * z) \in \mu_{\beta\delta\mu}$ and $v \in \mu_{\beta\delta\mu}$ for all u, v, z $\in$ A & $q \in Q$, $u *(v * z) \in \mu_{\beta\delta\mu}$ and $v \in \mu_{\beta\delta\mu}$ for all u, v, z $\in$A this implies that $\delta_\mu (u *(v * z), q) \leq \beta$ and $\delta_\mu (v, q) \leq \beta$. $\delta_\mu ((v * z), q) \leq \max\{ \delta_\mu (u *(v * z), q) , \delta_\mu (v, q)\} \leq \max\{\beta, \beta\} = \beta$, thus $v * z \in \mu_{\beta\delta\mu}$ and we get $\mu_{\beta\delta\mu}$ an KU – ideal of A for every $\beta \in [0, 1]$.

Theorem 3.9 Let μ be an intuitionistic Q – fuzzy set of KU – algebra A. If for each $\alpha, \beta \in [0, 1]$, and $\mu^{\alpha}{}_{\lambda\mu}$, $\mu_{\beta\delta\mu}$is a KU – ideal of A, then μ is an intuitionistic Q – fuzzy KU – ideal of A.

Proof: Straightforward.

Theorem 3.10 An intuitionistic Q – fuzzy set $\mu = (\delta_\mu , \lambda_\mu)$ is an intuitionistic Q – fuzzy KU – ideal of A if and only if for all $\alpha, \beta \in [0, 1]$, the set $\mu^\alpha{}_{\lambda\mu}$ and $\mu_{\beta\delta\mu}$ are either empty or KU – ideal of A.

Proof:

$\Rightarrow$ Let $\mu = (\delta_\mu , \lambda_\mu)$ is an intuitionistic Q – fuzzy KU – ideal of A and $\mu^\alpha{}_{\lambda\mu} \neq \phi \neq \mu_{\beta\delta\mu}$. Since $\lambda_\mu(0, q) \geq \alpha$ and $\delta_\mu (0, q) \leq \beta$, let $u, v, z \in A$ be such that $u *(v * z) \in \mu^\alpha{}_{\lambda\mu}$, $v \in \mu^\alpha{}_{\lambda\mu}$. Then $\lambda_\mu(u *(v * z), q) \geq \alpha$ and $\lambda_\mu(v, q) \geq \alpha$, it follows that $\lambda_\mu(u * z), q) \geq \min \{ \lambda_\mu(u *(v * z), q), \lambda_\mu(v, q)\} \geq \alpha$ thus $u * z \in \mu^\alpha{}_{\lambda\mu}$. Therefore $\mu^\alpha{}_{\lambda\mu}$ is an KU – ideal of A.

On the other hand, if $u, v, z \in A$ such that $u *(v * z) \in \mu_{\beta\delta\mu}$, then $\delta_\mu(u *(v * z), q) \leq \beta$ and $\delta_\mu(v, q) \leq \beta$ thus $\delta_\mu(u * z), q) \leq \max \{ \delta_\mu(u *(v * z), q), \delta_\mu(v, q)\} \leq \beta$ thus $u * z \in \mu_{\beta\delta\mu}$. Therefore $\mu_{\beta\delta\mu}$ is an KU – ideal of A.

$\Leftarrow$ Suppose that for each $\alpha, \beta \in [0, 1]$, the sets $\mu^\alpha{}_{\lambda\mu}$ and $\mu_{\beta\delta\mu}$ are either empty or KU – ideal of A. For any $u \in A$, let $\lambda_\mu(u, q) = \alpha$ and $\delta_\mu(u, q) = \beta$, then $u \in \mu^\alpha{}_{\lambda\mu} \cap \mu_{\beta\delta\mu}$ and $\mu^\alpha{}_{\lambda\mu} \neq \phi \neq \mu_{\beta\delta\mu}$. Since $\mu^\alpha{}_{\lambda\mu}$ and $\mu_{\beta\delta\mu}$ are KU – ideal of A, therefore $0 \in \mu^\alpha{}_{\lambda\mu} \cap \mu_{\beta\delta\mu}$ hence $\lambda_\mu(0, q) \geq \alpha = \lambda_\mu(u , q)$ and $\delta_\mu (0, q) \leq \beta = \delta_\mu (u, q)$ for all $u \in A$. If there exist $d, e, f \in A$ be such that $\lambda_\mu(d * f, q) \geq \min \{ \lambda_\mu(d *(e * f), q), \lambda_\mu(e, q)\}$ by taking $\alpha_0 = ½\{ \lambda_\mu(d * f, q) + \min \{ \lambda_\mu(d *(e * f), q), \lambda_\mu(e, q)\}\}$ we get $\lambda_\mu(d * f, q) < \alpha_0 < \min \{ \lambda_\mu(d *(e * f), q), \lambda_\mu(e, q)\}$and hence $d * e \notin \mu^{\alpha 0}{}_{\lambda\mu}$, $d *(e * f) \in \mu^{\alpha 0}{}_{\lambda\mu}$ and $e \in \mu^{\alpha 0}{}_{\lambda\mu}$, this means that $\mu^{\alpha 0}{}_{\lambda\mu}$ is not an KU – ideal of A and this is contradiction. Now, assume that there exist $u, v, z \in A$ such that $\delta_\mu((u * z), q) \geq \max \{ \delta_\mu(u *(v * z), q), \delta_\mu(v, q)\}$ by taking $\beta_0 = ½\{ \delta_\mu(u * z, q) + \max \{ \delta_\mu(u *(v * z), q), \delta_\mu(v, q)\}\}$ we get $\max \{ \delta_\mu(u *(v * z), q), \delta_\mu(v, q)\}\} < \beta_0 < \delta_\mu(u * z, q)$ thus $u *(v * z) \in \mu_{\beta 0\delta\mu}$ and $v \in \mu_{\beta 0\delta\mu}$ while $(x * z) \notin \mu_{\beta 0\delta\mu}$ which is contradiction and this complete proof.

Definition 3.11 Let A be an KU – algebra and $a, b \in A$, we can define a set $U(a, b) = \{ a \in A; a *(b *a) = 0\}$. It easy to see that $0, a, b \in U(a, b)$ for all $a, b \in A$.

Theorem 3.12 Let μ be an intuitionistic Q – fuzzy set in KU – algebra A. Then μ is an intuitionistic Q – fuzzy KU – ideal of A if and only if μ satisfies the following condition. For all $a, b \in A$; $\alpha, \beta \in[0, 1]$, $(a, b) \in \mu^\alpha{}_{\lambda\mu}$ thus $U(a, b) \subseteq \mu^\alpha{}_{\lambda\mu}$ and $(a, b) \in \mu_{\beta\delta\mu}$ thus $U(a, b) \subseteq \mu_{\beta\delta\mu}$.

Proof:

$\Rightarrow$ Suppose that μ is an intuitionistic Q – fuzzy KU – ideal of A, now let u, v $\in \mu^{\alpha}{}_{\lambda\mu}$. Then $\lambda_\mu(u, q) \geq \alpha$ and $\lambda_\mu(v, q) \geq \alpha$ let $a \in U(u, v)$. Then $u *(v * a) = 0$, now $\lambda_\mu(a, q) = \lambda_\mu(a * 0, q)$

$\geq \min \{ \lambda_\mu(0 *(v * a), q), \lambda_\mu(v, q)\}$

$= \min \{ \lambda_\mu(v * a), q), \lambda_\mu(v, q)\}$

$\geq \min \{ \min\{\lambda_\mu(v *(u * a), q), \lambda_\mu(u, q)\}, \lambda_\mu(v, q)\}$

$= \min \{ \min\{\lambda_\mu(u *(v * a), q), \lambda_\mu(u, q)\}, \lambda_\mu(v, q)\}$

$= \min \{ \min\{\lambda_\mu(0 , q), \lambda_\mu(u, q)\}, \lambda_\mu(v, q)\}$

$= \min\{\lambda_\mu(u , q), \lambda_\mu(v, q)\}$

$= \min \{\alpha, \alpha\}$

$= \alpha.$

Thus $\lambda_\mu(a, q) \geq \alpha$. And hence $a \in \mu^{\alpha}{}_{\lambda\mu}$ therefore $U(a, b) \subseteq \mu^{\alpha}{}_{\lambda\mu}$

And let u, v $\in \mu_{\beta\delta\mu}$. Then $\delta_\mu(u, q) \leq \beta$ and $\delta_\mu(v, q) \leq \beta$ let $a \in U(u, v)$. Then $u *(v * a) = 0$, now

$\delta_\mu(a, q) = \delta_\mu(a * 0, q)$

$\leq \max \{ \delta_\mu(0 *(v * a), q), \delta_\mu(v, q)\}$

$= \max \{ \delta_\mu(v * a), q), \delta_\mu(v, q)\}$

$\leq \max \{ \max\{\delta_\mu(v *(u * a), q), \delta_\mu(u, q)\}, \delta_\mu(v, q)\}$

$= \max \{ \max\{\delta_\mu(u *(v * a), q), \delta_\mu(u, q)\}, \delta_\mu(v, q)\}$

$= \max \{ \max\{\delta_\mu(0 , q), \delta_\mu(u, q)\}, \delta_\mu(v, q)\}$

$= \max\{\delta_\mu(u , q), \delta_\mu(v, q)\}$

$= \min \{\beta, \beta\}$

$= \beta.$

Thus $\delta_\mu(a, q) \leq \beta$. And hence $a \in \mu_{\beta\delta\mu}$ therefore $U(a, b) \subseteq \mu_{\beta\delta\mu}$

$\Leftarrow$ Assume that $U(a, b) \subseteq \mu^{\alpha}{}_{\lambda\mu}$, its clear that $0 \in U(a, b) \subseteq \mu^{\alpha}{}_{\lambda\mu}$ for all a, b $\in$ A

Now, let u, v, z $\in$ A such that $u *(v * z) \in \mu^{\alpha}{}_{\lambda\mu}$ and $v \in \mu^{\alpha}{}_{\lambda\mu}$ since $(u *(v * z)) * (v *(u * z)) = (v *(u * z)) * (u *(v * z)) = 0$ and we have $u * z \in U(u *(v * z), v) \subseteq \mu_{\beta\delta\mu}$. Thus $\mu^{\alpha}{}_{\lambda\mu}$ is an KU – ideal of A.

And, suppose that $U(a, b) \subseteq \mu_{\beta\delta\mu}$ its clear that $0 \in U(a, b) \subseteq \mu_{\beta\delta\mu}$ for all a, b $\in$ A

Now, let u, v, z $\in$ A such that $u *(v * z) \in \mu_{\beta\delta\mu}$ and $v \in \mu_{\beta\delta\mu}$ since $(u *(v * z)) * (v *(u * z)) = (v *(u * z)) * (u *(v * z)) = 0$ and we have $u * z \in U(u *(v * z), v) \subseteq \mu_{\beta\delta\mu}$. Thus $\mu_{\beta\delta\mu}$ is an KU – ideal of A.

Therefore, by Theorem 3.9 μ is an intuitionistic Q – fuzzy KU – ideal of A.

Definition 3.13 An intuitionistic Q – fuzzy set μ in KU- algebra A is said to be intuitionistic Q – fuzzy sub algebra of A if

1. $\lambda_\mu(a * b, q) \geq \min\{ \lambda_\mu(a, q), \lambda_\mu(b, q)\}$
2. $\delta_\mu(a * b, q) \leq \max \{ \delta_\mu(a, q), \delta_\mu(b, q)\}$

For all a, b $\in$ A & q $\in$ Q.

Theorem 3.14 Let μ be an intuitionistic Q – fuzzy sub algebra of a KU – algebra A then.

1. $\lambda_\mu(0 , q) \geq \lambda_\mu(a , q)$
2. $\delta_\mu(0, q) \leq \delta_\mu(a , q)$

For all a $\in$ A & q $\in$ Q.

Proof:

We know a $*$ a = 0 for any a $\in$ A, then

1. $\lambda_\mu(0 , q) = \lambda_\mu(a * a , q)$

$\geq \min\{ \lambda_\mu(a, q), \lambda_\mu(a, q)\}$

$= \lambda_\mu(a, q)$

And we get $\lambda_\mu(0 , q) \geq \lambda_\mu(a , q)$.

2. $\delta_\mu(0, q) = \delta_\mu(a * a , q)$

$\leq \max \{ \delta_\mu(a, q), \delta_\mu(a, q)\}$

$= \delta_\mu(a, q)$

Hence $\delta_\mu(0, q) \leq \delta_\mu(a , q)$.

Corollary 3.15 If A is a KU – algebra, then an intuitionistic Q – fuzzy set μ is an intuitionistic Q – fuzzy sub algebra if and only if for every $\alpha, \beta \in [0, 1]$, $\mu^{\alpha}{}_{\lambda\mu}$ and $\mu_{\beta\delta\mu}$ are either empty or KU – sub algebra of A.

Proof:

$\Rightarrow$ Assume that is an intuitionistic Q – fuzzy sub algebra and $\mu^{\alpha}{}_{\lambda\mu} \neq \phi \neq \mu_{\beta\delta\mu}$

for any u, v $\in \mu^{\alpha}{}_{\lambda\mu}$ and q $\in$Q , we have $\lambda_\mu(u * v, q) \geq \min\{ \lambda_\mu(u, q), \lambda_\mu(v, q)\} \geq \alpha$ then $u * v \in \mu^{\alpha}{}_{\lambda\mu}$ and hence $\mu^{\alpha}{}_{\lambda\mu}$ is a KU – sub algebra of A. On the other hand u, v $\in \mu_{\beta\delta\mu}$ and q $\in$Q , we have $\delta_\mu(u * v, q) \leq \max \{\delta_\mu(u, q), \delta_\mu(v, q)\} \leq \beta$ then $u * v \in \mu_{\beta\delta\mu}$ and hence $\mu_{\beta\delta\mu}$ is a KU – sub algebra of A.

$\Leftarrow$ Suppose that ${}^{\alpha}{}_{\lambda\mu}$ and $\mu_{\beta\delta\mu}$ are KU – sub algebra of A, for any u, v $\in \mu^{\alpha}{}_{\lambda\mu}$ then u $*$ v$\in \mu^{\alpha}{}_{\lambda\mu}$ take $\alpha = \min\{\ \lambda_\mu(u, q),\ \lambda_\mu(v, q)\}$therefore $\lambda_\mu(u * v, q) \geq \alpha = \min\{\ \lambda_\mu(u, q),\ \lambda_\mu(v, q)\}$ and for any u, v $\in \mu_{\beta\delta\mu}$ then u $*$ v$\in \mu_{\beta\delta\mu}$ take $\beta = \max\{\delta_\mu(u, q), \delta_\mu(v, q)\}$ thus $\delta_\mu(u * v, q) \leq \beta = \max\{\delta_\mu(u, q), \delta_\mu(v, q)\}$and hence μ is an intuitionistic Q – fuzzy sub algebra of A.

4. CONCLUSION

In this research, we have studied intuitionistic Q – fuzzy KU – sub algebra, KU – ideal and its level cuts. These notions can further be generalized.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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