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# APPLICATION OF INTUITIONISTIC FUZZY CRITICAL PATH METHOD ON AIRFREIGHT GROUND OPERATION SYSTEMS

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**Abstract.** The main objective of this paper is to express how intuitionistic fuzzy critical path method helps and improves the working process of the airfreight ground operation system. In this way, an example is discussed based on the application of intuitionistic fuzzy critical path method for airfreight ground operation systems. Instead of following the traditional method to find critical path and total completion duration of intuitionistic fuzzy critical path analysis, we have proposed a new approach that is easier and simple to understand when compared with existing methods. Here activity durations are represented as trapezoidal intuitionistic fuzzy numbers. Also, a new centroid based ranking grade of modal value, left fuzziness and right fuzziness index of membership and nonmembership functions of trapezoidal intuitionistic fuzzy numbers has been applied.

**Keywords:** intuitionistic fuzzy critical path; trapezoidal intuitionistic fuzzy number; Euclidean distance; location index; left fuzziness index; right fuzziness index.

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## 1. INTRODUCTION

In this competitive world, construction sectors, industrial organizations and government agencies have to plan their projects to make the best use of resources and to decrease the overall cost to gain profit. Generally, a project contains a set of activities that must be done in some specified manner so that some activities cannot begin before the completion of others. The critical path method is a more powerful and successful technique to handle such management problems effectively and also minimizes the project duration by using the resources optimally and identifies the critical path. There are many applications available for critical path method, some of them are aerospace and defense, software development, research projects, airfreight ground operation system, etc. In that, we focus on air freight ground operation system. In peak time customs officers in the airport have to face a huge quantity of goods from passengers and also from multinational companies who want to meet their customer needs from all over the world. If the airport authorities fail to handle these situations wisely then it will ultimately lead to the delay in cargo loading extending it to delay in departure and arrival of the airlines which is not good for the long run of airlines. These situations can be easily handled by slightly changing some work process or by following the suitable airport cargo management approaches. However, in real-life situations getting cargo clearance approval from airport officials within the expected time is quite difficult due to the long process. The time duration of each activity, like getting manuscript sanction followed by inspection or examination of cargo waiting for loading is not much accurate. At that moment of planning the activities and getting precise information regarding packing and loading the cargo, in-plane is difficult. Hence it leads to the development of intuitionistic fuzzy critical path method (IFCPM) which do scheduling as well as it helps in managing airfreight ground operations more effectively.

Intuitionistic fuzzy set theory was first introduced by Atanassov [1] in 1986. Since then more problems in intuitionistic fuzzy set theory has been produced and developed. Intuitionistic fuzzy set (IFS) theory is an extension of fuzzy set theory introduced by Zadeh [2]. IFS give space for both membership grade and non membership grade which helps the decision makers to get better results. Chanas and kamburowski [3] introduced fuzzy set theory to networking problems for the first time. Chanas and zielinski [4] proposed some relations between the notions of fuzzy criticality with application of the extension principle of zadeh.

Zhang and zhang [5] discussed about the financial break even for airports and insist to implement the policy which maximizes the social welfare in practical life. Anming zhang [6] discussed about the theoretical structure of the role of an international airfreight hub by considering Hong Kong air cargo case and analyzed the competitive factors in the industries of China and East Asia. Chen and Huang [7] have proposed a new model which combines fuzzy set theory with the PERT technique to determine the critical path. Elizabeth and Sujatha [8] developed a dynamic programming recursion formulation method to identify fuzzy critical path by defuzzifying each edge weights which are triangular fuzzy numbers.

In 2014, Jayagowri and Geetharamani [9, 10] suggested a novel approach to define the critical path in a project network whose task parameters are interpreted by trapezoidal intuitionistic fuzzy numbers. Graded mean integration formula has been defined to reduce trapezoidal intuitionistic fuzzy number to equivalent crisp number. Again in 2015, Jayagowri and Geetharamani analyzed the criticality in the project network by computing total slack time for each path under the intuitionistic fuzzy environment using the metric distance ranking method. In a fuzzy network, Elizabeth and Sujatha [11] developed two distinct algorithms to obtain the critical path, where the duration of each activity is represented as triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. Sophia porchelvi and Sudha [12, 13] proposed an algorithm to perform intuitionistic fuzzy critical path analysis, the length of which is the triangular intuitionistic fuzzy number.

By considering the above literatures, we tried to improve airfreight ground critical processes with the help of a proposed new intuitionistic fuzzy critical path algorithm. To show the efficacy of the proposed method we have considered an example solved by Jayagowri and nallathambi [14] and it is to be noticed that our method is more simple and produced vagueness reduced result.

The rest of the paper is organized as follows: In section 2, basic definitions and results of intuitionistic fuzzy set theory have been reviewed. In sections 3, algorithm for the proposed method is presented for the computation of the critical path, when the activity durations are taken as trapezoidal intuitionistic fuzzy numbers. Numerical example is provided to illustrate the efficiency of the proposed method in section 4.

## 2. PRELIMINARIES

Some notations, notions and results are discussed in this section which is useful for our further study.

**2.1. Intuitionistic Fuzzy Set**[1]. Let $\tilde{a}^I = \{(x, \mu_{\tilde{a}^I}(x), \gamma_{\tilde{a}^I}(x)) \ / \ x \in X\}$ be intuitionistic fuzzy set (IFS) in $X$ the universe of discourse. For all $x \in \tilde{a}^I$, we have $0 \leq \mu_{\tilde{a}^I}(x) + \gamma_{\tilde{a}^I}(x) \leq 1$ where the function $\mu_{\tilde{a}^I}(x) : X \longrightarrow [0,1]$ determines the degree of membership and the function $\gamma_{\tilde{a}^I}(x)$ : $X \longrightarrow [0,1]$ determines the degree of non membership of every element $x \in \tilde{a}^I$.

**2.2. Degree of Hesitancy.** For every common fuzzy subset $\tilde{a}^I \in X, \pi_{\tilde{a}^I}(x) = 1 - \mu_{\tilde{a}^I}(x) - \gamma_{\tilde{a}^I}(x)$ denotes intuitionistic fuzzy index of an element $x \in \tilde{a}^I$. For every $x \in \tilde{a}^I$, we have $0 \leq \pi_{\tilde{a}^I}(x) \leq 1$ is called as degree of hesitancy or degree of uncertainty.

**2.3. Intuitionistic Fuzzy Number.** An Intuitionistic Fuzzy Number (IFN) $\tilde{a}^I$ is

(i) An intuitionistic fuzzy subset of the real line,

(ii) Normal, that is there is any $x_0 \in R$, such that $\mu_{\tilde{a}^I}(x_0) = 1, \gamma_{\tilde{a}^I}(x_0) = 0$.

(iii) Convex for the membership function $\mu_{\tilde{a}^I}(x)$, that is,

$\mu_{\tilde{a}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{a}^I}(x_1), \mu_{\tilde{a}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

(iv) Concave for the non-membership function $\gamma_{\tilde{a}^I}(x)$, that is,

$\gamma_{\tilde{a}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\gamma_{\tilde{a}^I}(x_1), \gamma_{\tilde{a}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

**2.4. Trapezoidal Intuitionistic Fuzzy Number.** Trapezoidal intuitionistic fuzzy number (TRIFN) $\tilde{a}^I = ((a_1, a_2, a_3, a_4); (a_1^{'}, a_2^{'}, a_3^{'}, a_4^{'}))$ is a subset of IFS in R whose membership function $\mu_{\tilde{a}^I}(x) \in R \longrightarrow [0,1]$ and non membership function $\gamma_{\tilde{a}^I}(x) \in R \longrightarrow [0,1]$ has the following characteristics:

$$\mu_{\tilde{a}^I}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} & \text{for } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$\gamma_{\tilde{a}^I}(x) = \begin{cases} \dfrac{a_2^{'} - x}{a_2^{'} - a_1^{'}} & \text{for } a_1^{'} \leq x \leq a_2^{'} \\ 0 & \text{for } a_2^{'} \leq x \leq a_3^{'} \\ \dfrac{x - a_3^{'}}{a_4^{'} - a_3^{'}} & \text{for } a_3^{'} \leq x \leq a_4^{'} \\ 1 & \text{otherwise} \end{cases}$$

We use $F(R)$ to denote the set of all trapezoidal intuitionistic fuzzy numbers (TRIFN). Where $\tilde{a}_0 = [a_2, a_3]$, $\tilde{a}_0^{'} = [a_2', a_3']$ denotes the core of membership and non membership function respectively. $\alpha_1 = (a_2 - a_1)$, $\beta_1 = (a_4 - a_3)$ represents the left spread and right spread of membership function and $\alpha_1' = (a_2' - a_1')$, $\beta_1' = (a_4' - a_3')$ represents the left spread and right spread of non- membership function respectively.

**2.5.** $r - r^*$ **cuts.** Trapezoidal intuitionistic fuzzy number $\tilde{a}^I \in F(R)$ can also be represented as a pair $\tilde{a}^I = (\underline{a}, \overline{a}; \underline{a}^{'}, \overline{a}^{'})$ of functions $\underline{a}(r), \overline{a}(r), \underline{a}^{'}(r^*)$ and $\overline{a}^{'}(r^*)$ which satisfies the following requirements:

(i) The left legs $\underline{a}(r)$ and $\overline{a}'(r^*)$ are bounded monotonic increasing left continuous functions for membership and nonmembership functions respectively.

(ii) The right legs $\overline{a}(r)$ and $\underline{a}'(r^*)$ are bounded monotonic decreasing left continuous functions for membership and nonmembership functions respectively.

(iii) $\underline{a}(r) \leq \overline{a}(r), 0 \leq r \leq 1$.

(iv) $\underline{a}^{'}(r^*) \leq \overline{a}^{'}(r^*), 0 \leq r^* \leq 1$.

**2.6. Parametric Form of TRIFN.** The modal value (location index) of membership and nonmemership function of trapezoidal intuitionistic fuzzy number $\tilde{a}$ are represented as $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right), a_0^{'} = \left(\frac{\underline{a}^{'}(0)+\overline{a}^{'}(0)}{2}\right)$ respectively. The non-decreasing left continuous functions $a_* = (a_0 - \underline{a}), a'^* = (\overline{a}' - a_0')$ represents the left fuzziness index function and right fuzziness index function of membership and nonmembership function respectively. And in the same way the non-increasing left continuous function $a^* = (\overline{a} - a_0)$ *and* $a_*^{'} = (a_0^{'} - \underline{a}^{'})$ represents the right fuzziness index function and left fuzziness index function of membership and nonmembership function respectively. Hence every trapezoidal intuitionistic fuzzy number $\tilde{a}^I$ can also be represented by $\tilde{a}^I = (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle)$.

**2.7. Arithmetic Operations on Trapezoidal Intuitionistic Fuzzy Numbers.** In particular for any two trapezoidal intuitionistic fuzzy numbers $\tilde{a}^I = (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle)$ and $\tilde{b}^I = (\langle b_0, b_*, b^* \rangle, \langle b_0^{'}, b_*^{'}, b^{'*} \rangle)$, we define

*Addition:*

$$\begin{aligned}\tilde{a}^I + \tilde{b}^I =& (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle) \\ &+ (\langle b_0, b_*, b^* \rangle, \langle b_0^{'}, b_*^{'}, b^{'*} \rangle) \\ =& (\langle a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\} \rangle, \\ &\langle a_0^{'} + b_0^{'}, \max\{a_*^{'}, b_*^{'}\}, \max\{a^{'*}, b^{'*}\} \rangle)\end{aligned}$$

*Subtraction:*

$$\begin{aligned}\tilde{a}^I - \tilde{b}^I =& (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle) \\ &- (\langle b_0, b_*, b^* \rangle, \langle b_0^{'}, b_*^{'}, b^{'*} \rangle) \\ =& (\langle a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\} \rangle, \\ &\langle a_0^{'} - b_0^{'}, \max\{a_*^{'}, b_*^{'}\}, \max\{a^{'*}, b^{'*}\} \rangle)\end{aligned}$$

**2.8. Ranking of Trapezoidal Intuitionistic Fuzzy Numbers.** Arun Prakash [15] discussed about ranking of intuitionistic fuzzy numbers using centroid concept. We extend this ranking index [16] in such a way to compare the parametric representation of trapezoidal intuitionistic fuzzy numbers. To compare any two trapezoidal intuitionistic fuzzy numbers, the centroid point of the trapezoidal intuitionistic fuzzy number is considered. Each trapezoidal intuitionistic fuzzy number is reduced to its corresponding crisp value by centroid index which uses the geometric center of a trapezoidal intuitionistic fuzzy number. The geometric center corresponds to $x(\tilde{a}^I)$ value on the horizontal axis and $y(\tilde{a}^I)$ value on the vertical axis.

$$x(\tilde{a}^I_\mu) = \frac{1}{3}\left[3a_0 + \frac{(a^* - a_*)}{1-r}\right]$$

$$y(\tilde{a}^I_\mu) = \frac{1}{3}$$

$$x(\tilde{a}^I_\gamma) = \frac{1}{3}\left[3a_0^{'} + \frac{2(a^{*'} - a_*^{'})}{r^*}\right]$$

$$y(\tilde{a}^I_\gamma) = \frac{2}{3}$$

where $(x(\tilde{a}^I_\mu), y(\tilde{a}^I_\mu), x(\tilde{a}^I_\gamma), y(\tilde{a}^I_\gamma))$ represents the centroid of the generalized trapezoidal intuitionistic fuzzy number.

The ranking function of the trapezoidal intuitionistic fuzzy number $\tilde{a}^I$ is defined by $R(\tilde{a}^I) = \sqrt{\frac{1}{2}\left(\left[x(\tilde{a}^I_\mu) - y(\tilde{a}^I_\mu)\right]^2 + \left[x(\tilde{a}^I_\gamma) - y(\tilde{a}^I_\gamma)\right]^2\right)}$.

Consider any two trapezoidal intuitionistic fuzzy numbers $\tilde{a}^I = (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle)$ and $\tilde{b}^I = (\langle b_0, b_*, b^* \rangle, \langle b_0^{'}, b_*^{'}, b^{'*} \rangle)$ in F(R), we have the following Comparison. we define the ranking of $\tilde{a}^I$ and $\tilde{b}^I$ by comparing the $R(\tilde{a}^I)$ and $R(\tilde{b}^I)$ on R as follows:

(i) If $R(\tilde{a}^I) < R(\tilde{b}^I)$, then $\tilde{a}^I$ is smaller than $\tilde{b}^I$ (i.e. $\tilde{a}^I \prec \tilde{b}^I$).

(ii) If $R(\tilde{a}^I) > R(\tilde{b}^I)$, then $\tilde{a}^I$ is greater than $\tilde{b}^I$ (i.e. $\tilde{a}^I \succ \tilde{b}^I$).

(iii) If $R(\tilde{a}^I) = R(\tilde{b}^I)$, then $\tilde{a}^I$ is equivalent to $\tilde{b}^I$ (i.e. $\tilde{a}^I \approx \tilde{b}^I$).

## 3. INTUITIONISTIC FUZZY CRITICAL PATH ANALYSIS

A Trapezoidal intuitionistic fuzzy project network is an acyclic digraph, where the vertices and directed edges represent the events and activities respectively, to be performed in a project. We denote it by $\tilde{N} = (\tilde{V}, \tilde{A}^I, \tilde{T}^I)$ where $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \ldots \tilde{v}_n\}$ be the set of all nodes, $\tilde{A}^I = \left\{\tilde{a}^I_{ij} = (\tilde{v}_i, \tilde{v}_j) \text{ for } \tilde{v}_i, \tilde{v}_j \varepsilon \tilde{V}\right\}$ be the set of all activities (directed edges) that joins each node in the project network and $\tilde{t}^I_{ij} \in \tilde{T}^I$ denotes the time duration of each activity in a project network. An intuitionistic fuzzy critical path is a longest path from the initial node $\tilde{v}_1$ to the terminal node $\tilde{v}_n$ of the project network, and an activity $\tilde{a}^I_{ij}$ on a critical path is called an intuitionistic fuzzy critical activity.

**Theorem 1.** [4] *A path $p \in P$ is critical if and only if it is the longest path in the network $\tilde{N}$ with the lengths of arcs equal to $\tilde{t}^I_{ij}, \tilde{a}^I_{ij} \in \tilde{A}^I$. The length of this path is equal to $\tilde{D}^I_n$.*

**Theorem 2.** [4] *An activity $\tilde{a}^I_{ij} \in \tilde{A}^I$ or an event $\tilde{v}_i \in \tilde{V}$ is critical if and only if it belongs to a certain critical path $p \in P$.*

### 3.1. Notation and Meanings.

$V(j)$ : The set of all predecessor nodes of node j.

$\tilde{D}^I_i$ : Trapezoidal intuitionistic fuzzy distance between the node i and source node.

$\tilde{D}^I_{ij}$ : Trapezoidal intuitionistic fuzzy distance between the node i and node j.

$P_i$ : The i-th path of the trapezoidal intuitionistic fuzzy project network.

$P$ : The set of all paths in a trapezoidal intuitionistic fuzzy project network.

**3.2. Modified Critical Path Algorithm.** A new algorithm is proposed for finding the intuitionistic fuzzy longest path.

**Step 1:** Construct the intuitionistic fuzzy project network based on the precedence relationships and numbering all the events.

**Step 2:** Express all the trapezoidal intuitionistic fuzzy activity duration $\tilde{a}^I = ((a_1, a_2, a_3, a_4); (a_1^{'}, a_2^{'}, a_3^{'}, a_4^{'}))$ in the form $\tilde{a}^I = (\langle a_0, a_*, a^* \rangle, \langle a_0^{'}, a_*^{'}, a^{'*} \rangle)$.

**Step 3:** Let there are n nodes and assume that the start time of the initial node of the project network be $\tilde{D}_1^I = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle)$.

**Step 4:** Find the start time of the jth node using $\tilde{D}_j^I = \max\{\tilde{D}_i^I + \tilde{t}_{ij}^I\}$ with $i \in V(j)$ the set of all predecessor nodes of node j, $j \neq 1, i = 1,2,3,\cdots,n-1$ and $j = 2,3,\cdots,n$.

**Step 5:** If $\tilde{D}_j^I = \max\{\tilde{D}_i^I + \tilde{t}_{ij}^I\}$ is unique, say for $i = k$ then label node j as $[\tilde{D}_j^I, k]$. If there is a tie in selecting i, break the tie arbitrarily.

**Step 6:** Let the destination node (node n) be labeled as $[\tilde{D}_n^I, L]$, and then the intuitionistic fuzzy longest duration between initial node and destination node is $\tilde{D}_n^I$.

**Step 7:** To find the longest path between initial node and destination node, check the label of node L. Let it be $[\tilde{D}_L^I, p]$ now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

**Step 8:** The longest path between initial node and terminal node will be the intuitionistic fuzzy critical path that can be obtained by combining all the labeled nodes obtained by step 7.

## 4. NUMERICAL EXAMPLE

To show the efficiency of the proposed method, an example based on the application of intuitionistic fuzzy critical path method for airfreight ground operation system has been discussed in this section.

**Example 1.** *Consider an example discussed by Jayagowri and Nallathambi* [14].

TABLE 1. TRAPEZOIDAL INTUITIONISTIC FUZZY ACTIVITY DURATION OF EACH ACTIVITY

| Activity | Description | Intuitionistic fuzzy activity duration |
|---|---|---|
| 1-2 | Customs office cargo clearance with manuscript approval | $(\langle 1,1,2,3\rangle;\langle 1,2,2,3\rangle)$ |
| 1-3 | Customs office cargo clearance with examination | $(\langle 2,2,3,4\rangle;\langle 1,2,3,4\rangle)$ |
| 1-4 | Customs office cargo sanction with document approval and inspection | $(\langle 3,3,3,4\rangle;\langle 2,4,5,7\rangle)$ |
| 2-5 | Customs office inspection exempt cargo clearance | $(\langle 2,3,3,4\rangle;\langle 2,4,5,5\rangle)$ |
| 3-4 | Customs office investigation released cargo clearance | $(\langle 6,7,7,8\rangle;\langle 1,1,1,2\rangle)$ |
| 3-5 | Customs office after discharging cargo and packing cargo coming up for packing | $(\langle 1,3,3,4\rangle;\langle 1,1,3,5\rangle)$ |
| 4-6 | Customs office after cargo clearance with inspection and coming up for loading | $(\langle 2,3,4,5\rangle;\langle 1,3,4,6\rangle)$ |
| 5-6 | Customs office after check-up relieved cargo clearance, releasing cargo and packing cargo waiting for packing | $(\langle 1,1,1,2\rangle;\langle 2,2,3,3\rangle)$ |
| 5-8 | Customs office after cargo clearance with file approval, releasing cargo waiting for heaping | $(\langle 1,3,4,5\rangle;\langle 3,3,4,6\rangle)$ |
| 6-7 | Customs headquarters after cargo clearance with document agreement, stuffing cargo to come for stacking | $(\langle 1,2,2,2\rangle;\langle 3,4,5,5\rangle)$ |
| 6-8 | Customs office after cargo consent with inspection, discharging cargo in store for loading | $(\langle 4,5,6,7\rangle;\langle 1,1,2,2\rangle)$ |
| 7-8 | Customs office after cargo approval with scrutiny, filling cargo for heaping | $(\langle 2,3,3,4\rangle;\langle 3,3,4,5\rangle)$ |

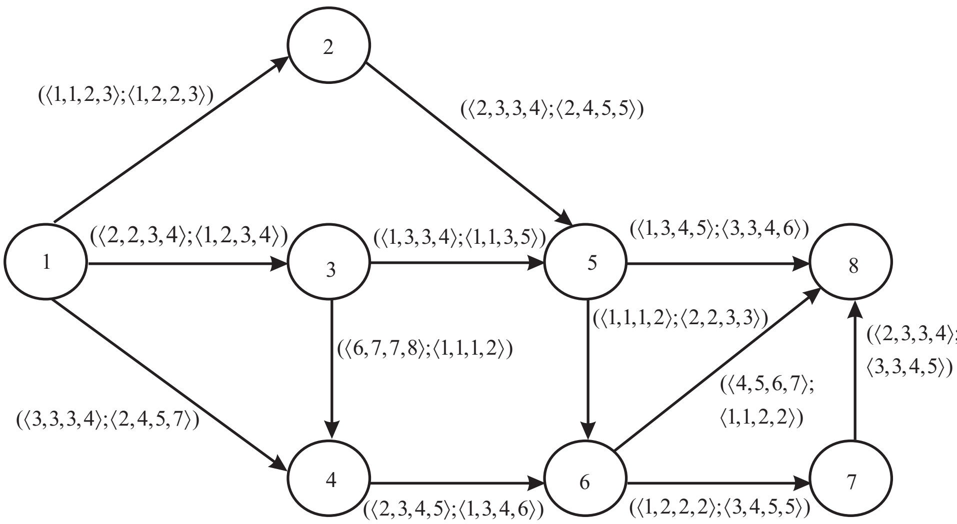


FIGURE 1. Computational Process

Given trapezoidal intuitionistic fuzzy numbers can be expressed as

$$\begin{aligned}\tilde{a}^I &= (\langle a_1, a_2, a_3, a_4\rangle);(\langle a_1^{'}, a_2^{'}, a_3^{'}, a_4^{'}\rangle)\\ &= (\langle a_0, a_*, a^*\rangle, \langle a_0^{'}, a_*^{'}, a^{'*}\rangle)\end{aligned}$$

TABLE 2. TRAPEZOIDAL INTUITIONISTIC FUZZY ACTIVITY DURATION OF EACH ACTIVITY IN THE FORM $(\langle a_0, a_*, a^*\rangle, \langle a_0^{'}, a_*^{'}, a^{'*}\rangle)$

| Activity | Description | Intuitionistic fuzzy activity duration |
|---|---|---|
| 1-2 | Customs office cargo clearance with manuscript approval | $(\langle 1.5, 0.5, 1.5-r\rangle, \langle 2, r^*, r^*\rangle)$ |
| 1-3 | Customs office cargo clearance with examination | $(\langle 4.5, 0.5, 1.5-r\rangle, \langle 2.5, 0.5+r^*, 0.5+r^*\rangle)$ |
| 1-4 | Customs office cargo sanction with document approval and inspection | $(\langle 3, 0, 1-r\rangle, \langle 4.5, 0.5+2r^*, 0.5+2r^*\rangle)$ |

| Activity | Description | Intuitionistic fuzzy activity duration |
|---|---|---|
| 2-5 | Customs office inspection exempt cargo clearance | $(\langle 3,1-r,1-r\rangle,\ \langle 4.5,0.5+2r^*,0.5\rangle)$ |
| 3-4 | Customs office investigation released cargo clearance | $(\langle 7,1-r,1-r\rangle,\ \langle 1,0,r^*\rangle)$ |
| 3-5 | Customs office after discharging cargo and packing cargo coming up for packing | $(\langle 3,2-2r,1-r\rangle,\ \langle 2,1,1+2r^*\rangle)$ |
| 4-6 | Customs office after cargo clearance with inspection and coming up for loading | $(\langle 3.5,1.5-r,1.5-r\rangle,\ \langle 3.5,0.5+2r^*,0.5+2r^*\rangle)$ |
| 5-6 | Customs office after check-up relieved cargo clearance, releasing cargo and packing cargo waiting for packing | $(\langle 1,0,1-r\rangle,\ \langle 2.5,0.5,0.5\rangle)$ |
| 5-8 | Customs office after cargo clearance with file approval, releasing cargo waiting for heaping | $(\langle 3.5,\ 2.5-2r,1.5-r\rangle,\ \langle 3.5,0.5,0.5+2r^*\rangle)$ |
| 6-7 | Customs headquarters after cargo clearance with document agreement, stuffing cargo to come for stacking | $(\langle 2,1-r,0\rangle,\ \langle 4.5,0.5+r^*,0.5\rangle)$ |
| 6-8 | Customs office after cargo consent with inspection, discharging cargo in store for loading | $(\langle 5.5,1.5-r,1.5-r\rangle,\ \langle 1.5,0.5,0.5\rangle)$ |
| 7-8 | Customs office after cargo approval with scrutiny, filling cargo for heaping | $(\langle 3,1-r,1-r\rangle,\ \langle 3.5,0.5,0.5+r^*\rangle)$ |

Here node 8 is the destination node, so n=8. Let the start time of the initial node as $\tilde{D}_1^I = (\langle 0,0,0\rangle,\langle 0,0,0\rangle)$ and label it as $[\tilde{D}_1^I,-]$. The values of $\tilde{D}_j^I, j = 2,3,4,5,6,7,8$ can be obtained as follows:

**Iteration 1:** For j=2, check the predecessor nodes of node 2. It is identified that there is only one activity heading towards the node 2 from node 1. That is node 1 is the only predecessor node of node 2, put i=1 in step 5, then the value of $\tilde{D}_2^I$ is

$$\begin{aligned}\tilde{D}_2^I &= \max\{\tilde{D}_1^I + \tilde{t}_{12}^I\} \\ &= \max\{(\langle 0,0,0\rangle,\langle 0,0,0\rangle) + (\langle 1.5,0.5,1-r\rangle,\langle 2,r^*,r^*\rangle)\} \\ &= \max\{(\langle 1.5,0.5,1-r\rangle,\langle 2,r^*,r^*\rangle)\}\end{aligned}$$

Hence maximum obtains equivalent to i=1, so label node 2 as $[\tilde{D}_2^I,1]$.

(i.e.) $[(\langle 1.5,0.5,1-r\rangle,\langle 2,r^*,r^*\rangle),1]$

**Iteration 2:** For j=3, check the predecessor nodes of node 3. It is identified that there is only one activity heading towards the node 3 from node 1. That is node1 is the only predecessor node of node 3, put i=1 in step 5, then

$$\begin{aligned}\tilde{D}_3^I &= \max\{\tilde{D}_1^I + \tilde{t}_{13}^I\} \\ &= \max\{(\langle 0,0,0\rangle,\langle 0,0,0\rangle) + (\langle 2.5,0.5,1-r\rangle,\langle 2.5,0.5+r^*,0.5+r^*\rangle)\} \\ &= \max\{(\langle 2.5,0.5,1-r\rangle,\langle 2.5,0.5+r^*,0.5+r^*\rangle)\}\end{aligned}$$

Hence label node 3 as $[\tilde{D}_3^I,1]$.

(i.e.) $[(\langle 2.5,0.5,1-r\rangle,\langle 2.5,0.5+r^*,0.5+r^*\rangle),1]$

**Iteration 3:** For j=4, check the predecessor nodes of node 4. It is identified that there are two activities heading towards the node 4 from node 1 and node 3. That is nodes 1 and 3 are the predecessor nodes of node 4, put i=1 and 3 in step 5, then

$$\begin{aligned}\tilde{D}_4^I &= \max\{\tilde{D}_1^I + \tilde{t}_{14}^I, \tilde{D}_3^I + \tilde{t}_{34}^I\} \\ &= \max\left\{\begin{array}{l}(\langle 3,0,1-r\rangle,\langle 4.5,0.5+2r^*,0.5+2r^*\rangle); \\ (\langle 9.5,1-r,1.5-r\rangle,\langle 4,0.5+r^*,0.5+r^*\rangle)\end{array}\right\} \\ &= \max\{(\langle 9.5,1-r,1.5-r\rangle,\langle 4,0.5+r^*,0.5+r^*\rangle)\}\end{aligned}$$

By the ranking function proposed above in section 1, the maximum value obtains equivalent to i=3, hence label the node 4 as $[\tilde{D}_4^I, 3]$.

(i.e.) $[(\langle 9.5, 1-r, 1.5-r\rangle, \langle 4, 0.5+r^*, 0.5+r^*\rangle), 1]$

**Iteration 4:** For j=5, check the predecessor nodes of node 5. It is identified that there are two activities heading towards the node 5 from node 2 and node 3. That is nodes 2 and 3 are the predecessor nodes of node 5, put i=2 and 3 in step 5, then

$$\begin{aligned}
\tilde{D}_5^I &= \max\{\tilde{D}_2^I + \tilde{t}_{25}^I, \tilde{D}_3^I + \tilde{t}_{35}^I\} \\
&= \max\left\{\begin{array}{l} (\langle 4.5, 1-r, 1.5-r\rangle, \langle 5.5, -0.5+2r^*, 1.5\rangle); \\ (\langle 5.5, 2-2r, 1.5-r\rangle, \langle 5.5, 2, 2r^*\rangle) \end{array}\right\} \\
&= \max\{(\langle 5.5, 2-2r, 1.5-r\rangle, \langle 5.5, 2, 2r^*\rangle)\}
\end{aligned}$$

Here maximum obtains equivalent to i=3, hence label the node 5 as $[\tilde{D}_5^I, 3]$.

(i.e.) $[(\langle 5.5, 2-2r, 1.5-r\rangle, \langle 5.5, 2, 2r^*\rangle), 1]$

**Iteration 5:** For j=6, check the predecessor nodes of node 6. It is identified that there are two activities heading towards the node 6 from node 4 and node 5. That is nodes 4 and 5 are the predecessor nodes of node 6, put i=4 and 5 in step 5, then

$$\begin{aligned}
\tilde{D}_6^I &= \max\{\tilde{D}_4^I + \tilde{t}_{46}^I, \tilde{D}_5^I + \tilde{t}_{56}^I\} \\
&= \max\left\{\begin{array}{l} (\langle 13, 1.5-r, 1.5-r\rangle, \langle 7.5, 0.5+2r^*, 0.5+2r^*\rangle); \\ (\langle 6.5, 2-2r, 1.5-r\rangle, \langle 8, 2, 2r^*\rangle) \end{array}\right\} \\
\tilde{D}_6^I &= \max\{(\langle 13, 1.5-r, 1.5-r\rangle, \langle 7.5, 0.5+2r^*, 0.5+2r^*\rangle)\}
\end{aligned}$$

Here maximum obtains equivalent to i=4, hence label node 6 as $[\tilde{D}_6^I, 4]$.

(i.e.) $[(\langle 13, 1.5-r, 1.5-r\rangle, \langle 7.5, 0.5+2r^*, 0.5+2r^*\rangle), 4]$

**Iteration 6:** For j=7, check the predecessor nodes of node 7. It is identified that there is only one activity heading towards the node 7 from node 6. That is node 6 is the only predecessor node of node 7, put i=6 in step 5, then

$$\tilde{D}_7^I = \max\{\tilde{D}_6^I + \tilde{t}_{67}^I\}$$
$$= \max\left\{\begin{array}{l} (\langle 13, 1.5-r, 1.5-r\rangle, \langle 7.5, 0.5+2r^*, 0.5+2r^*\rangle) \\ +(\langle 2, 1-r, 0\rangle, \langle 4, r^*, 1\rangle) \end{array}\right\}$$
$$\tilde{D}_7^I = \max\{(\langle 15, 1.5-r, 1.5-r\rangle, \langle 11.5, 0.5+2r^*, 0.5+2r^*\rangle)\}$$

Here maximum obtains equivalent to i=6, hence label node 7 as $[\tilde{D}_7^I, 6]$.

(i.e.) $[(\langle 15, 1.5-r, 1.5-r\rangle, \langle 11.5, 0.5+2r^*, 0.5+2r^*\rangle), 6]$

**Iteration 7:** For j=8, check the predecessor nodes of node 8. It is identified that there are three activities heading towards the node 8 from the nodes 5, 6 and 7. That is nodes 5, 6 and 7 are the predecessor nodes of node 8, put i=5, 6 and 7 in step 5, then

$$\tilde{D}_8^I = \max\{\tilde{D}_5^I + \tilde{t}_{48}^I, \tilde{D}_6^I + \tilde{t}_{68}^I, \tilde{D}_7^I + \tilde{t}_{78}^I\}$$
$$= \max\left\{\begin{array}{l} (\langle 9, 2.5-2r, 1.5-r\rangle, \langle 10, 2, 2r^*\rangle); \\ (\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle); \\ (\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle) \end{array}\right\}$$
$$\tilde{D}_8^I = \max\{(\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle)\}$$

Here maximum obtains equivalent to i=6, hence label node 8 as $[\tilde{D}_8^I, 6]$.

(i.e.) $[(\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle), 6]$

Since node 8 is the destination node of the given network, trapezoidal intuitionistic fuzzy longest duration of the project network between node 1 and 8 is

$[(\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle), 6]$

Now the intuitionistic fuzzy longest path between node 1 and node 8 can be obtained by using the following procedure:

Since node 8 is labeled as

$[(\langle 18.5, 1.5-r, 1.5-r\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle), 6]$

It means that this activity starts from node 6. That is cargo is discharging in store for loading after customs office cargo consent with inspection. Node 6 is labeled by

$[(\langle 13.5, 1.5-r, 1.5-r\rangle, \langle 7.5, 0.5+2r^*, 0.5+2r^*\rangle), 4]$

It means that this activity starts from node 4. That is cargo coming up for loading after getting customs office clearance with inspection on cargo. Now node 4 is labeled by

$[(\langle 9.5, 1-r, 1.5-r\rangle, \langle 4, 0.5+r^*, 0.5+r^*\rangle), 3]$

It means that this activity starts from node 3. That is getting cargo clearance after customs office investigation. Now node 3 is labeled by

$[(\langle 2.5, 0.5, 1.5-r\rangle, \langle 2.5, 0.5+r^*, 0.5+r^*\rangle), 1]$

It means that this activity starts from node 1. That is customs office clearance with examination. Now the intuitionistic fuzzy longest path between node 1 and node 8 is obtained by joining all the labeled nodes. Hence the trapezoidal intuitionistic fuzzy longest path is $1 \to 3 \to 4 \to 6 \to 8$. By applying our proposed method, intuitionistic fuzzy completion duration for project network is

$[(\langle 18.5, 1.5-r, 1.5-r^*\rangle, \langle 9, 0.5+2r^*, 0.5+2r^*\rangle), 6] = (\langle 17, 18, 19, 20\rangle; \langle 6.5, 8.5, 9.5, 11.5\rangle)$.

## 5. RESULT AND DISCUSSION

In this paper, we have proposed a new modified intuitionistic fuzzy critical path algorithm instead of following the traditional method. We have used a new centroid based ranking grade of location index, left fuzziness and right fuzziness index of membership and nonmembership function of a trapezoidal intuitionistic fuzzy number. We tried to find the critical path and total completion duration of an intuitionistic fuzzy project network just by using only the earliest starting time of each node. To show the efficiency of our proposed method, we compared our method with one of the existing methods. We considered a problem discussed by Jayagowri and Nallathambi [14], they followed the traditional method and calculated total slack time for all possible paths in the project network. From that, they have concluded that the path $1 \to 3 \to 4 \to 6 \to 8$ is a critical path. The drawback of jayagowri and nallathambi's method is that they have not discussed any ranking method to find the intuitionistic fuzzy longest duration. It is to be noticed that our method is simple and produced vagueness reduced result when comparing with Jayagowri and nallathambi [14] proposed method.

## 6. Conclusion

A new algorithm has been proposed to find intuitionistic fuzzy critical path and it is used in the airfreight ground operation decision analysis. The above study suggests that by remodeling airfreight ground operation technique, the performance of the airlines freight service can be improved in terms of cargo handling speed, airlines service quality and cost of cargo handling in hub airports.

## Conflict of Interests

The author(s) declare that there is no conflict of interests.

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