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A SLIDING MODEL CONTROL FOR AN EULER-BERNOULLI BEAM WITH BOUNDARY SHEAR FORCE FEEDBACK

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Abstract. A flexible Euler-Bernoulli beam formulated by partial differential equations subject to the boundary shear force feedback is investigated in this paper. First, an abstract evolution equation corresponding to the beam system is established in an appropriate Hilbert space. Then, a spectral analysis and semigroup generation of the operator of the beam system are discussed. Finally, a sliding model control is proposed associated with an equivalent control equation, and a significant result that the solution of the beam system can be approximated by the ideal sliding mode under a sliding model control is proposed and proved.

Keywords: Euler-Bernoulli beam; semigroup of linear operators; sliding model control.

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1. Introduction

Let us consider the following Euler-Bernoulli beam with the boundary shear force feedback control [1]−[6]

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$$\begin{cases} y_{tt}(x,t)+y_{xxxx}(x,t)=0,\ 0<x<1,t>0, \\ y(0,t)=y_x(0,t)=y_{xx}(1,t)=0, \\ y_{xxx}(1,t)=ky_t(1,t)=0,\ k>0, \\ y(x,0)=y_0=,y_t(x,0)=y_1(x),0\le x\le 1, \end{cases} \tag{1.1}$$

where $y(x,t)$ is the displacement of the beam at the locatin x and the time t, $y_t(x,t)$ is the instantaneous velocity, $y_x(x,t)$ is the rotation of the beam at the location x and the time t, $y_{xt}(x,t)$ is the angular velocity of the beam, $y_{xx}(x,t)$ is the bending moment, and $y_{xxx}(x,t)$ is the shear force.

For the sake of simplicity, let $z(x,t)=y_x(x,t)$, and it is easy to see that $z(x,t)$ satisfies

$$\begin{cases} z_{tt}(x,t)+z_{xxxx}(x,t)=0,\ 0<x<1,t>0, \\ z(0,t)=z_{xxx}(0,t)=z_x(1,t)=0, \\ z_{xxt}(1,t)=-kz_{xxx}(1,t),\ k>0, \\ z(x,0)=z_0(x)=,z_t(x,0)=z_1(x),0\le x\le 1. \end{cases} \tag{1.2}$$

It should be noted that the boundary conditions specified in (1.2) are different from any one of the five kind beams: clamped, embeded, simply supported, roller suported and two end free. In present paper, we are going to investigate a sliding model control problem for the Euler-Bernoulli bean systen (1.1)-(1,2) above in an appropriate Hilbert space. First, let us establish an abstract equation corresponding to the system (1.1)-(1.2) in next section.

2. An abstract evolution equation of the system and its properties

Now, we can rewrite (1.1) and (1.2) in terms of a system operator.

First, let us define a system operator $\mathscr{A}_0 : D(\mathscr{A}_0)(\subset \mathscr{H}_0) \to \mathscr{H}_0$ as follows:

$$\begin{cases} \mathscr{A}_0(f,g)=(g,-f^{(4)}),\forall(f,g)\in D(A_0), \\ D(A_0)=\{(f,g)\in(H^4\cap H_c^2\times H_c^(2)|, \\ \ f'''(1)=kg(1),f''(1)=0\}, \\ \mathscr{H}_0=H_c^2(0,1)\times L^2(0,1), \\ H_C^2=\{f\in H^2(0,1)|f(0)=0,f'(0)=0\}. \end{cases} \tag{2.1}$$

It has been shown in [6] that $\mathscr{A}_0$ is a dissipative discrete operator:$\mathscr{A}_0^{-1}$ exists and it is compact on $\mathscr{H}_0$.Hence the spectrum $\sigma(\mathscr{A}_0)$ consists of isolated eigenvalues only and Re $\lambda < 0$ for all $\lambda \in \sigma(\mathscr{A}_0)$. Moreover, $\lambda \in (\mathscr{A}_0)$ if and only if there exists an $f \neq 0$ such that (f, λ) satisfies the differential equation of the following :

$$\lambda^2 f(x) + f^{(4)}(x) = 0,\ 0 < x < 1,$$

$$f(0) = f'(0) = f''(1) = 0\ f'''(1) = k\lambda(1). \tag{2.2}$$

Define the underlying state space for (1.2) as the natural energy space $\mathscr{H} = H_E^2(0,1) \times L^2(0,1) \times C$, where $H_E^2(0,1) = \{f \in H^2(0,1) | f(0) = 0\ f'(1) = 0\}$.The space $\mathscr{H}$ is a Hilbert space with inner product induced norm of the following:

$$\|(f,g,\alpha)\|^2 = \int_0^1 [|f''(x)|^2 + |(g(x)|^2]dx + \frac{1}{k}|\alpha|^2$$

for all $(f,g,\alpha) \in \mathscr{H}$.

The system operator $\mathscr{A} : \mathscr{D}(\mathscr{A})(\subset \mathscr{H}) \rightarrow$ is defined as follows

$$\begin{cases} \mathscr{A}(f,g,\alpha) = (g, -f^{(4)}, -kf'''(1)), \\ \mathscr{D}\mathscr{A} = \{(f,g,\alpha) \in (H^4 \bigcap H_E^2 \times H_E^2 \times C|, \\ \alpha = f''(1), f'''(0) = 0\}. \end{cases} \tag{2.3}$$

In terms of the setting above, system (1.2) can be formulated as

$$\frac{dZ(t)}{dt} = \mathscr{A}Z(t),\ Z(0) = Z_0 \in \mathscr{H}, \tag{2.4}$$

where $Z(t) = (z(\cdot,t), z_t(\cdot,t), z''(1,t))$ and Z_0 is the initial value.

Next, we are going to discuss the spectral properties of the system operator $\mathscr{A}$. Some results can be found in the reference [3].

Recall that a nonzero $Y \in \mathscr{H}$ is called a eigenvalues of $\mathscr{A}$, corresponding to eigenvalues λ, if there is a positive integer n such that$(\lambda - \mathscr{A})^n Y = 0$. Let $\mathrm{S}p(\mathscr{A})$ denote the root subspace of $\mathscr{A}$, that is a closed linear span of all generalized eigenfunctions of $\mathscr{A}$. A sequence in $\mathscr{H}$ is said to be complete in $\mathscr{H}$ if its linear span is dense in $\mathscr{H}$.

Theorem 2.1. *The root subspaces of both $\mathscr{A}$ and $\mathscr{A}^*$ are complete in $\mathscr{H}$:$Sp(\mathscr{A})$ =$Sp(\mathscr{A}^*) = \mathscr{H}$.*

Proof. We only show the completeness for the root subspace of $\mathscr{A}$ because the proof for that of $\mathscr{A}^*$ is almost the same. It follows from [4, lemma 5, p.2355] that the following orthogonal decomposition holds:

$$\mathscr{H} = \sigma_\infty(\mathscr{A}^*) \oplus Sp(\mathscr{A}),$$

where $\sigma_\infty(\mathscr{A}^*)$ consists of those $Z \in \mathscr{A}$ so that $R(\lambda,\mathscr{A}^*)Z$ is an analytic function of λ in the whole complex plane. Hence, $\mathrm{S}p(\mathscr{A}) = \mathscr{H}$, if and only $\sigma_\infty(\mathscr{A}^*) = \{0\}$. Now suppose that $Z \in \sigma_\infty(\mathscr{A}^*)$. Since $R(\lambda,\mathscr{A}^*)Z$ is an analytic function in λ, it is also analytic in ρ. By means of the maximum modulus principle (or the Phragemé-Lindelöf theorem), [5,Lemma 2.5] and the fact that $\|R(\lambda,\mathscr{A}^*)\| = \|R(\bar{\lambda},\mathscr{A})\|$, it can seen that

$$R(\|\lambda,\mathscr{A}^*)\| \leq C(1+|\lambda|)\|Z\|),\ \forall\lambda \in C$$

for some constant $C > 0$. By [7,Th.1,P.3], we conclude that $R(\lambda,\mathscr{A}^*Z)$ is a polynomial in λ of degree ≤ 1, that is $R(\lambda,\mathscr{A}^*)Z = \mathrm{Z}_2 + \lambda \mathrm{Z}_1$ for some $\mathrm{Z}_2, \mathrm{Z}_1 \in \mathscr{H}$ Therefore,$Z = (\lambda - \mathscr{A}^*)(\mathrm{Z}_2 + \lambda Z_1)$, or

$$-\mathscr{A}^*\mathrm{Z}_2 + \lambda(\mathrm{Z}_2 - \mathscr{A}^*\mathrm{Z}_1) + \lambda^2\mathrm{Z}_1 = \mathrm{Z},\ \forall\lambda \in c.$$

Thus, $Z_1 = Z_2 = Z = 0$. The proof is complete.

We now study peroperty of basis of Hilber space. A sequence in a Hilbert space H is called *minimal* if each element of this sequence lies outside the closed linear span of the remaining elements.Two sequences$\{e_i\}$ and $\{e_i^*\}$in H are said to be biorthogonal if

$$\langle e_i, e_j^*\rangle = \delta_{ij} =^{1,i=j}_{0,i\neq j}$$

for every i and j. It is well know that for a given sequence $\{e_i\}$ a biorthogonal sequence $\{e_i^*\}$ exists if and only if $\{e_i\}$ is minimal,and that $\{e_i^*\}$ is uniquely determined if and only if $\{e_i\}$ is complete.A sequence $\{e_i\}i =^{\infty}_{1}$ is called a *Bessel sequence* in H if for any $x \in H$, the series $\{\langle x, e_i\rangle\}_{i=1}^{\infty} \in l^2$. A sequence $\{e_i\}_{i=1}^{\infty}$ is called a basis for H if any element $x \in H$ has a unique representation

$$x = \sum_{i=1}^{\infty} a_i e_i\ . \tag{2.5}$$

and the convergence of the series is in the norm of H.$\mathscr{A}$ sequence $\{e_i\}_{i=1}^{\infty}$ with birorthogonal sequence $\{e_i^*\}_{i=1}^{\infty}$ is called a *Riesz basis* for H if $\{e_i^*\}_{i=1}^{\infty}$ is an approximately normalized basis for H and the series in (2.5) converges unconditionally in the norm of H. Equivalently, $\{e_i\}_{i=1}^{\infty}$(or$\{e_i^*\}_{i=1}^{\infty}$ is a *Riesz basis* for H if and only if the following two conditions are satisfied [16,p.27]:

a) both $\{e_i\}_{i=1}^{\infty}$ and $\{e_i^*\}_{i=1}^{\infty}$ are complete in H;

b) both $\{e_i\}_{i=1}^{\infty}$ and $\{e_i^*\}_{i=1}^{\infty}$ are *Bessel sequence* in H.

Theorem 2.2. *The generalized eigenfunctions of $\mathscr{A}$ form a Riesz basis for $\mathscr{H}$.*

Proof. In view of the [5, Lemma 2.2] ,we may assume without loss of generality that $\sigma(\mathscr{A}) = \sigma(\mathscr{A}^*) = \{\lambda_n, \bar{\lambda}_n\}_{n=1}^{\infty}$. Corollary 2.1 tells us that there exists an integer $N > 0$ such that all $\lambda_n, \bar{\lambda}_n, n \geq N$ are algebraically simple.Assume that the algebraic multiplicity of each λ_n is m_n for $n \leq N$.We say that $\Phi_{n,1}$ is the highest-order generalized eigenvector of $\mathscr{A}$ if

$$(\mathscr{A} - \lambda_n)^{m_n}\Phi_{n,1} = 0 \text{ but } (\mathscr{A} - \lambda_n)^{m_n-1}\Phi_{n,1} \neq 0.$$

Then, the other lower order linearly independent generalized eigenvectors associated with λ can be found through

$$\Phi_{n,j} = (\mathscr{A} - \lambda_n)^{j-1}\Phi_{n,1},\ j = 2,3,\cdots m_n.$$

Assume Φ_n is an eigenfunction of $\mathscr{A}$ corresponding to λ_n with $n \geq N$. Then $\{\{\Phi_{n,j}\}_{j=1}^{m_n}\}_{n<N} \cup \{\Phi_n\}_{n\geq N} \cup \{\text{their conjugates}\}$ are all linearly independent generalized eigenfunctions of $\mathscr{A}$. Let $\{\{\Phi_{n,j}^*\}_{j=1}^{m_n}\}_{n<N} \cup \{\Phi_n^*\}_{n\geq N}$ be the biorthogonal sequence of $\{\{\Phi_{n,j}\}_{j=1}^{m_n}\}_{n<N} \cup \{\Phi_n\}_{n\geq N}$. Then, $\{\{\Phi_{n,j}\}_{j=1}^{m_n}\}_{n<N} \cup \{\Phi_n^*\}_{n\geq N} \cup \{\text{their conjugates}\}$ are all linearly independent generalized eigenfunctions of $\mathscr{A}^*$. It is well-known that these two sequences are minimal in $\mathscr{H}$ and from Theorem 2.1, they are also complete in $\mathscr{A}$.

According to the definite of a Riesz basis,it suffices to show that both $\{\Phi_n\}_{n\geq N}$ and $\{\Phi_n^*\}_{n\geq N}$ are Bessel sequences in $\mathscr{H}$ Since $1 \leq \|\Phi_n\|\|\Phi_n^*\| \leq M$ for some constant M independent of n [8,p.19],we may assume without loss of generality that $\Phi_n = (\phi_n, \lambda_n\phi_n, \alpha_n)$ [5, (2.10)] and $\Phi_n^* = (f_n, g_n\beta_n)$[5, (2.13)] for all $n \geq N$. Then,it follows from [5,Lemma 2.6] that all sequences $\{\phi_n''\}_{n=N}^{\infty}, \{\lambda_n\phi_n\}_{n=N}^{\infty}$ and $\{f_n''\}_{n=N}^{\infty}, \{g_n''\}_{n=N}^{\infty}$ are Bessel sequences in $L^2(0,1)$ and $\{\alpha_n\}_{n=N}^{\infty}$ and $\{\beta_n\}_{n=N}^{\infty}$ are Bessel sequences in $\mathscr{C}$,so are $\{\phi_n\}_{n\geq N}$ and $\{\phi_n^*\}_{n\geq N}$ in $\mathscr{H}$. The result follows.

The following theorem is immediately a corollary of the theorem 2.2 above, which provides important semigroup generation by the system operator.

Theorem 2.3. *The operator $\mathscr{A}$ generates a C_0-semigroup $e^{\mathscr{A}}$ on $\mathscr{H}$.*

The next theorem shows that the beam system (2.4) is exponential stable.

Theorem 2.4. *The beam trajectories system (2.4) converges exponentially to the zero eigenspace. Precisely, there exist constants $M, \omega > 0$, such that any mild solution $Z(t)$ [13]to the (2.4) with initial value $Z_0 = (f, g, \alpha) \in \mathscr{H}$ satisfies*

$$\|Z(t) - \langle Z_0, \Psi_0^*\rangle \Phi_0\| \le Me^{-\omega t}\|Z_0\|,$$

where

$$\langle Z_0, \Psi_0^*\rangle \Phi_0 = (x^2 - 2x, 0, 2) \int_0^1 g(\tau) d\tau. \tag{2.6}$$

Proof. From the Theorem 2.2 and the [5,Corollary 2.1], the eigenvalue λ_n for n large enough is simple and we can choose a sequence

$$\{\Phi_0\} \cup \{(\Phi_{n,j}, \bar{\Phi}_{n,j}) : \ 1 \le j \le m_n, \ n = 1, 2, \cdots\}$$

of generalized eigenfunctions of $\mathscr{A}$ to form a Riesz basis for $\mathscr{H}$, Where m_n denotes the algebraic multiplicity of eigenvalue λ_n.Denote by

$$\{\Psi_0^*\} \cup \{(\Psi_{n,j}^*, \bar{\Psi}_{n,j}^*) : \ 1 \le j \le m_n, \ n = 1, 2, \cdots\}$$

its corresponding biorthogonal system. Without loss of generality, we may assume that all λ_n for $n \ge 1$ are simple. Then for any initial state $Z_0 = (f, g, \alpha) \in \mathscr{H}$, we have

$$Z_0 \langle Z_0, \Phi_0^*\rangle \Phi_0 + \sum_{n=1}^{\infty} \langle Z_0, \Phi_n^*\rangle \Phi_n + \sum_{n=1}^{\infty} \langle Z_0, \bar{\Phi}_n^*\rangle \bar{\Phi}_n.$$

Let $e^{\mathscr{A}t}$ be the semigroup generated by $\mathscr{A}$.Then,we get

$$\begin{cases} Z(t) &= e^{\mathscr{A}t} Z_0 = \langle Z_0, \Phi_0^*\rangle e^{\mathscr{A}t}\Phi_0 + \sum_{n=1}^{\infty} \langle Z_0, \Phi_n^*\rangle e^{\mathscr{A}t}\Phi_n \\ &+ \sum_{n=1}^{\infty} \langle Z_0, \bar{\Phi}_n^*\rangle e^{\mathscr{A}t}\bar{\Phi}_n = \langle Z_0, \Phi_0^*\rangle \Phi_0 \\ &+ \sum_{n=1}^{\infty} e^{\lambda_n t} \langle Z_0, \Phi_n^*\rangle + \sum_{n=1}^{\infty} \langle Z_0, \bar{\Phi}_n^*\rangle e^{\bar{\lambda}_n t} \bar{\Phi}_n. \end{cases}$$

Since all nonzero eigenvalues satisfy Re $\lambda_n < 0$,it follows that there exists a constant $\delta > 0$ such that

$$\|Z(t) - \langle Z_0, \Phi_0^* \rangle \Phi_0\| = \mathscr{O}(e^{-\delta t}) \to 0,\ t \to \infty.$$

Furthermore,we conclude from $\Phi_0 = (x^2 - 2x, 0, 2)$ in [5, Lemma 2.1] and $\Psi_0^* = (0, 1, 0)$ in [5, Lemma 2.4] that

$$\langle Z_0, \Psi_0^* \rangle \Phi_0 = \Phi_0 \int_0^1 g(\tau) d\tau.$$

This is (2.6). The proof is complete.

3. Sliding model control

Let us establish a sliding model control for the Euler-Bernoulli beam system (2.4):

$$\begin{cases} \dfrac{dZ(t)}{dt} = \mathscr{A} Z(t) + Bw(Z,t), \\ Z(0) = Z_0, \end{cases} \tag{3.1}$$

where B is a bounded linear operator from $\mathscr{H}$ to $\mathscr{H}$, $w(Z,t)$ is the control of the beam system (2.4) that is not continuous on the manifold $S = CZ = 0$, and C is a bounded linear operator with $S = S(Z) = CZ \in R^n$.

Now, we consider the δ-neighborhood of sliding mode $S = CZ = 0$, where $\delta > 0$ is an arbitrary given positive number. Using a continuous control $\tilde{w}(z,t)$ to replace $w(Z,t)$ in the system (3.1) yields

$$\begin{cases} \dot{Z} = \mathscr{A} Z + B\tilde{w}(Z,t), \\ Z(0) = Z_0, \end{cases} \tag{3.2}$$

where $\dot{Z} = \partial Z / \partial t$, and the solution of (3.2) belongs to the boundary layer $\|S(Z)\| \leq \delta$.

Let $\dot{S}(Z) = c\dot{Z} = 0$. Applying C to the first equation of (3.2) leads to the following the equivalent control:

$$w_{eq}(Z,t) = -(CB)^{-1} C(\mathscr{A} Z)$$

with assumption that $(CB)^{-1}$ exists. Substitute $w_{eq}(Z,t)$ into (3.1) to find

$$\dot{Z} = [I - B(CB)^{-1} C] \mathscr{A} Z \ \cdot \tag{3.3}$$

Denote $P = B(CB)^{-1}C$ and $\mathscr{A}_1 = (I-P)\mathscr{A}$, then (3.1) becomes

$$\dot{z} = \mathscr{A}_1 z, \qquad z(0) = z_0. \tag{3.4}$$

In the rest part of this paper, we are going to show that the actual sliding mode $Z(t)$ will approach uniformly to the ideal sliding mode $\bar{Z}(t)$ under certain conditions.

Lemma 3.1 *If $(CB)^{-1}$ is a compact operator and $P\mathscr{A} = \mathscr{A}P$, then $\mathscr{A}_1 = (I-P)\mathscr{A}$ generates a C_0-semigroup $T_2(t)$ in $\mathscr{H}$ and $T_2(t) = (I-P)T_1(t)$, where $T_1(t)$ is the C_0-semigroup generated by $\mathscr{A}$.*

Proof. Since $(CB)^{-1}$ is a compact operator, B and C are bounded linear operators, we see from the definition of P that P is compact, and therefor the range of $I-P$ is a closed subspace of $\mathscr{H}$. Since $P^2 = P$ and $(1-P)^2 = I-P$, $I-P$ can be viewed as the identity operator on $(I-P)\mathscr{H}$. It can be easily seen that $T_2(t) = (I-P)T_1(t)$ is a C_0-semigroup in $(I-P)\mathscr{H}$.

Next, we shall prove that the infinitesimal generator of $T_2(t)$ is $(I-P)\mathscr{A}$ and $\mathscr{D}((I-P)\mathscr{A}) = (I-P)\mathscr{D}(\mathscr{A})$.

In fact, for every $x \in (I-P)\mathscr{D}(\mathscr{A})$, there is a $x_1 \in \mathscr{D}(\mathscr{A})$ such that $x = (I-P)x_1$. It should be noted that $T_1(t)$ and $I-P$ are commutative because $\mathscr{A}$ and P are commutative. We see that

$$\begin{aligned}
\lim_{t\to 0^+} T_2(t)x - x &= \lim_{t\to 0^+} (I-P)T_1(t)(I-P)x_1 - (I-P)x_1 \\
&= \lim_{t\to 0^+} (I-P)^2 T_1(t)x_1 - (I-P)x_1 \\
&= \lim_{t\to 0^+} (I-P)T_1(t)x_1 - (I-P)x_1 \\
&= (I-P)\lim_{t\to 0^+} T_1(t)x_1 - x_1 \\
&= (I-P)\mathscr{A}x_1.
\end{aligned}$$

Let $\tilde{\mathscr{A}}$ be the infinitesimal generator of $T_2(t)$. Since the limit on the left exists, we can assert that $x \in \mathscr{D}(\tilde{\mathscr{A}})$ and $(I-P)\mathscr{D}(\mathscr{A}) \subseteq \mathscr{D}(\tilde{\mathscr{A}})$.

On the other hand, for any $x \in \mathscr{D}(\tilde{\mathscr{A}})$, since $\mathscr{D}(\tilde{\mathscr{A}}) \subseteq (I-P)\mathscr{H}$, there exists $\tilde{x} \in \mathscr{H}$, such that $x = (I-P)\tilde{x}$, and

$$\begin{aligned}\lim_{t\to 0^+} T_2(t)x - x &= \lim_{t\to 0^+} T_2(t)(I-P)\tilde{x} - (I-P)\tilde{x} \\ &= \lim_{t\to 0^+} (I-P)T_1(t)\tilde{x} - (I-P)\tilde{x} \\ &= (I-P)\lim_{t\to 0^+} T_1(t)\tilde{x} - \tilde{x} \\ &= (I-P)\mathscr{A}\tilde{x}.\end{aligned}$$

Since the limit of the left hand side exists, and so the limit of the right hand side exists, and $\tilde{x} \in \mathscr{D}(\mathscr{A})$ which implies that $\mathscr{D}(\tilde{\mathscr{A}}) \subseteq (I-P)\mathscr{D}(\mathscr{A})$. Thus, $\mathscr{D}(\tilde{\mathscr{A}}) = (I-P)\mathscr{D}(\mathscr{A})$ and $\tilde{\mathscr{A}}$, the infinitesimal generator of $T_2(t)$, is $(I-P)\mathscr{A}$. The proof of the lemma is complete.

Theorem 3.2 *Suppose that in the beam system (3.1),*

(1) $(CB)^{-1}$ *exists and it is compact,*

(2) $P\mathscr{A} = \mathscr{A}P$, *where* $P = B(CB)^{-1}C$.

Then for any solution $Z(t)$ *of the system (3.4) satisfying* $S(\bar{Z}_0) = 0$, $\bar{Z}_0 \in \mathscr{D}(\mathscr{A}_1)$ *and* $\|Z_0 - \bar{Z}_0\| \leq \delta$, $Z_0 \in \mathscr{D}(\mathscr{A})$, *we have*

$$\lim_{\delta\to 0} \|Z(t) - \bar{Z}(t)\| = 0$$

uniformly on $[0,T]$ *for any positive number* T.

Proof. We see from the Theorem 2.3 and Lemma 3.1 that $\mathscr{A}$ and $\mathscr{A}_0 = (I-P)\mathscr{A}$ are infinitesimal generators of C_0-semigroups $T_1(t)$ and $T_2(t)$ respectively. It follows from theory of semigroup of linear operators that there are positive constants M_1, M_2, ω_1 and ω_2 such that

$$\|T_1(t)\| \leq M_1 e^{\omega_1 t}, \quad \|T_2(t)\| \leq M_2 e^{\omega_2 t}. \quad (0 \leq t \leq T) \tag{3.5}$$

In the boundary layer $\|T_1(t)\| \leq \delta$, the equivalent control is

$$w_{eq}(Z,t) = -(CB)^{-1}C\mathscr{A}Z + (CB)^{-1}C\dot{Z}. \tag{3.6}$$

Substitute (3.6) into (3,1) to find

$$\dot{Z} = (I-P)\mathscr{A}Z + P\dot{Z}. \tag{3.7}$$

Hence, the solution of (3.7) can be expressed as follows:

$$Z(t) = T_2(t)Z_0 + \int_0^t T_2(t-s)P\dot{Z}(s)ds, \tag{3.8}$$

and therefore, the solution of (3.4) can be written as

$$\overline{Z}(t) = T_2(t)\overline{Z}_0. \tag{3.9}$$

Substracting (3.9) into (3.8) yields

$$\begin{aligned} & Z(t) - \overline{Z}(t) \\ = \;& T_2(t)(Z_0 - \overline{Z}_0) + \int_0^t T_2(t-s)P\dot{Z}(s)ds. \end{aligned} \tag{3.10}$$

Since $P\mathscr{A} = \mathscr{A}P$, we see that $PT_1(t) = PT_1(t)$. It should be emphasized that $(I-P)P = 0$ and $T_2(t) = (I-P)T_1(t)$, and consequently,

$$\begin{aligned} \int_0^t T_2(t-s)P\dot{Z}(s)ds &= \int_0^t (I-P)T_1(t-s)P\dot{Z}(s)ds \\ &= \int_0^t T_1(t-s)(I-P)P\dot{Z}(s)ds \\ &= 0. \end{aligned}$$

It can be obtained from (3.10) and (3.5) that

$$\|Z(t) - \bar{Z}(t)\| \le \|T_2(t)\|\|Z_0 - \bar{Z}_0\| \le M_2 e^{\omega_2 T}\|Z_0 - \bar{Z}\|.$$

Since $\|Z_0 - \bar{Z}_0\| \le \delta$, we have

$$\|Z(t) - \bar{Z}(t)\| \le M_2 e^{\omega_2 T}\delta.$$

Thus,

$$\lim_{\delta \to 0} \|Z(t) - \bar{Z}_0\| = 0.$$

The proof of the theorem is complete.

We see from the Theorem 3.2 that the solution of the beam system can be approximated by ideal sliding mode in any accuracy.

4. Conclusion

In this paper, the sliding model control problem for a flexible Euler-Bernoulli beam formulated by partial differential equations subject to the boundary shear force feedback is investigated. An evolution equation corresponding to the beam system is established in an appropriate Hilbert space. A spectral analysis and semigroup generation of the system operator of the beam system are discussed. Finally, a sliding model control is proposed, and a significant result that the solution of the beam system can be approximated by ideal sliding model under the control is obtained.

Conflict of Interests

The authors declare that there is no conflict of interests.

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