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ORTHOGONAL REVERSE DERIVATIONS- SEMIPRIME GAMMA RINGS

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Abstract: This paper presents the definition of orthogonal reverse derivations; some characterizations of semi prime gamma rings are obtained by using of orthogonal reverse derivations. This paper also investigates conditions for two reverse derivations to be orthogonal.

Keywords: derivation; reverse derivation; semi prime gamma ring; orthogonal reverses derivation.

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1. INTRODUCTION:

In this section review of results on reverse derivations were presented. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [9]. The authors obtain some characterizations of semi prime rings by using reverse derivations. The commutatively properties of a gamma ring with the derivations investigated [9] .The notion of orthogonality for two derivations on a semi prime ring initiated by Bresar and Vukman [6]. Some necessary and sufficient conditions for two orthogonal derivations to be are obtained. They also obtained a counterpart of a result of Posner [8]. Argac, on orthogonal generalized derivations on a semi prime ring and they established some results concerning two generalized derivations on a semi prime ring are worked by Nakajima and Albas [1] Ozturk, Jun and Kim [7] worked on prime -rings by means of derivations.

This paper extends the results mentioned above to gamma rings case. The notions of orthogonality of two reverse derivations and conditions of two reverse derivations to be orthogonal are provided. We also obtain some characterizations of a semi prime gamma rings with orthogonal reverse derivations.

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2. PREMILINARIES:

Let us consider M and Γ be additive abelian groups. If there exists a mapping $M\times\Gamma\times M \to M : (x, \alpha, y) \to x\alpha y$ which satisfies the conditions: for all a, b, c $\in$ M, and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring

1. $(a\alpha b)\beta c = a\alpha(b\beta c)$

2. $a(\alpha + \beta)b = a\alpha b + a\beta b$,

   $(a + b)\alpha c = a\alpha c + b\alpha c$,

   $a\alpha(b + c) = a\alpha b + a\alpha c$,

The concepts of sub ring and ideal are imitated from the classical case. Throughout the paper, M denotes a Γ-ring with center Z(M). If $2x = 0$ for $x \in M$ implies $x = 0$A ring, then M is said to be 2-torsion free.We write $[x, y]\alpha$ for $x\alpha y - y\alpha x$. Recall that a Γ-ring, If $a\Gamma M\Gamma b = 0$ implies $a = 0$ or $b = 0$, and it is called semi prime if $a\Gamma M\Gamma a = 0$ implies $a = 0$. A prime Γ-ring is obviously semi prime. A Γ-ring M is called commutative if $[x, y]\alpha = 0$ for every $x, y \in M$ and $\alpha \in \Gamma$. If $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$,for all $x, y \in M$, $\alpha \in \Gamma$ then an additive mapping d from M is called a derivation.

We consider an assumption (*) $x\alpha y\beta z = x\beta y\alpha z$ for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$. The basic commentator identities given by $[x\beta y, z]\alpha = x\beta[y, z]\alpha + [x, z]\alpha\beta y + x[\beta, \alpha]zy$ & $[x, y\beta z]\alpha = y\beta[x, z]\alpha + [x, y]\alpha\beta z + y[\beta, \alpha]xz$, for all $x, y, z \in M$ and for all $\alpha, \beta \in \Gamma$. Taking the above assumption (*) the basic commentator identities reduce to $[x\beta y, z]\alpha = x\beta[y, z]\alpha + [x, z]\alpha\beta y$ and $[x, y\beta z]\alpha = y\beta[x, z]\alpha + [x, y]\alpha\beta z$, for $x, y, z \in M$ and for all $\alpha, \beta \in \Gamma$ which are used extensively in our results.

3. REVERSE DERIVATION AND ORTHOGONAL REVERSE DERIVATION:

An additive mapping d from a Γ-ring M satisfying $d(x\alpha y) = d(y)\alpha x + y\alpha d(x)$, for all $x, y \in M$, $\alpha \in \Gamma$, is called a reverse derivation. Cleary, if M is commutative, then both derivation and reverse derivation are the same. An additive mapping $d: M \to M$ is called a Jordan derivation if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$ for all $a \in M$ and $\alpha \in \Gamma$. It can be easily seen that the reverse derivation is not a general derivation in, but it is a Jordan derivation

Example 3.1: Let R be an associative ring with 1, d: R → R be a reverse derivation. Consider M = $M_{1,2}(R)$ and $\Gamma = \{\binom{n.1}{0}: n \in Z\}$.It is clear that M is a Γ-ring. Let N = {(x, x): x ∈ R} ⊂ M. Then N is a sub ring of M. Define D: N → N by D ((x, x)) = (d(x), d(x)). If a = (x_1, x_1), b = (x_2, x_2) and $\alpha = \binom{n.1}{0} \in \Gamma$. Then we have

$$D(a\alpha b) = D\left((x_1, x_1)\binom{n.1}{0}(x_2, x_2)\right)$$

$$= D(x_1 n x_2, x_1 n x_2)$$

$$= (d(x_1 n x_2), d(x_1 n x_2))$$

$$= (d(x_2)nx_1 + x_2 n d(x_1), d(x_2)nx_1 + x_2 n d(x_1))$$

$$= (d(x_2)nx_1, d(x_2)nx_1) + (x_2 n d(x_1), x_2 n d(x_1))$$

$$= (d(x_2), d(x_2))\binom{n.1}{0}(x_1, x_1) + (x_2, x_2)\binom{n.1}{0}(d(x_1), d(x_1))$$

$$= D((x_2, x_2))\alpha a + b\alpha D((x_1, x_1))$$

$$= D(b)\alpha a + b\alpha D(a).$$

Hence D is a reverse derivation on Γ-ring N. Now we give the definition of orthogonality of two reverse derivations.

Definition 3.2: Let d and g be two reverse derivations on M. If d(x) Γ M Γ g(y) = 0 = g(y) Γ M Γ d(x) for all x, y ∈ M. --------------------(1)

Then d and g are said to be orthogonal. Also note that a non-zero reverse derivation cannot be orthogonal on itself.

Example 3.3: Let M_1 be a Γ_1-ring and let M_2 be a Γ_2-ring. Consider $M = M_1 \times M_2$ and $\Gamma = \Gamma_1 \times \Gamma_2$. The addition and multiplication on M and Γ are defined as follows:

(a, b) + (c, d) = (a + c, b + d), (a, b) (α, β) (c, d) = (aαc, bβd) for every a, b ∈ M_1, c, d ∈ M_2, α ∈ Γ_1 and β ∈ Γ_2.

Under these operations M is a Γ-ring. Let d_1 be a reverse derivation on M_1. Define a derivation d on M by d((a, b)) = (d_1(a), 0). Then d is a reverse derivation on M. Let d_2 be a reverse derivation on M_2. Define a derivation g on M by $g((a, b)) = (0, d_2(b))$. Then g is a reverse derivation on M. It is clear that d and g are orthogonal reverse derivation on M.

4. RESULTS:

The following result has been given in [2],

Lemma 4.1: Let M be a 2-torsion free semi prime Γ-ring and a, b ∈ M. Then the following conditions are equivalent:

1. $a\Gamma x\Gamma b = 0$, for all $x \in M$.

2. $b\Gamma x\Gamma a = 0$, for all $x \in M$.

(3) $a\Gamma x\Gamma b + b\Gamma x\Gamma a = 0$, for all $x \in M$. If one of these conditions is fulfilled then $a\Gamma b = 0 = b\Gamma a$

Lemma 4.2: Let M be a semi prime Γ-ring and suppose that additive mappings d and g of M into itself satisfy $d(x)\Gamma M\Gamma g(x) = 0$ for $x \in M$. Then $d(x)\Gamma M\Gamma g(y) = 0$, for all $x \in M$.

Proof: Suppose that $d(x)\alpha m\beta g(x) = 0$, for all $x, m \in M$, $\alpha, \beta \in \Gamma$. By Replacing x by x + y in the above relation, we get

$0 = d(x + y)\alpha m\beta g(x + y)$

$= (d(x) + d(y))\alpha m\beta(g(x) + g(y))$

$= d(x)\alpha m\beta g(x) + d(x)\alpha m\beta g(y) + d(y)\alpha m\beta g(x) + d(y)\alpha m\beta g(y)$

$= d(x)\alpha m\beta g(y) + d(y)\alpha m\beta g(x)$.T hus$d(x)\alpha m\beta g(y)$

$= -d(y)\alpha m\beta g(x)$.

Now

$(d(x)\alpha m\beta g(y))\gamma n\delta(d(x)\alpha m\beta g(y)) = (d(x)\alpha m\beta g(y))\gamma n\delta(-d(y)\alpha m\beta g(x))$

$= -(d(x)\alpha m\beta g(y)\gamma n\delta d(y)\alpha m\beta g(x)$

$= 0$, for all $x, y, m, n \in M$, $\alpha, \beta, \delta \in \Gamma$.

Thus $d(x)\Gamma M\Gamma g(y) = 0$, for all $x, y \in M$

Lemma 4.3: Let M be a 2-torsion free semi prime Γ-ring. Let d and g be reverse derivations of M. Then $d(x)\Gamma g(y) + g(x)\Gamma d(y) = 0$, for all $x, y \in M$. if and only if d and g are orthogonal. (2)

Proof:

Let us Suppose that $d(x)\alpha g(y) + g(x)\alpha d(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$. Consider the substitution $y = x\beta y$ in (2). Then,

$$0 = d(x)\alpha g(x\beta y) + g(x)\alpha d(x\beta y),$$

$$= d(x)\alpha(g(y)\beta x + y\beta g(x)) + g(x)\alpha(d(y)\beta x + y\beta d(x),$$

$$= (d(x)\alpha g(y) + g(x)\alpha d(y))\beta x + d(x)\alpha y\beta g(x) + g(x)\alpha y\beta d(x)$$

By using (2), we have $d(x)\alpha y\beta g(x) + g(x)\alpha y\beta d(x) = 0$. Then due to Lemma 4.2,

we get $d(x)\alpha y\beta g(x) = 0$, which gives the orthogonality of d and g. Conversely, if d and g are orthogonal, we get $d(x)\alpha m\beta g(y) = g(x)\alpha m\beta d(y) = 0$ for all $m \in M$, $\alpha, \beta \in \Gamma$. Then by Lemma 4.1, we obtain $d(x)\alpha g(y) = g(x)\alpha d(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$. Thus $d(x)\alpha g(y)+ g(x)\alpha d(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$ this completes the proof. Suppose that d and g are reverse derivations of a Γ-ring M.

The following identities are from the definition of reverse derivation.

$$(d\,g)(x\alpha y) = d\,(g\,(x\alpha y))$$

$$= d\,(g(y)\alpha x + y\alpha g(x))$$

$$= (d\,g)\,(x)\alpha y + d(x)\alpha g(y) + g(x)\alpha d(y) + x\alpha(dg)(y) \text{ for } x, y \in M, \alpha \in \Gamma. \text{ ------} \quad (3)$$

Similarly,

$$(g\,d)(x\alpha y) = g(d(x\alpha y))$$

$$= (gd)(x)\alpha y + g(x)\alpha d(y) + d(x)\alpha g(y) + x\alpha(gd)(y) \text{ for } x, y \in M, \alpha \in \Gamma \text{ -------} \quad (4)$$

The following theorem gives criteria on orthogonality of reverse derivations. It is an extension of Theorem 2.1 from [2] to the reverse derivations case.

Theorem 4.4: Let M be a 2-torsion free semi prime Γ-ring. Let d and g be reverse derivations on M. Then the following conditions are equivalent:

(i) d and g are orthogonal. (ii) $dg = 0$. (iii) $gd = 0$. (iv) $dg + gd = 0$. (v) dg is a derivation. (vi) gd is a derivation.

Proof: (ii) ⇒ (i). Suppose $dg = 0$. Then by using the identity (3) above we obtain $d(x)\alpha g(y) + g(x)\alpha d(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$. Therefore by Lemma 4.3, d and g are orthogonal

1. ⇒ (ii). Consider $d(x)\alpha y\beta g(z) = 0$, for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$.

Then

$0 = d(d(x)\alpha y\beta g(z))$

$= d(y\beta g(z))\alpha d(x) + y\beta g(z)\alpha d2\ (x)$

$= (dg)(z)\beta y\alpha d(x) + g(z)\beta d(y)\alpha d(x) + y\beta g(z)\alpha d(d(x))$.

Owing to (i), the second and third summands are zero. Therefore we obtain $(dg)(z)\beta y\alpha d(x) = 0$ for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$. Now take $x = g(z)$ and we obtain $(dg)(z)\beta y\alpha(dg)(z) = 0$, for all $z \in M$, $\alpha, \beta \in \Gamma$.

Since M is semi prime, we get $(dg)(z) = 0$, for all $z \in M$., that is $dg = 0$. The proof of the parts (iii) ⇒ (i) and (i) ⇒ (iii) are similar. (iv) ⇒ (i). If d and g are any reverse derivations, then by (ii) and (iii), $dg = 0$ and $gd = 0$. Now using the equation (3), we obtain,

$$\begin{aligned}(dg + gd)(x\alpha y) &= (dg)(x\alpha y) + (gd)(x\alpha y)\\ &= (dg)(x)\alpha y + d(x)\alpha g(y) + g(x)\alpha d(y) + x\alpha(dg)(y) + (gd)(x)\alpha y + g(x)\alpha d(x) +\\ &\quad d(x)\alpha g(y) + x\alpha(gd)(y)\\ &= (dg + gd)(x)\alpha y + 2d(x)\alpha g(y) + 2g(x)\alpha d(y) + x\alpha((dg)(y) + (gd)(y))\end{aligned}$$

for all $x, y \in M$, $\alpha \in \Gamma$.

Thus, if $dg + gd = 0$, then the above relation reduces to $2(d(x)\alpha g(y) + g(x)\alpha d(y)) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$. Since M is 2-torsion free, we get $d(x)\alpha g(y) + g(x)\alpha d(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$.

By Lemma 4.3, we get that d and g are orthogonal. (i) ⇒ (iv). From the parts (ii) and (iii) of Theorem 4.1, we get $dg + gd = 0$.

(v) ⇒ (i). Since dg is a derivation, we have $(dg)(x\alpha y) = (dg)(x)\alpha y + x\alpha(dg)(y)$. Comparing this expression with (3) we obtain $d(x)\alpha g(y) + g(x)\alpha d(y) = 0$. The proof of (vi) ⇒ (i) is the similar to that of (v) ⇒ (i). (iii) ⇒ (vi). This completes the proof.

Corollary 4.5: Let M be a prime 2-torsion free Γ-ring. Suppose that d and g are orthogonal reverse derivations of M. Then either $d = 0$ or $g = 0$. The proof is immediate from Theorem 4.4.

Theorem 4.6: Let M be a 2-torsion free semi prime Γ-ring such that $x\alpha y\beta z = x\beta y\alpha z$ for all x, y, z ∈ M and α, β ∈ Γ.

Let d and g be reverse derivations on M. Then the conditions are equivalent to:

(i)d and g are orthogonal. (ii) $d(x)\Gamma g(x) = 0$, for all x ∈ M. (iii) $g(x)\Gamma d(x) = 0$, for all x ∈ M.

(iv) $d(x)\Gamma g(x) + g(x)\Gamma d(x) = 0$, for all x ∈ M.

Proof: (ii) ⇒ (i) The linearization of $d(x + y)\alpha g(x + y) = 0$ gives $d(x)\alpha g(y) + d(y)\alpha g(x) = 0$, for all x, y ∈ M, α ∈
(5)

Take yβz as yin (5), we obtain $d(x)\alpha g(y\beta z) + d(y\beta z)\alpha g(x) = 0$, for all x, y, z ∈ M, α, β ∈ Γ.

$d(x)\alpha g(z)\beta y + d(x)\alpha z\beta g(y) + d(z)\beta y\alpha g(x) + z\beta d(y)\alpha g(x) = 0$, for all x, y, z ∈ Γ, α, β ∈ Γ.

Since , $d(x)\alpha g(z) = -d(z)\alpha g(x)$ and $d(y)\alpha g(x) = -d(x)\alpha g(y)$ and so the above relation becomes

$-d(z)\alpha g(x)\beta y + d(x)\alpha z\beta g(y) + d(z)\beta y\alpha g(x) - z\beta d(x)\alpha g(y) = 0$, for all x, y, z ∈ M, α, β ∈ Γ.

Now we make use the condition (*) then $d(z)\beta[y, g(x)]\alpha + [d(x), z]\alpha\beta g(y) = 0$.

Replacing z by d(x) in the above identity we obtain $d\ 2\ (x)\beta[y, g(x)]\alpha = 0$ for all x, y ∈ M, α, β ∈ Γ.

Letting $y = y\delta w$ in the last relation and using the condition (*), we get,

$0 = d\,2\,(x)\beta[y\delta w, g(x)]\alpha$

$= d\,2\,(x)\beta y\delta[w, g(x)]\alpha + d\,2\,(x)\beta y\delta[w, g(x)]\alpha$

$= d\,2\,(x)\beta y\delta[w, g(x)]\alpha$ for all $x, y, w \in M$, $\alpha, \beta, \delta \in \Gamma$.

Then by Lemma 4.2, we obtain $d\,2\,(x)\beta y\delta[w, g(y)]\alpha = 0$, for all $x, y, w \in M$, $\alpha, \beta, \delta \in \Gamma$. (6)

Replacing x by $x\lambda u$ in (6) and using (3) yields.

$0 = d\,2\,(x\lambda u)\beta y\delta[w, g(y)]\alpha$

$= (d\,2\,(x)\lambda u + 2d(x)\lambda d(u) + x\lambda d2\,(u))\beta y\delta[w, g(y)]\alpha$ for all $x \in M$, $\alpha, \beta, \delta, \lambda \in \Gamma$.

By (6) the above relation reduces to $2d(x)\lambda d(u)\beta y\delta[w, g(y)]\alpha = 0$

Since M is 2-torsion free, we have

$d(x)\lambda d(u)\beta y\delta[w, g(y)]\alpha = 0$, for all $x,y \in M$, $\alpha, \beta, \delta, \lambda \in \Gamma$ (7)

It is obvious from the definition of K that d leaves K Taking $x\gamma z$ for x in (7), we get

$0 = d(x\gamma z)\lambda d(u)\beta y\delta[w, g(y)]\alpha$

$= d(z)\gamma x\lambda d(u)\beta y\delta[w, g(y)]\alpha + z\gamma d(x)\lambda d(u)\beta z\delta[w, g(y)]\alpha$ and

$d(z)\gamma x\lambda d(u)\beta y\delta[w, g(y)]\alpha = 0$. (By using (7))

In particular, $d(z)\gamma x\lambda d(x)\beta y\delta[w, g(y)]\alpha = 0$.

The replacement $d(z) = d(x)\beta y\delta[w, g(y)]\alpha$, gives $d(x)\beta y\delta[w, g(y)]\alpha\gamma x\lambda d(x)\beta y\delta[w, g(y)]\alpha = 0$.

Since M is semi prime, we get $d(x)\beta y\delta[w, g(y)]\alpha = 0$. Using (6) and (7) we obtain by replacing $d(x)$ for w, $[d(x), g(y)]\,\alpha\gamma y\delta[d(x), g(y)]\alpha = 0$, for all $x, y \in M$, $\alpha, \beta, \delta, \gamma \in \Gamma$.

Hence, $d(x)\alpha g(y) = g(y)\alpha d(x)$, for all $x, y \in M$, $\alpha \in \Gamma$.

Thus (5) can be written in the form $g(y)\alpha d(x) + d(y)\alpha g(x) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$.

Now use Lemma 4.3 to get the required relation. (i) $\Rightarrow$ (iii). If d and g are orthogonal then we have $d(x)\Gamma M\Gamma g(x) = 0$, for all $x \in M$.

Then due to Lemma 4.1, we get $d(x)\alpha g(x) = 0$, for all $x \in M$, $\alpha \in \Gamma$.

(ii) $\Rightarrow$ (ii). Take $y = x$ in (3). Then we see that $(dg)(x\alpha x) = (dg)(x)\alpha x + d(x)\alpha g(x) + g(x)\alpha d(x) + x\alpha(dg)(x)$.

Thus we obtain $(dg)(x\alpha x) = (dg)(x)\alpha x + x\alpha(dg)(x)$, for all $x \in M$, $\alpha \in \Gamma$.

The above relation implies that dg is a Jordan derivation. We know that if M is semi prime Γ-ring, then every Jordan derivation is a derivation. (i) $\Rightarrow$ (ii). This follows from Lemma 4.3.

Corollary 4.7: Let M be a 2-torsion free semi prime Γ-ring and let d be a reverse derivation of M. If d 2 is also a derivation, then $d = 0$.

The proof follows from part (ii) of Theorem 4.6.

Theorem 4.8: Let M be a 2-torsion free semi prime Γ-ring. Let d and g be reverse derivations on M. Then the following conditions are equivalent: (i) d and g are orthogonal. (ii) There exist ideals K_1 and K_2 of M such that: (a) $K_1 \cap K_2 = 0$ and $K = K_1 \oplus K_2$ is a nonzero ideal of M.

(b) d maps M into K_1 and g maps M into K_2. (c) The restriction of d to $K = K_1 \oplus K_2$ is a direct sum $d_1 \oplus 0_2$, where $d_1 : K_1 \rightarrow K_1$ is a reverse derivation of K_1 and $0_2 : K_2 \rightarrow K_2$ is zero. If $d_1 = 0$ then $d = 0$. (d) The restriction of g to $K = K_1 \oplus K_2$ is a direct sum $0_1 \oplus g_2$, where $0_1 : K_1 \rightarrow K_1$ is zero and $g_2 : K_2 \rightarrow K_2$ is a reverse derivation of K_2. If $g_2 = 0$ then $g = 0$.

Proof: (ii) $\Rightarrow$ (i). Obvious. (i) $\Rightarrow$ (ii).

Let K_1 be an ideal of M generated by all $d(x)$, $x \in M$, and let K_2 be $Ann(K_1)$, the annihilator of K_1. From (1) we see that $g(x) \in K_2$, for all $x \in M$. Whenever K_1 is an ideal in a semi prime Γ-ring, we have $K_1 \cap K_2 = 0$ and $K = K_1 \oplus K_2$ is a nonzero ideal. Thus (a) and (b) are proved.

To show that d is zero on K_2. Take $k_2 \in K_2$.

Then $k_1\alpha k_2 = 0$, for all $k_1 \in K_1$, $\alpha \in \Gamma$. Hence $0 = d(k_1\alpha k_2) = d(k_2)\alpha k_1 + k_2\alpha d(k_1)$. It is obvious from the definition of K that d leaves K_1 invariant and hence $k_2\alpha d(k_1) = 0$. Then the above relation reduces to $d(k_2)\alpha k_1 = 0$. Since in a semi prime Γ-ring the left, right and two-sided annihilators of an ideal coincide, we then have $d(k_2) \in Ann(K_1) = K_2$. But on the other hand $d(k_2)$ belongs to the set of generating elements of K_1. Thus $d(k_2) \in K_1 \cap K_2 = 0$, which means that d is zero on K_2. As we have mentioned above d leaves K_1 invariant. Therefore we may define a mapping $d_1: K_1 \rightarrow K_1$ as a restriction of d to K_1.

Suppose that $d_1 = 0$. Then d is zero on $K = K_1 \oplus K_2$. Take $k \in K$ and $y \in M$, we have $d(y\alpha k) = d(k)\alpha y + k\alpha d(y)$ But $d(y\alpha k) = d(k) = 0$ since $k\alpha y, k \in K, \alpha \in \Gamma$. Consequently $k\alpha d(y) = 0$, for all $y \in M, \alpha \in \Gamma$. Thus $d(y) \in Ann(K)$. But ideal K is nonzero and therefore $Ann(K) = 0$. Hence $d(y) = 0$, for all $y \in M$. Then (c) is thereby proved.

It remains to prove (d). First we show that g is zero on K_1. Take $x, y, z \in M$, $\alpha, \beta \in \Gamma$ and set $k_1 = z\alpha d(y)\beta x$. Then,

$g(k_1) = g(x)\beta(z\alpha d(y)) + x\beta g(z\alpha d(y)) = g(x)\beta z\alpha d(y) + x\beta(gd)(y)\alpha z + x\beta d(y)\alpha g(z)$.

Since d and g are orthogonal we have $g(x)\alpha z\beta d(y) = 0$, $d(y)\alpha g(z) = 0$ and $gd = 0$. Hence $g(k_1) = 0$. Similarly $g(z\alpha d(y)) = 0$, $g(d(y)\alpha x) = 0$ and $g(d(y))= 0$. Then h is zero on K_1. Recall that g maps M into K_2. In particular, it leaves K_2 invariant. Thus we may define $g_2: K_2 \rightarrow K_2$ as a restriction of g to K_2. The proof that $g_2 = 0$ implies $g = 0$ is the same as the proof that $d_1 = 0$ implies $d = 0$. This completes the proof.

Corollary 4.9: Let M be a 2-torsion free semi prime Γ-ring and let d be a reverse derivation of M. If $d(x)\alpha d(x) = 0$ for all $x \in M, \alpha \in \Gamma$, then $d = 0$. If $d_2 = g_2$ or if $d(x)\alpha d(x) = g(x)\alpha g(x)$, for every $x \in M, \alpha \in \Gamma$, then we obtain the relation between the reverse derivations d and g of a Γ-ring.

Theorem 4.10: Let M be a 2-torsion free semi prime Γ-ring. Let d and g be reverse derivations of M. Suppose that $d_2 = g_2$, then $d + g$ and $d – g$ are orthogonal. Thus, there exist ideals K_1 and K_2 of M such that $K = K_1 \oplus K_2$ is a nonzero ideal which is direct sum in M, $d = g$ on K_1 and $d = – g$ on K_2.

Proof: From $d_2 = g_2$ it follows immediately that $(d + g)(d – g)+ (d – g)(d+g) = 0$. Hence $d+g$ and $d – g$ are orthogonal by the part (iii) of Theorem 4.4. Another part of Theorem 4.10, follows from (iii) of Theorem 4.8.

From Theorem 4.10 we get the following;

Corollary 4.11: Let M be a prime 2-tosion free Γ-ring. Let d and g be derivations of M.

If $d_2 = g_2$ then either $d = -g$ or $d = g$.

Theorem 4.12: Let M be a 2-torsion free semi prime Γ-ring. Let d and g be reverse derivations of M. If $d(x)\alpha d(x) = g(x)\alpha g(x)$, for all $x \in M$, $\alpha \in \Gamma$, then $d + g$ and $d - g$ are orthogonal. Thus, there exist ideals K_1 and K_2 of M such that $K = K_1 \oplus K_2$ is an essential direct sum in M, $d = g$ on K_1 and $d = -g$ on K_2.

Proof: Note that $(d + g)(x)\alpha(d - g)(x) + (d - g)(x)\alpha(d + g)(x) = 0$, for all $x \in M$, $\alpha \in \Gamma$. Now applying parts (ii) and (iii) of Theorem 4.6, we obtain the required result.

Corollary 4.13: Let M be a prime 2-torsion free Γ-ring. Let d and g be reverse derivations of M. If $d(x)\alpha d(x) = g(x)\alpha g(x)$, for all $x \in M$, $\alpha \in \Gamma$, then either $d = g$ or $d = -g$.

The proof is immediate from Theorem 4.12.

Conflict of Interests

The author declares that there is no conflict of interests.

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