Available online at http://scik.org
J. Semigroup Theory Appl. 2014, 2014:3
ISSN: 2051-2937

ON FUZZY POINTS IN TERNARY SEMIGROUPS

ESSAM. H. HAMOUDA

Department of basic sciences, faculty of industrial education, Beni-Suef University, Egypt

Abstract. We consider the ternary semigroup $\underline{S}$ of the fuzzy points of a ternary semigroup S, and discuss the relation between some fuzzy ideals of a ternary semigroup S and the subsets of $\underline{S}$.

Keywords: Fuzzy set; fuzzy point; fuzzy ideal; ternary semigroup.

2010 AMS Subject Classification: 20N10; 20N25; 20M12.

1. Introduction

The concept of fuzzy set was initiated by L. Zadeh [1]. The study of fuzzy algebraic structures started with the introduction of the concepts of fuzzy groups in the pioneering paper of Rosenfeld [2]. Kuroki [3, 4, 5, 6] defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. M. Santiago and S. Bala developed the theory of ternary semigroups[7]. Recently, S. Kar and P. Sarkar defined fuzzy left (right, lateral) ideals of ternary semigroups and characterize regular and intra-regular ternary semigroups by using the concept of fuzzy ideals of ternary semigroups[8]. Kim in [9], considered the semigroup $\underline{S}$ of the fuzzy points of a semigroup S, and discussed the relation between some fuzzy ideals of a semigroup S and the subsets of $\underline{S}$. In the present paper, we consider the ternary semigroup $\underline{S}$ of the fuzzy points of a ternary semigroup S, and discuss the relation between some fuzzy ideals of a ternary semigroup S and the subsets of $\underline{S}$.

Received May 29, 2014

2. Preliminaries

Definition 2.1 [7] *A ternary semigroup is a nonempty set* S *together with a ternary operation* $(a,b,c) \to abc$ *satisfying* $(abc)de = a(bcd)e = ab(cde)$ *for all* $a; b; c; d; e \in S$.

Example 2.2 [8] *Let* $\mathbb{Z}^-$ *be the set of all negative integers. Then with the usual ternary multiplication,* $\mathbb{Z}^-$ *forms a ternary semigroup.*

Definition 2.3 [7,10] *A non-empty subset* A *of a ternary semigroup is called*

1) *A ternary subsemigroup if* $A^3 = AAA \subseteq A$.
2) *A left ideal of* S *if* $SSA \subseteq A$.
3) *A lateral ideal of* S *if* $SAS \subseteq A$.
4) *A right ideal of* S *if* $ASS \subseteq A$.
5) *An ideal of* S *if* A *is a left ideal, a lateral ideal and a right ideal of* S.

Definition 2.4 [10] *A ternary subsemigroup* B *of a ternary semigroup* S *is said to be a bi-ideal of* S *if* $BSBSB \subseteq B$.

Definition 2.5 [11] *A ternary subsemigroup* B *of a ternary semigroup* S *is called an interior ideal of* S *if* $SSBSS \subseteq B$.

Example 2.6 Let $S = \{(0,0),(0,1),(1,0),(1,1)\}$. Then S is a ternary semigroup with respect to ternary multiplication defined by

$$(i,j)(k,l)(m,n) = (i,n).$$

Let $A = \{(0,0),(0,1)\}$ be a subset of S. Then A is a right ideal of S, but not a lateral ideal nor a left ideal because

in SAS,

$$(1,0)(0,1)(1,1) = (1,1) \notin A,$$

in SSA,

$$(1,0)(1,1)(0,0) = (1,0) \notin A.$$

Let $B = \{(0,1),(1,1)\}$ be a subset of S. Then B is a left ideal of S, but not a lateral ideal nor a right ideal because

in SBS,

$$(1,0)(1,1)(1,0) = (1,0) \notin B,$$

in BSS,

$$(0,0)(1,1)(0,0) = (0,0) \notin B.$$

A function f from S to the closed interval [0, 1] is called a *fuzzy set* in S [1]. The ternary semigroup S itself is a fuzzy set in S such that $S(x) = 1$ for all $x \in S$, denoted also by C_S .

Definition 2.7 [1] *Let f be a fuzzy set in a nonempty set S. For any $t \in [0,1]$; the subset $f_t = \{x \in S: f(x) \geq t\}$ of S is called a level subset of f.*

Let A and B be two fuzzy sets in S. Then the inclusion relation $A \subseteq B$ is defined by $A(x) \leq B(x)$ for all $x \in S$. $A \cap B$ and $A \cup B$ are fuzzy sets in S defined by $(A \cap B)(x) = min\{A(x), B(x)\} = A(x) \wedge B(x)$, $(A \cup B)(x) = max\{A(x), B(x)\} = A(x) \vee B(x)$, for all $x \in S$.

Definition 2.8 [10] *Let S be a non-empty set and $x \in S, t \in (0,1]$. A fuzzy point x_t of S is a fuzzy set in S, defined by,*

$$x_t(y) = \begin{cases} t & if\ x = y, \\ 0 & otherwise, \end{cases}$$

for all $y \in S$.

The fuzzy point x_t is said to be contained in a fuzzy set A, denoted by $x_t \in A$, iff $t \leq A(x)$.

Definition 2.9 [8] *A non-empty fuzzy set A in a ternary semigroup S is called a fuzzy ternary subsemigroup of S if $A(xyz) \geq A(x) \wedge A(y) \wedge A(z)$ for all $x, y, z \in S$.*

Definition 2.10 [8] *A non-empty fuzzy set A in a ternary semigroup S is called a fuzzy left (resp. lateral, right) ideal of S if $A(xyz) \geq A(z)$ (resp. $A(xyz) \geq A(y), A(xyz) \geq A(x)$) for all $x, y, z \in S$.*

If A is a fuzzy left ideal, a fuzzy lateral ideal and a fuzzy right ideal of S, then A is called a fuzzy ideal of S.

It is clear that A is a fuzzy ideal of a ternary semigroup S if and only if $A(xyz) \geq$ A(x) ∨ A(y) ∨ A(z) for all $x, y, z \in S$, and that every fuzzy left (lateral, right) ideal is a fuzzy ternary semigroup of S.

Definition 2.11 [11] *A fuzzy ternary subsemigroup B in a ternary semigroup S is called a fuzzy interior ideal of S if $B(xsary) \geq B(a)$ for all $x, a, r, s, y \in S$.*

Example 2.12 In example 2.6, $S = \{(0,0), (0,1), (1,0), (1,1)\}$ is a ternary semigroup and $A = \{(0,0), (0,1)\}$ is a right ideal of S. Define a fuzzy set f in S as follows:

$$f(x) = \begin{cases} 0.6 & if\ x \in A; \\ o & otherwise. \end{cases}$$

It is clear that f is a fuzzy left ideal, not a fuzzy lateral ideal nor a fuzzy right ideal. Similarly, for the left ideal $B=\{(0,1),(1,1)\}$ we can define a fuzzy right ideal f which is neither a fuzzy lateral ideal nor a fuzzy left ideal.

3. Some ideals of fuzzy points

Let $\mathcal{F}(S)$ be the set of all fuzzy sets in a ternary semigroup S. For each $A,B,C\in\mathcal{F}(S)$, the product of $A\,,B,C$ is a fuzzy set $A\circ B\circ C$ defined as follows:

$$(A\circ B\circ C)(x)=\begin{cases}\bigvee\limits_{x=abc}\{A(a)\wedge B(b)\wedge C(c)\} & if\ abc=x\\ 0 & otherwise.\end{cases}$$

for each $x\in S$. Since $(A\circ B\circ C)\circ D\circ E=A\circ(B\circ C\circ D)\circ E=A\circ B(\circ C\circ D\circ E)$ [9], then $\mathcal{F}(S)$ is a ternary semigroup with the product "$\circ$".

Let $\underline{S}$ be the set of all fuzzy points in a ternary semigroup S. Then $x_\alpha\circ y_\beta\circ z_\gamma=(xyz)_{\alpha\wedge\beta\wedge\gamma}\in\underline{S}$ [8] and $(x_\alpha\circ y_\beta\circ z_\gamma)\circ w_\sigma\circ u_\tau=x_\alpha\circ\left(y_\beta\circ z_\gamma\circ w_\sigma\right)\circ u_\tau=x_\alpha\circ y_\beta\circ(z_\gamma\circ w_\sigma\circ u_\tau)$ for $x_\alpha,y_\beta,z_\gamma,w_\sigma,u_\tau\in\underline{S}$. Thus $\underline{S}$ is a ternary subsemigroup of $\mathcal{F}(S)$. For any $A\in\mathcal{F}(S)$, $\underline{A}$ denotes the set of all fuzzy points contained in A, that is, $\underline{A}=\{\,x_\alpha\in\underline{S}: A(x)\geq\alpha\}$. for any $A,B,C\subseteq\underline{S}$, we define the product of A , B and C as $A\circ B\circ C=\left\{x_\alpha\circ y_\beta\circ z_\gamma : x_\alpha\in A, y_\beta\in B, z_\gamma\in C\right\}$.

Lemma 3.1. *Let A, B and C be fuzzy sets in a ternary semigroup S. Then*

a) $\underline{A\cup B\cup C}=\underline{A}\cup\underline{B}\cup\underline{C}$.

b) $\underline{A\cap B\cap C}=\underline{A}\cap\underline{B}\cap\underline{C}$.

c) $\underline{A\circ B\circ C}\supseteq\underline{A}\circ\underline{B}\circ\underline{C}$.

Proof. (a) Let $z_\alpha\in\underline{A\cup B\cup C}$, then

$$(A\cup B\cup C)(z)=A(z)\vee B(z)\vee C(z)\geq\alpha.$$

Hence, $A(z)\geq\alpha\ or\ B(z)\geq\alpha\ \ or\ C(z)\geq\alpha$, and consequently, $z_\alpha\in\underline{A}\cup\underline{B}\cup\underline{C}$. This implies that $\underline{A\cup B\cup C}\subseteq\underline{A}\cup\underline{B}\cup\underline{C}.$ Let $z_\alpha\in\underline{A}\cup\underline{B}\cup\underline{C}$, then $(z)\geq\alpha\ or\ B(z)\geq\alpha$, $or\ C(z)\geq\alpha$ and hence $(A\cup B\cup C)(z)\geq\alpha$. This implies that $z_\alpha\in\underline{A\cup B\cup C}$ and consequently, $\underline{A}\cup\underline{B}\cup\underline{C}\subseteq\underline{A\cup B\cup C}$. Hence $\underline{A\cup B\cup C}=\underline{A}\cup\underline{B}\cup\underline{C}.$

(b) is similar to (a).

(c) Let $z \in S$ and $z_\omega \in \underline{A} \circ \underline{B} \circ \underline{C}$, then $z_\omega = a_\alpha \circ b_\beta \circ c_\gamma$ such that $a_\alpha \in \underline{A}, b_\beta \in \underline{B}$ and $c_\gamma \in \underline{C}$. If $z = pqr$ for some $p, q, r \in S$, then $A(p) \geq a_\alpha(p)$, $B(q) \geq b_\beta(q)$ and $C(r) \geq c_\gamma(r)$. From the definition of fuzzy points we have $A(p) \geq \bigvee_{a_\alpha \in A} a_\alpha(p)$, $B(q) \geq \bigvee_{b_\beta \in B} b_\beta(q)$ and $C(r) \geq \bigvee_{c_\gamma \in C} c_\gamma(r)$. Thus

$$(A \circ B \circ C)(z) = \bigvee_{z=pqr} A(p) \wedge B(q) \wedge C(r)$$
$$\geq \bigvee_{z=pqr} \bigvee_{a_\alpha \in \underline{A},\, b_\beta \in \underline{B},\, c_\gamma \in \underline{C}} a_\alpha(p) \wedge b_\beta(q) \wedge c_\gamma(r)$$
$$= \bigvee_{a_\alpha \in \underline{A},\, b_\beta \in \underline{B},\, c_\gamma \in \underline{C}} \bigvee_{z=pqr} a_\alpha(p) \wedge b_\beta(q) \wedge c_\gamma(r)$$
$$= \bigvee_{a_\alpha \in \underline{A},\, b_\beta \in \underline{B},\, c_\gamma \in \underline{C}} (a_\alpha \circ b_\beta \circ c_\gamma)(z) = \bigvee_{a_\alpha \in \underline{A},\, b_\beta \in \underline{B},\, c_\gamma \in \underline{C}} z_\omega(z) = \omega.$$

This implies that $z_\omega \in \underline{A \circ B \circ C}$, and hence $\underline{A \circ B \circ C} \supseteq \underline{A} \circ \underline{B} \circ \underline{C}$. □

Theorem 3.2. *Let A be a fuzzy set in a ternary semigroup S. then the following conditions are equivalent:*

a) A is a fuzzy left (lateral, right) ideal of S.

b) $\underline{A}$ is a left (lateral, right)ideal of $\underline{S}$.

Proof. Let A is a fuzzy left ideal in S, and let $x_p \in \underline{A}$ and $y_q, z_r \in \underline{S}$. Then $y_q \circ z_r \circ x_p = (yzx)_{q \wedge r \wedge p} \in \underline{S} \circ \underline{S} \circ \underline{A}$. Since A is a fuzzy left ideal, we have $A(yzx) \geq A(x) \geq p \geq q \wedge r \wedge p$. Hence $y_q \circ z_r \circ x_p = (yzx)_{q \wedge r \wedge p} \in \underline{A}$. This implies that $\underline{S} \circ \underline{S} \circ \underline{A} \subseteq \underline{A}$, thus $\underline{A}$ is a left ideal of $\underline{S}$. conversely, assume that $\underline{A}$ is a left ideal of $\underline{S}$. Let $x, y, z \in S$, if $A(z) = 0$, then $A(xyz) \geq 0 = A(z)$. If $A(z) \neq 0$, then $z_{A(z)} \in \underline{A}$ and $x_{A(z)}, y_{A(z)} \in \underline{S}$. Since $\underline{A}$ is a left ideal of $\underline{S}$, we have $x_{A(z)} \circ y_{A(z)} \circ z_{A(z)} = (xyz)_{A(z)} \in \underline{S} \circ \underline{S} \circ \underline{A} \subseteq \underline{A}$. This implies that $A(xyz) \geq A(z)$, and hence A is a fuzzy left ideal of S. By a similar argument, one can prove the other cases. □

Lemma 3.3. *Let A and B be any fuzzy interior ideals of a ternary semigroup S. Then*

a) $A \cap B$ is also a fuzzy interior ideal of S (provided $A \cap B \neq \emptyset$) .

b) $\underline{A} \cap \underline{B}$ is also an interior ideal of $\underline{S}$.

Proof. a) Since A and B are fuzzy ternary subsemigroups of S , $A \cap B$ is a fuzzy ternary subsemigroup of S [8, lemma 2.3]. Let $x, a, r, s, y \in S$, be arbitrary elements of S. Since A and B are fuzzy interior ideals of S, then

$$(A \cap B)(xsary) = A(xsary) \wedge B(xsary)$$

$$\geq A(a) \wedge B(a) = (A \cap B)(a).$$

Hence $A \cap B$ is a fuzzy interior ideal of S .

b) At first, it is an easy exercise to show that: A is a fuzzy ternary subsemigroup of S if and only if $\underline{A}$ is a ternary subsemigroup of $\underline{S}$. From lemma 3.1, we have $\underline{A} \cap \underline{B} = \underline{A \cap B}$ and so it is a ternary subsemigroup of $\underline{S}$. Let $a_\alpha \in \underline{A \cap B}$ and $x_p, \acute{x}_r, \acute{y}_s, y_q \in \underline{S}$, then

$$(x\acute{x}a\acute{y}y)_{p \wedge r \wedge \alpha \wedge s \wedge q} = x_p \circ \acute{x}_r \circ a_\alpha \circ \acute{y}_s \circ y_q \in \underline{S} \circ \underline{S} \circ \underline{A \cap B} \circ \underline{S} \circ \underline{S}.$$

Since A ∩ B is a fuzzy interior ideal of S, then

$$(A \cap B)(x\acute{x}a\acute{y}y) \geq (A \cap B)(a) = A(a) \wedge B(a) \geq \alpha \wedge \alpha = \alpha$$
$$\geq p \wedge r \wedge \alpha \wedge s \wedge q.$$

This implies that

$$x_p \circ \acute{x}_r \circ a_\alpha \circ \acute{y}_s \circ y_q = (x\acute{x}a\acute{y}y)_{p \wedge r \wedge \alpha \wedge s \wedge q} \in \underline{A \cap B}.$$

Therefore, $\underline{A} \cap \underline{B}$ is also an interior deal of $\underline{S}$. □

Theorem 3.4. *Let A be a fuzzy set in a ternary semigroup S. Then* $\underline{A}$ *is an interior ideal of* $\underline{S}$ *if and only if A is a fuzzy interior ideal of S.*

Proof. Let A is a fuzzy interior ideal of S, then $\underline{A}$ is a ternary subsemigroup of $\underline{S}$. Suppose that $x_p, \acute{x}_r, \acute{y}_s, y_q \in \underline{S}$ and $z_\alpha \in \underline{A}$. Then $A(z) \geq \alpha$, and $A(x\acute{x}z\acute{y}y) \geq A(z) \geq \alpha \geq p \wedge r \wedge \alpha \wedge s \wedge q$. Hence $\underline{S} \circ \underline{S} \circ \underline{A} \circ \underline{S} \circ \underline{S} \ni (x_p \circ \acute{x}_r \circ z_\alpha \circ \acute{y}_s \circ y_q) = (x\acute{x}z\acute{y}y)_{p \wedge r \wedge \alpha \wedge s \wedge q} \in \underline{A}$. This implies that $\underline{S} \circ \underline{S} \circ \underline{A} \circ \underline{S} \circ \underline{S} \subseteq \underline{A}$, thus $\underline{A}$ is an interior ideal of $\underline{S}$. Conversely, suppose that $\underline{A}$ is an interior ideal of $\underline{S}$. For all $, y, z \in S$, the elements $x_{A(x)}, y_{A(y)}, z_{A(z)}$ belong to $\underline{A}$. Since $\underline{A}$ is an interior ideal of $\underline{S}$, we have

$$x_{A(x)} \circ y_{A(y)} \circ z_{A(z)} = (xyz)_{A(x) \wedge A(y) \wedge A(z)} \in \underline{A}.$$

Thus $A(xyz) \geq A(x) \wedge A(y) \wedge A(z)$ and so A is a fuzzy ternary subsemigroup in S. Let $x, \acute{x}, z, \acute{y}, y \in S$, if $A(z) \neq 0$, then $z_{A(z)} \in \underline{A}$ and $x_{A(z)}, \acute{x}_{A(z)}, y_{A(z)}, \acute{y}_{A(z)} \in \underline{S}$. Since $\underline{A}$ is an interior ideal of $\underline{S}$, we get $(x\acute{x}z\acute{y}y)_{A(z)} = (x\acute{x}z\acute{y}y)_{A(z) \wedge A(z) \wedge A(z) \wedge A(z) \wedge A(z)} = x_{A(z)} \circ x'_{A(z)} \circ z_{A(z)} \circ' y_{A(z)} \circ y_{A(z)} \in \underline{A}$. This implies that $A(x\acute{x}z\acute{y}y) \geq A(z)$, and hence A is a fuzzy interior ideal of S. □

Let S be a ternary semigroup. An element $x \in S$ is called *regular* if there exists an element $a \in S$ such that $x = xax$. A ternary semigroup is called *regular* if all its elements are regular [7].

Theorem 3.6. *Let A be a fuzzy set in a regular ternary semigroup S. Then the following conditions are equivalent:*

a) A is a fuzzy ideal of S.

b) $\underline{A}$ is an interior ideal of $\underline{S}$.

Proof. Let A be a fuzzy ideal of S. Then A is a fuzzy ternary subsemigroup of S, and consequently $\underline{A}$ is a ternary subsemigroup of $\underline{S}$. Since any fuzzy ideal of S is a fuzzy interior ideal of S[7], then theorem 3.4 implies that $\underline{A}$ is an interior ideal of $\underline{S}$. Assume that (b) holds. Let $x \in S$, then there exists $a \in S$ such that $x = xax$ (since S is regular). If $A(x) = 0, A(xyz) \geq 0 = A(x)$. If $A(x) \neq 0$, then $x_{A(x)} \in \underline{A}$ and $y_{A(x)}, z_{A(x)} \in \underline{S}$. Since $\underline{A}$ is an interior ideal of $\underline{S}$, we have $(xyz)_{A(x)} = (xaxyz)_{A(x)} = x_{A(x)} \circ a_{A(x)} \circ x_{A(x)} \circ y_{A(x)} \circ z_{A(x)} \in \underline{A}$. This implies that $A(xyz) \geq A(x)$, and hence A is a fuzzy right ideal of S. In a similar argument we prove that A is a fuzzy left ideal of S. It remains to show that A is a fuzzy lateral ideal of S. For this purpose, assume that $y, a \in S$ such that $y = yay$ (since S is regular). By theorem, $A(y) = A(yay) \geq A(y) \wedge A(a) \wedge A(y)$ which implies that $A(a) \geq A(y)$. If $A(y) \neq 0$, then $y_{A(y)}, a_{A(y)} \in \underline{A}$ and $x_{A(y)}, z_{A(y)} \in \underline{S}$. Since $\underline{A}$ is an interior ideal of $\underline{S}$, we have $(xyz)_{A(y)} = (xyayz)_{A(y)} = x_{A(y)} \circ y_{A(y)} \circ a_{A(y)} \circ y_{A(y)} \circ z_{A(y)} \in \underline{A}$. This implies that $A(xyz) \geq A(y)$, and hence A is a fuzzy lateral ideal of S. This completes that A is a fuzzy ideal of S. □

A ternary semigroup S is called *intra-regular* if for each element $a \in S$, there exist elements $x, y \in S$ such that $a = xa^3y$ [8]. For example, let $S = \{i, 0, -i\}$. Then S is a ternary semigroup under the multiplication over complex numbers. In S, we have $(-i)(i^3)(-i) = i$, $(i)(0^3)(-i) = 0$ and $(i)(-i)^3(i) = -i$. Therefore, $S = \{i, 0, -i\}$ is intra-regular.

Theorem 3.7. *A ternary semigroup S is intra-regular if and only if $\underline{S}$ is intra-regular.*

Proof. (⇒) Let a_α be an element is $\underline{S}$. Since S is intra-regular and $a \in S$, there exist $x, y \in S$ such that $a = xa^3y$. Thus $x_\alpha, y_\alpha \in \underline{S}$ and $x_\alpha \circ a_\alpha \circ a_\alpha \circ a_\alpha \circ y_\alpha = (xa^3y)_\alpha = a_\alpha$. Hence $\underline{S}$ is intra-regular.

(⇐) Assume $\underline{S}$ is intra-regular and $a \in S$. Then for any $\alpha \in (0,1]$, there exist $x_\beta, y_\gamma \in \underline{S}$ such that $a_\alpha = x_\beta \circ a_\alpha \circ a_\alpha \circ a_\alpha \circ y_\gamma = (xa^3y)_{\beta\wedge\alpha\wedge\gamma}$. This implies that $a = xa^3y$ for $x, y \in S$, hence S is intra-regular.

A fuzzy ternary subsemigroup A of a ternary semigroup S is called a *fuzzy bi-ideal* of S if $A(xaybz) \geq A(x) \wedge A(y) \wedge A(z)$ for all $x; a; y; b; y \in S$[10].

Theorem 3.8 (see [10, Theorem 4.4]). *A fuzzy ternary subsemigroup B of a ternary semigroup S is a fuzzy bi-ideal of S if and only if* $(B \circ S \circ B \circ S \circ B) \subseteq B$.

Theorem 3.9 (see [10, Theorem 4.5]). *A fuzzy ternary subsemigroup* f *of a semigroup* S *is a fuzzy bi-ideal of* S *if and only if the level set of* f, f_t *is a bi-ideal of* S *for* $t \in Im\, f$.

Theorem 3.10 *Let A be a fuzzy set in a ternary semigroup S. Then A is a fuzzy bi- ideal of S if and only if* $\underline{A}$ *is a bi- ideal of* $\underline{S}$.

Proof. Let A be a fuzzy bi- ideal of S, then by theorem 3.8, $A \circ S \circ A \circ S \circ A \subseteq A$. This implies that $\underline{A \circ S \circ A \circ S \circ A} \subseteq \underline{A}$ and by lemma 3.1, $\underline{A} \circ \underline{S} \circ \underline{A} \circ \underline{S} \circ \underline{A} \subseteq \underline{A \circ S \circ A \circ S \circ A} \subseteq \underline{A}$. Since $\underline{A}$ is a ternary subsemigroup of $\underline{S}$, we conclude that $\underline{A}$ is a bi-ideal of $\underline{S}$. Conversely, let $\underline{A}$ is a bi-ideal of $\underline{S}$, then $\underline{A} \circ \underline{S} \circ \underline{A} \circ \underline{S} \circ \underline{A} \subseteq \underline{A}$. For some $t \in Im\, A$, let $A_t = \{x \in S: A(x) \geq t\}$ be the level set of A. It is clear that $x_t, y_{t,} z_t \in \underline{A}$, for $x, y, z \in A_t$. Now let $w_t = (xaybz)_t = x_t \circ a_t \circ y_t \circ b_t \circ z_t \in \underline{A} \circ \underline{S} \circ \underline{A} \circ \underline{S} \circ \underline{A}$, since $\underline{A}$ is a bi-ideal of $\underline{S}$, then $w_t \in \underline{A}$. Hence, $A(xaybz) \geq t$ and implies that $(xaybz) \in A_t$ for $a, b \in S$. Then $A_t S A_t S A_t \subseteq A_t$, that is, A_t is a bi-ideal of S. Now by theorem 3.9 and the fact that A is a fuzzy ternary semigroup of S , it follows that A is a fuzzy bi- ideal of S.

Conflict of Interests

The author declares that there is no conflict of interests.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, Inform. Control, 8 (1965) 338-353.

[2] A. Rosenfeld, Fuzzy Groups, J. Math. Anal. Appl. 35 (1971), 512-517.

[3] N. Kuroki, Fuzzy bi-ideals in semigroups, Comment. Math. Univ. St. Pauli 28 (1979) 17-21.

[4] N. Kuroki, On fuzzy ideals and bi-ideals in semigroups, Fuzzy Sets and Systems, 5 (1981) 203-215.

[5] N. Kuroki, Fuzzy semiprime ideals in semigroups, Fuzzy Sets and Systems 8 (1982), 71-79.

[6] N. Kuroki, On fuzzy semigroups, Inform. Sci. 53 (1991), 203-236.

[7] M. Santiago and S. Bala, Ternary semigroups , Semigroup Forum, 81 (2010) 380–388.

[8] S. Kar and P. Sarkar, fuzzy ideals of ternary semigroups, Fuzzy Inf. Eng. 2 (2012) 181-193.

[9] K. H. Kim, On fuzzy points in semigroups, Int. J. Math. Math. Sci. 26(11) (2001) 707-712.

[10] S. Kar and P. Sarkar, Fuzzy quasi-ideals and fuzzy bi-ideals of ternary semigroups, Annals of Fuzzy Mathematics and Informatics. 4 (2012), 407- 423.

[11] S. Lekkoksung , Fuzzy interior ideals in ordered ternary semigroups, Int. J. of Math. Analysis, 5 (2011), 2035 – 2039.