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DRIFT TERM AND VERTEX POINT IN SINGLE FACTOR INTEREST RATE MODEL

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Abstract: By using a stochastic differential equation in the form of the Ornstein-Uhlenbeck process, the drift term that is obtained plays a crucial role in determining the vertex point of monetary policy. The drift term, essentially is the same in its structural form for both single factor interest rate models of Vasicek and Cox Ingersoll Ross. Both single factor interest rate models are useful when it comes to measuring the vertex point and vertex point gap of monetary policy transmission.

Keywords: Vasicek, Cox Ingersoll Ross Interest Rate Models, Stochastic Differential Equation, Ornstein-Uhlenbeck process, Drift Term, Vertex Point, Vertex Point Gap, Speed of Monetary Policy Transmission.

2000 AMS Subject Classification: 91G30

1. Introduction

The transmission of monetary policy to financial markets and the general public is crucial in identifying imperfections in interest rate markets. The different interest rate instrument in each sector of the economy provides inference on the transmission flow. A breakdown of this transmission flow disrupts effectiveness of monetary policy. The main instrument of monetary policy is the policy interest rate and by adjusting it, the impact is transmitted to interest rates in financial markets and to public-related

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interest rate instruments. Shocks in financial markets in the form of sharply reduced interest rates elevate financial market volatility and this in turn widens the dispersion of modelled interest rates from policy interest rates. In enhancing the capability of single factor interest rate models that incorporate the behavioral economics tool, Zoubi (2009) identified the drift function for mean reversion of interest rates as beneficial. The function captures expected instantaneous change in interest rate at time *t*. The drift function and instantaneous volatility parameters complement each other in designing optimal financial market strategies to hedge against interest rate risk. Findings by Nautz and Scheithauer (2011) indicate when the gap between modelled interest rates and policy interest rates reduce, it is due to appropriate monetary policy communication in a transparent and efficient manner. The reduction in the gap between modelled interest rates and policy interest rates allows equilibrium in money markets to occur in a rapid manner and reduces friction in interest rates markets, therefore increasing the speed of interest rates mean reversion capability.

This research paper uses two single factor interest rate models, the Vasicek and Cox Ingersoll Ross models to capture the vertex point gap of monetary policy. It essentially measures the speed of monetary policy transmission using a stochastic differential differential equation in the framework of Ornstein-Uhlenbeck process to identify the drift term. The objective, to be able to measure the vertex point of monetary policy and to show as the vertex point gap distance increases, the speed of monetary policy transmission slows down while as the vertex point gap distance reduces the speed of monetary policy transmission increases.

2. Preliminaries

The Vasicek model is represented as

$$dr_t = \alpha(\beta - r_t)d_t + \sigma dw_t \tag{1}$$

and the Cox Ingersoll Ross model is represented as

$$dr_t = \alpha(\beta - r_t)d_t + \sigma\sqrt{r_t}dw_t \tag{2}$$

Where α is speed of mean reversion, β is the long term mean of short term interest rates, σdw_t is the instantaneous volatility with a Weiner process, r is the short term interest rate and $\alpha(\beta - r_t)d_t$ is the drift term. The coefficients in the Cox Ingersoll Ross model are the same as with the Vasicek model with the exception of the instantaneous volatility which is $\sigma\sqrt{r_t}dw_t$ .In calibrating the single factor mean reversion interest rate models for estimation of coefficients, this exercise uses a stochastic differential equation in the framework of Ornstein-Uhlenbeck process. The process involves computing the following equation,

$$r_{t+1} = r_t e^{-\alpha\delta} + \beta\left(1 - e^{-\alpha\delta}\right) + \sigma\sqrt{\frac{1-e^{-2\alpha\delta}}{2\alpha}} \qquad (3)$$

Where in equation (3) it shows that the short term interest rates in the period ahead r_{t+1} is determined by actual short term interest rates at period r_t with respect to a mathematical constant of e approximately equal to 2.71828. This is adjusted to the speed of mean reversion parameter and the fixed time step of δ. The fixed time step is measured as a single financial trading day divided by the 252 financial trading days in a year. The long term mean of short term interest rates β, is adjusted by the difference against the mathematical constant of 2.71828 that is powered to the mean reversion coefficient and the time step. The stochastic differential equation takes into account of instantaneous volatility in the framework of the mathematical constant, the speed of mean reversion and the fixed time step. The relationship between r_t and r_{t+1} is linear with a normal random error of ε with a constant ƀ. The coefficient $\acute{\alpha}$ reflects the relationship between current short term interest rates r_t and short term interest rates at period r_{t+1}, therefore allowing for the estimation of the following equation

2.1 Calibration Process

The identified coefficients from equation (4) are calibrated into the Ornstein-Uhlenbeck process in the following form,

$$\acute{\alpha} = e^{-\alpha\delta} \qquad (5)$$

$$ƀb = \beta(1 - e^{-\alpha\delta}) \qquad (6)$$

and standard deviation of

$$\varepsilon = \sigma \sqrt{\frac{1-e^{-2\alpha\delta}}{2\alpha}} \quad (7)$$

Rewriting in the form of a quantifiable method to find the values of α, β and σ for both the single factor mean reversion interest rate models of Vasicek and Cox Ingersoll Ross, shows,

$$\alpha = \frac{-ln\acute{a}}{\delta} \quad (8)$$

$$\beta = \frac{b}{1-\acute{a}} \quad (9)$$

and

$$\sigma = std.dev\ (\varepsilon)\ \sqrt{\frac{-2ln\acute{a}}{\delta(1-\acute{a}^2)}} \quad (10)$$

2.2 Drift Term

The framework for Vasicek and Cox Ingersoll Ross single factor mean reversion of short term interest rates indicate the drift term is a function of short term interest rates in the form of

$$\alpha(\beta - r_t)d_t = f\,(dr_t) \quad (11)$$

Equation (11) is integrated into a second order polynomial regression with a quadratic function[1], where the coefficient of φ_2 is obtained to reflect the inverse or positive relationship against the drift term, where

$$\alpha(\beta - r_t)d_t = \varphi_0 + \varphi_1 dr_t + \varphi_2 dr^2_t + \epsilon_t \quad (12)$$

and setting the intercept $\varphi_0 = 0$ the equation is transformed to

$$\alpha(\beta - r_t)d_t = \varphi_1 dr_t + \varphi_2 dr^2_t + \epsilon_t \quad (13)$$

The drift term captures the behavior of short term interest rates to either move up or down towards equilibrium. A negative coefficient φ_2 of the drift term generates the tendency for short term interest rates to move downwards towards equilibrium while a

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[1] The quadratic function plots as a parabola, a curve with a single built in bump or wiggle.

positive coefficient φ_2 of the drift term generates the tendency for short term interest rates to move upwards towards equilibrium.

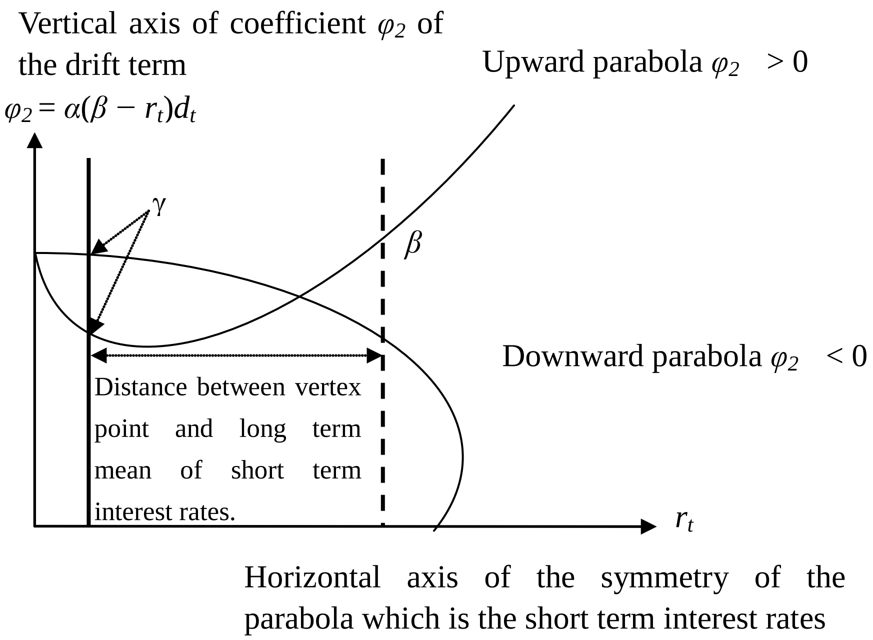
2.3 Speed of Monetary Policy Transmission

The drift coefficient of $\varphi_2 < 0$ from equation (13) indicates the curve in the polynomial regression is reflective of a downward parabola while $\varphi_2 > 0$ indicates the curve in the polynomial regression is reflective of an upward parabola. The coefficient φ_2 of the drift term $\alpha(\beta - r_t)d_t$ controls the speed of decrease or increase in the polynomial function from the vertex. The larger the negative or positive coefficient φ_2, the function will decrease rapidly. The vertex of the parabola is the turning point of financial market expectations of interest rate behavior. It is the maximum point when coefficient of the drift term is $\varphi_2 < 0$ and is the minimum point when coefficient of the drift term is $\varphi_2 > 0$. The vertex point is measured as the vertical line that passes through the vertex point and is computed as

$$\gamma = -\frac{\varphi 1}{2\varphi 2} \qquad (14)$$

The horizontal axis of the symmetry of the parabola is identified as γ which is measured based on the coefficient φ_1 and φ_2 from equation (13) and is substituted into equation (14) to obtain the vertex point of financial market expectations of interest rate changes. The distance between γ and β is the vertex point gap. The vertex point gap reflects the speed of monetary policy transmission, whereby as the vertex point gap distance increases, the speed of monetary policy transmission slows down while as vertex point gap distance reduces, the speed of monetary policy transmission increases (see Figure 1).

Figure 1 –Vertex Point Gap – Speed of Monetary Policy Transmission



Source: Authors compilation.

Where γ is the vertex point with, $\varphi_2 > 0$ as an upward parabola and $\varphi_2 < 0$ as an downward parabola. The distance between γ and β, reflects the distance from vertex to long term mean of short term interest rates, also known as the vertex point gap of speed of monetary policy transmission.

3. Main results

Important aspects identified from this exercise, by using a stochastic differential equation in the form of the Ornstein-Uhlenbeck process, the drift term that is obtained plays a crucial role in determining the vertex point of monetary policy. The drift term, essentially is the same in its structural form for both single factor interest rate models of Vasicek and Cox Ingersoll Ross. Both single factor interest rate models are useful when it comes to measuring the vertex point and vertex point gap of monetary policy transmission. This essentially provides central banks and financial regulators an added tool in deliberating short term policy interest rates within the framework of monetary policy.

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