Available online at http://scik.org
J. Semigroup Theory Appl. 2015, 2015:4
ISSN: 2051-2937

SOME COMBINATORIAL RESULTS ON GREEN'S RELATION OF PARTIAL INJECTIVE TRANSFORMATION SEMIGROUP

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Abstract: In this paper, we characterized the Green's relation on CI_n; the contraction mapping injective partial transformation semigroup on n-objects. Using two parameters $F(n,p)$. We found that the order of L-classes and $R-class$ are the same but $D-class$ is different.

Keywords: Semigroup; injective; contraction mapping; Green's relation.
2010 AMS Subject Classification: 20M18.

1. Introduction and Preliminaries

A Semigroup is an algebraic structure consisting of a nonempty set S together with an associative binary operation. A transformation on X is a function from X to itself. Transformation semigroups are one of the most fundamental mathematical objects. They occur in theoretical computer science, where properties of language depend on algebraic properties of various transformation semigroups related to them.

Let I_n be the symmetric inverse semigroup on $X_n = \{1,2,\dots n\}$. Let $X_n = \{1,2,\dots,n\}$, then a (partial) transformation $\alpha : Dom\alpha \subseteq X_n \to \operatorname{Im}\alpha$ is said to be full or total, if $Dom\alpha = X_n$. Otherwise it is called strictly partial.

The set of all partial transformations on n-objects form a semigroup under the usual composition of transformation, which is denoted by P_n when it is partial, T_n when it is full or total and I_n when it is partial one-one. The elements I_n are usually called chart. All these are the three fundamental transformation semigroups which were introduced by [1]. The semigroup I_n form

Received April 22, 2015

the basis of our study in this research. A transformation α for which $|x\alpha - y\alpha| \leq |x - y|\ \forall\ x, y \in CI_n$ is said to be a contraction mapping.

The combinatorial and algebraic properties of transformation semigroup have been studied and interesting results have emerged. Many researchers have studied combinatorial and algebraic properties of different classes of transformation semigroup.

[4] studied some semigroups of full contraction mapping on a finite chain and [3] studied identity difference transformation semigroups.

Green's relations.

The notation of ideals natural [1] to the consideration of certain equivalence relations on a semigroup, these equivalences have played a fundamental role in the development of semigroup theory. Each D-class in a semigroup S is a union of L-classes and R-classes. The intersection of L-classes and R-classes is either empty or is an H-class. Hence it is convenient to visualize a D-class as egg box, in which each row represents an R-class, each column represent an L-class and each cell represents an H-class (it is possible for the egg box to contain a single row or a single column of cells, or even to contains only cell).

2. Method and Procedure

If $\alpha \in CI_n$, using $ker\alpha$ (i.e the partition of domain α) and I$mage\ \alpha$, we classify the elements of CI_n.

Example 2.1

For$n = 3, CI_3$ has 27 elements, element were arranged as follow: Each row represent an R-class and each column represent and L-class:

$$|Im\alpha| = 3$$

$Ker\alpha \| \text{Im}\,\alpha$	$\{1,2,3\}$
$1\|2\|3$	$\begin{pmatrix}123\\123\end{pmatrix} \begin{pmatrix}123\\321\end{pmatrix}$

$$|\operatorname{Im}\alpha| = 2$$

$Ker\alpha \mid \operatorname{Im}\alpha$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$
$1\|2$	$\begin{pmatrix}12\\12\end{pmatrix}$ $\begin{pmatrix}12\\21\end{pmatrix}$		$\begin{pmatrix}12\\23\end{pmatrix}$ $\begin{pmatrix}12\\32\end{pmatrix}$
$1\|3$	$\begin{pmatrix}13\\12\end{pmatrix}$ $\begin{pmatrix}13\\21\end{pmatrix}$	$\begin{pmatrix}13\\13\end{pmatrix}$ $\begin{pmatrix}13\\31\end{pmatrix}$	$\begin{pmatrix}13\\23\end{pmatrix}$ $\begin{pmatrix}13\\32\end{pmatrix}$
$2\|3$	$\begin{pmatrix}23\\12\end{pmatrix}$ $\begin{pmatrix}23\\21\end{pmatrix}$		$\begin{pmatrix}23\\23\end{pmatrix}$ $\begin{pmatrix}23\\32\end{pmatrix}$

$$|Im\alpha| = 1$$

$Ker\alpha \mid \operatorname{Im}\alpha$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}1\\2\end{pmatrix}$	$\begin{pmatrix}1\\3\end{pmatrix}$
2	$\begin{pmatrix}2\\1\end{pmatrix}$	$\begin{pmatrix}2\\2\end{pmatrix}$	$\begin{pmatrix}2\\3\end{pmatrix}$
3	$\begin{pmatrix}3\\1\end{pmatrix}$	$\begin{pmatrix}3\\2\end{pmatrix}$	$\begin{pmatrix}3\\3\end{pmatrix}$

$$|Im\alpha| = 0$$

$Ker\alpha \mid \operatorname{Im}\alpha$	$\{0\}$
ϕ	$\begin{pmatrix}1 & 2 & 3\\ - & - & -\end{pmatrix}$

From the above example the following result we obtained. We have 8 rows and 8 columns which implies that $|L_a|, |R_a|$ and $|H_a|$ has 8 elements each classes.

3. Main Result

Based on our observation in the procedure used in section 2 above and ideals in [15], we propose the following results:

Lemma: 2.1 Let$\alpha, \beta \in CI_n$. Then

(i) $\alpha R\beta \; if \; and \; only \; if \; im(\alpha) = im(\alpha)$;

(ii) $\alpha L\beta \; if \; and \; only \; if \; Dom(\alpha) = Dom(\beta)$;

(iii) $\alpha H\beta \; if \; and \; only \; if \; im(\alpha) = im(\beta) and \; Dom(\alpha) = Dom(\beta)$;

(iv) $\alpha D\beta \; if \; and \; only \; if \; rank(\alpha) = rank(\beta)$;

(v) $\alpha J\beta \; if \; and \; only \; if \; rank(\alpha) = rank(\beta)$

Theorem 2.2 Let $, \beta \in CI_n$. Then the following result were obtained

(I) $|L_a| = 2^n$ for all $n \geq 1$

(II) $|R_a| = 2^n$ For all $n \geq 1$

(III) $(D_a) = n+1$ For all $n \geq 1$

(IV) $(J_a) = n+1$ For all $n \geq 1$

Proof: It follows directly from Ganyushkin & Mazorechuk [15].

Remark 2.3: It has being observed that the cardinality of L-classes, R-classes, D- classes and H-classes in contraction mapping injective partial transformation semigroup are the same as in partial one-to-one transformations semigroup for all $n \geq 1$.

For some computed values of $F(n ; p)$ for$|L_a| and |R_a|$ see table 1 and for $|D_a|$ see table 2.

Table 1. Number of elements of $|L_a| and |R_a|$ in contraction mapping injective partial transformation semigroup.

$n \| p$	0	1	2	3	4	5	$\sum F(n ; p) = \|L_a\| = \|R_a\|$
0	1						1
1	1	1					2
2	1	2	1				4
3	1	3	3	1			8
4	1	4	6	4	1		16
5	1	5	10	10	5	1	32

Table 2. Number of elements of $|D_a|$ in contraction mapping injective partial transformation semigroup

$n \vert p$	0	1	2	3	4	5	$\sum F(n,p) = \vert D_a \vert = \vert J_a \vert$
0	1						1
1	1	1					2
2	1	1	1				3
3	1	1	1	1			4
4	1	1	1	1	1		5
5	1	1	1	1	1	1	6

Conclusion

In this paper, combinatorial results of Green's relation on semigroup of class of transformation called contraction mapping of injective partial transformation semigroup were studied using two parameters $F(n,p)$, through the height and kernel of α we classify elements of CI_n into $R-class, L-class,\ and\ D-class$.We found that the Semigroup CI_n contains 2^n different $L-classes, 2^n different\ R-classes, and\ n+1\ different\ D-classes\ which\ is\ equal\ to\ J-classes.$

Conflict of Interests

The author declares that there is no conflict of interests.

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