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DISTORTIONARY TAXES AND GLOBAL INDETERMINACY IN AN ENDOGENOUS GROWTH MODEL WITH ELASTIC LABOR SUPPLY

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Abstract. This paper describes the global properties of the Ben-Gad (2013) economy where goverment public spending is financed by a distortionary taxation, in an endogenous growth framework with sector specific external effects. The application of the original Bogdanov-Takens theorem allows us to characterize the regions of the parametric space where the model exhibits a global indeterminate equilibrium and a low-growth trapping region.

Keywords: disotrtionary taxation; Bogdanov-Takens singularity; global indeterminacy; poverty trap.

2010 AMS Subject Classification: 91G80.

1. Introduction

A recent article due to Ben-Gad (2003) examines the distributional effects on wealth and income when changes in tax policies are introduced to finance a rise in government spending, in an endogenous growth model à la Uzawa-Lucas with elastic labor supply and factor taxation, which represents the standard framework to study the role of human capital accumulation in the long run growth performance of different countries (see, [8]). In particular, he shows that, for a wider range of parameters than those in the standard model, a policy shock might differently affect the steady state values of each predetermined variable, such as physical capital, in the transitional dynamics towards the long run equilibrium. This study is conducted through a local

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stability analysis, which confines the problem in the vicinity of the steady state, therefore losing the possibility to draw a complete picture of the policy implications, once we move outside such a small neighborhood of investigation where, given the initial condition on the predetermined variables, it is possible to find regions of the parameters space which lead to the emergence of multiple of equilibria, and the existence of a poverty trap. This result is referred to as *global* indeterminacy of the equilibrium (see, [19]).

The point is that, if global indeterminacy occurs, public intervention becomes not sufficient to drive the economy towards a unique long-run steady state, even if the equilibrium is locally determinate. A local analysis might be thus misleading, and the derived balanced growth path with active fiscal policy would be only a part of a more complex dynamics. Therefore, governmental interventions could lead to undesired fluctuations in economic activity, implying that the equilibrium, even though locally unique, might exhibit sources of indeterminacy, and potentially unstable dynamics in the large, where multiple transitional allocations (either a saddle connection or a stable limit cycle) can signal the presence of a possible low-growth trapping region (see, among others, [7], [10], [11], [17], [18]). In this case, the agents' decisions, despite the initial conditions or other economic fundamentals, will locate the economy in a particular optimal converging path that could not coincide with the one corresponding to the lowest optimal taxation level, or one with a more equal distribution of income (see also, [16], [23], [24]).

As discussed in [1], however, some authors use an alternative definition of global indeterminacy, where multiple equilibrium trajectories, departing from a given initial condition, approach different equilibrium points. In the field of economic growth theory, [11] prove the possibility of this kind of global indeterminacy in a generalized variant of the continuous-time two-sector model.

A large strand of analysis demonstrates how a *continuum* of equilibrium trajectories, existing in the neighborhood of the steady state, can emerge whenever some parametric conditions are verified. This phenomenon is commonly known as "*local* indeterminacy" ([2], [3], [4], [5], [6], [12], [25]). However, only very few attempts have been made to analyze the conditions under which these indeterminacy problems arise outside the small neighborhood of the steady state, to whom we refer to as "*global* indeterminacy" (as in [21]). The latter seems an innovative field to work on, even though it is usually related to very complicated nonlinear functions which increase the mathematical difficulties in handling these models.

The aim of this paper is to investigate the conditions which guarantee the emergence of t-wo steady state, one characterized by a relatively high growth rate and one with a relatively low growth level in the Ben-Gad (2003) model. A global stability analysis allows us to establish that the larger the effect of distortionary taxes on the productivity in accumulating human capital, the more likely is the low-growth steady state to be a non-saddle equilibrium. To validate our results, we also calibrate the model by using the standard parameters found for the U.S. economy in the related literature on the Uzawa-Lucas model (see, [9],[13], [20], [27]). Finally, we show that the model undergoes a co-dimension 2 Bogdanov-Takens bifurcation, for a plausible region of the parameters space.

The interesting point is that a generic vector field undergoing a Bogdanov-Takens singularity can be put in correspondence of a simple planar system, entirely preserving stability and bifurcation characteristics of the original vector field. The unfolding of this planar system is fully known, and permits the derivation of very useful details regarding the dynamics of any highly nonlinear dynamical system in proximity of the bifurcation. For the scopes of the paper, we are particularly interested in the determination of the regions in the parameters space implying a particular type of global phenomenon, namely the homoclinic bifurcation, by which orbits growing around the non-saddle steady state collide with the saddle steady state, which only exists outside the small neighborhood of the steady state valid for the local analysis.

The rest of the paper is organized as follows. In section 2, we present the model, derive the steady state conditions, and study the local dynamics. In section 3, we analyze the Bogdanov-Takens singularity and derive the conditions for global indeterminacy and the poverty trap to emerge, and confirm our results with a practical example. A final section concludes, and a subsequent Appendix provides all the necessary proofs.

2. The model

Consider the optimal control problem proposed in Ben-Gad (2003), where the representative agent maximizes the intertemporal utility function

$$(P) \qquad U = \int_0^\infty \left(\frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1-\varepsilon}}{1-\varepsilon} \right) e^{-\rho t} dt$$

subject to the following constraints

$$\begin{aligned}\dot{K} &= (1-\tau_l)wuh + (1-\tau_k)rK - C,\\ \dot{h} &= v(L-u)^{1-\gamma}h^{1-\gamma}(L_a-u_a)^{\gamma}h_a^{\gamma}\end{aligned}$$

and given initial positive values

$$K(0) = K_0 \quad h(0) = h_0,$$

where C is consumption, L is time devoted to non-leisure, and $u \in [0, L]$ is the fraction of time spent in the working sector (see, [8]). Moreover, K is physical capital, and h measures the stock of human capital, whereas r is the rental rate, and w is the wage rate. Additionally, τ_l and τ_k represent the tax rates on labor and capital income, respectively; while σ is the inverse of the intertemporal elasticity of substitution, and ρ is the time preference rate. Noticeably, human capital can be accumulated at a rate v, depending also on the externalities, u_a, L_a and h_a, whose magnitude is measured by the parameter γ.

In particular, the production function is assumed as

$$y = K^{\alpha}\phi^{1-\alpha}\phi_a^{\beta} \qquad 0 < \alpha < 1,\ \beta \geq 0, \tag{1}$$

where $\phi = uh$, and ϕ_a captures the contribution of external effects to total factor productivity.

Since, in equilibrium, rental rates must equal their marginal product, it follows consequently that

$$r = \alpha K^{\alpha-1}\phi^{1-\alpha}\phi_a^{\beta}, \tag{2.1}$$

$$w = (1-\alpha)K^{\alpha}\phi^{-\alpha}\phi_a^{\beta}. \tag{2.2}$$

Assume also that the government budget constraint is always binding, that is

$$gy = \tau_l w\phi + \tau_k rK, \tag{3}$$

where g is the percentage of public spending on national income.

The current value Hamiltonian of problem P is given by

$$H_c = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1-\varepsilon}}{1-\varepsilon} + \lambda\left[(1-\tau_l)wuh + (1-\tau_k)rK - C\right] + \mu\left[v(L-u)^{1-\gamma}h^{1-\gamma}(L_a-u_a)^{\gamma}h_a^{\gamma}\right],$$

where λ and μ are the costate variables associated with the accumulation of physical capital and human capital, respectively.

Solution to this optimal control problem implies the following necessary first order conditions

$$C^{-\sigma} = \lambda, \tag{4.1}$$

$$L^{-\varepsilon} = \mu(1-\gamma)v(L-u)^{-\gamma}h^{1-\gamma}(L_a-u_a)^{\gamma}h_a^{\gamma}, \tag{4.2}$$

$$\lambda(1-\alpha)(1-\tau_l)K^{\alpha}(uh)^{-\alpha}(u_a h_a)^{\beta} = \mu\,(1-\gamma)\,v(L-u)^{-\gamma}h^{-\gamma}(L_a-u_a)^{\gamma}h_a^{\gamma} \tag{4.3}$$

accompanied by the equations of motion for each costate variable:

$$\frac{\dot{\lambda}}{\lambda} = \rho - \alpha(1-\tau_k)K^{\alpha-1}(uh)^{1-\alpha}(u_a h_a)^{\beta}, \tag{4.4}$$

$$\frac{\dot{\mu}}{\mu} = \rho - (1-\gamma)\,v(L-u)^{-\gamma}h^{-\gamma}(L_a-u_a)^{\gamma}h_a^{\gamma} \tag{4.5}$$

and the transversality condition for a free terminal state

$$\lim_{t\to\infty}\left[\lambda_t K_t + \mu_t h_t\right]e^{-\rho t} = 0 \tag{4.6}$$

that jointly constitute the so-called canonical system.

Since all necessary and sufficient conditions that guarantee concavity and the existence of a non-degenerate steady state have been studied in [8], the next section directly moves to the study of the transitional dynamics around the equilibrium solution.

2.1. The reduced model

To begin with, we reduce the dimension of the canonical system in (4.i) through the following convenient variable substitution

$$\begin{aligned} c &= C\phi^{-\frac{1-\alpha+\beta}{1-\alpha}}, \\ k &= K\phi^{-\frac{1-\alpha+\beta}{1-\alpha}} \end{aligned}$$

and consequently end up with a three-dimensional system of first order differential equations

$$\begin{aligned}
\dot{u} &= \left\{ \frac{\alpha}{\alpha-\beta}\left[(\tau_k - g)k^{\alpha-1} - \frac{c}{k}\right] + v\frac{1-\gamma-\alpha+\beta}{\alpha-\beta}L + vu \right\} u \\
\dot{c} &= \left\{ \frac{1}{\sigma}\left[\alpha(1-\tau_k)k^{\alpha-1} - \rho\right] - \vartheta\left[\frac{(1-\gamma)v}{\alpha}L - \frac{c}{k} + (\tau_k - g)k^{\alpha-1}\right] \right\} c \\
\dot{k} &= \left\{ (1-g)k^{\alpha-1} - \frac{c}{k} - \vartheta\left[\frac{(1-\gamma)v}{\alpha}L - \frac{c}{k} + (\tau_k - g)k^{\alpha-1}\right] \right\} k,
\end{aligned} \tag{S}$$

where $\vartheta = \frac{\alpha(1-\alpha+\beta)}{(1-\alpha)(\alpha-\beta)}$ and

$$L = \left[\frac{uck^{-\alpha}}{1-\alpha-g+\alpha\tau_k}\right]^{\frac{1}{\varepsilon}}. \tag{5}$$

The steady state values are easily computed in terms of the growth rate, κ, by solving $\dot{u} = 0$, $\dot{c} = 0$ and $\dot{k} = 0$. That is,

$$c^* = \frac{(1-g)\rho + \kappa[1-g-\alpha(1-\tau_k)]}{(1-\tau_k)}k^*, \tag{6.1}$$

$$k^* = \left[\frac{\alpha(1-\tau_k)}{\rho+\kappa}\right]^{\frac{1}{1-\alpha}}. \tag{6.3}$$

Furthermore, let

$$\dot{u} = \psi(u) \tag{7}$$

be defined in the domain $D = \{u : u \in (0, L)\}$, with $\psi(u^*) = 0$. Given that ψ_u changes sign in D, and $\psi_{uu} > 0$, with $d\psi/d\tau_k > 0$, multiple intersections with the u-axis (multiple steady states) can emerge.

Then, picking τ_k as a bifurcation parameter, we have that

Proposition 1. *There exists a critical value of the bifurcation parameter, such that if* (i) $\tau_k < \bar{\tau}_k$, *system S admits two steady states, one with low investment in human capital* (u^*_{low}), *and one with high investment* (u^*_{high}) *(see, Figure 1);* (ii) $\tau_k = \bar{\tau}_k$, *system S admits a unique steady state, with* $u^* = \hat{u}^*$; (iii) $\tau_k > \bar{\tau}_k$, *system S does not admit a steady state.*

Proof. At $\tau_k = \bar{\tau}_k$, a particular value of the steady state, $u^* = \hat{u}^*$, exists at which both $\psi(\hat{u}^*) = \psi^*_u = 0$ Therefore, at this critical value of the tax rate, $\psi(u)$ is tangential to the u-axis. Additionally,

Since ϕ includes u, which is a jump variable chosen by the household at any point in time, this means that k can be seen as a forward-looking variable.

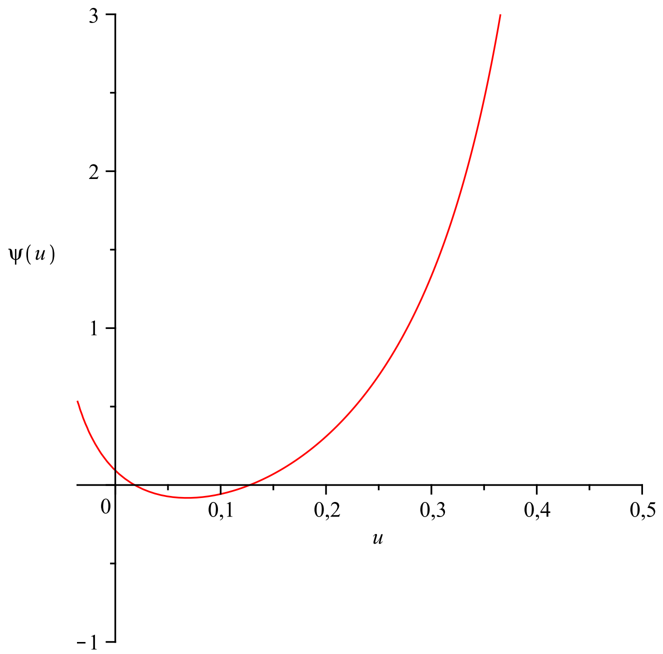

FIGURE 1. Multiple steady states

if $\tau_k > \bar{\tau}_k$ we have an upward shift of $\psi(u)$, and therefore no intersections with the u-axis. Conversely, $\tau_k < \bar{\tau}_k$ implies a downward shift of $\psi(u)$ and therefore two intersections with the u-axis.

In presence of a dual steady state, the local stability analysis solely provides information of the dynamic behavior of our economy in the vicinity of one of the two equilibria. Thus, we might derive erroneous conclusions and lack detecting global indeterminacy, even though local determinacy is found.

Normally, the whole set of necessary and sufficient conditions for the emergence of periodic fluctuations can be derived by applying the well-known Andronov-Hopf bifurcation theorem (see, Wiggins (1991)). Unfortunately, this theorem is not able to tell the full story on the global properties of the equilibrium outside the small neighborhood of the steady state, where a more complicate picture may emerge, due to the occurrence of some degenerate conditions, and the particular configuration of bifurcation parameters (see, for example, Antoci et al. (2011), Mattana and Venturi (1999), Nishimura and Shigoka (2006)). The next section is devoted to this end.

3. The Bogdanov-Takens bifurcation

In this section, we discuss the application of the Bogdanov-Takens bifurcation theorem (*BT*, henceforth) to system *S*. The theorem has the advantage to detect a particular global bifurcation phenomenon, namely the homoclinic bifurcation, where orbits surrounding the non-saddle steady state may start at some point to collapse to the saddle steady state. The emergence of this singularity can be used to establish the possibility of global indeterminacy of the equilibrium.

The *BT* theorem is a powerful mathematical tool used to simplify highly non-linear dynamical systems. In what follows, we apply all the necessary steps of the systematic procedure described in [26].

Let us first provide the following

Definition 1. Given the fixed point $P(u^*, c^*, k^*)$, and the associated Jacobian matrix $\mathbf{J}^*$, system *S* undergoes a *BT* bifurcation if the linearization of $\mathbf{J}^*$ around that point has a double-zero eigenvalue.

It would be of great interest to uncover the stability properties of the two steady states, when system *S* undergoes a *BT* singularity.

Proposition 2. *Recall Proposition 1 and Definition 1. Then, the low-growth steady state is always a non saddle equilibrium, whereas the high-growth steady state is a saddle, within the feasible set of parameters space.*

Proof. From system *S* and (*6.i*) it derives that $dc/dk > 0$, that is for a plausible range of the set of parameters $\blacksquare = \{\alpha, \beta, \tau_k, \gamma, \sigma, v, \rho, g\}$, the model admits at least two steady states, one with low consumption and physical capital $P_L(u^*_{low}, c^*_{low}, k^*_{low})$, and one with high consumption and high physical capital $P_H(u^*_{high}, c^*_{high}, k^*_{high})$.

Of course, a comprehensive view of the implications in Proposition 1 would require us to complement the analysis by establishing the conditions under which the non-saddle steady state is attracting or repelling. The issue is rather appealing. Indeed, an attracting low-growth steady

The Bogdanov-Takens bifurcation theorem is largely used in mathematics, physics and biology, but has found limited application in economics. To the best of our knowledge, only [7] seems to have applied the methodology in a monetary model in a R^2 ambient space.

state would imply the existence of a low-growth trapping region. Moreover, establishing the possibility of a repelling high-growth steady state is of high interest from the perspective of policy modelling.

Next, we use the principles of normal form theory to put system S in a more convenient Jordan form, and introduce the following auxiliary variables $\mu = g - \bar{g}$ and $\eta = \tau_k - \bar{\tau}_k$. We use τ_k and g as the bifurcation parameters. Consequently, we have that

Lemma 1. *for parameter values (τ_k, g) sufficiently close to $(\bar{\tau}_k, \bar{g})$ the vector field in S is topologically equivalent to the following system (joint with $\dot{\mu} = 0$ and $\dot{\eta} = 0$)*

$$\begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + M(\mu, \eta) \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \tag{8}$$

where $M(\mu, \eta)$ is a matrix that vanishes at $(\mu, \eta) = (0,0)$, while F_1 and F_2 contain the high order non linear terms.

Proof. See [26].

The system can be further simplified via normal form theory and time rescaling. A transverse family of this vector field (i.e., a versal deformation) can be used to infer the presence of the global bifurcation in the original vector field, S. Therefore, following the procedure described in [14] and [15], we obtain

Proposition 3. *the transverse family*

$$\begin{aligned} \dot{w}_1 &= w_2 \\ \dot{w}_2 &= \varepsilon_1 + \varepsilon_2 w_2 + w_1^2 + s w_1 w_2 \quad s = \pm 1 \end{aligned} \tag{9}$$

is a topologically equivalent versal deformation of (8), where the unfolding parameters $\varepsilon_i(\mu, \eta)$ are functions of $\mu = g - \bar{g}$ and $\eta = \tau_k - \bar{\tau}_k$ parameters, and satisfy the transversality condition $\frac{\partial(\varepsilon_1, \varepsilon_2)}{\partial(\mu, \nu)}|_{\mu=\nu=0} \neq 0$. Therefore, the change of the original parameters g and τ_k, through ε_1 and ε_2, is a local diffeomorphism.

In our case, versatility means that S, in the neighborhood of the bifurcation point, can be mapped onto a phase portrait of the unfolding parameters $(\varepsilon_1, \varepsilon_2)$. This is, in fact, a homeomorphism, since it captures all the possible dynamics of the original vector field. sufficiently close to the bifurcation point.

Proof. The system (9) is obtained via the versal deformation matrix $\mathbf{V}(\mu,\eta)=\begin{pmatrix} 0 & 1 \\ \varepsilon_1 & \varepsilon_2 \end{pmatrix}$, where

$$\begin{aligned} \varepsilon_1 &= -\left(\xi_1\mu-\frac{\eta}{\sigma}\right)\xi_4\mu+\left(\xi_3\mu+\frac{\eta}{v_2\sigma}\right)(\xi_2\mu+1) \\ \varepsilon_2 &= (\xi_1+\xi_4)\mu-\frac{\eta}{\sigma} \end{aligned}$$

and $\xi_1=\left(\frac{\vartheta k^{\alpha-1}}{u_1}+\vartheta\left[(\alpha-1)k^{*\alpha-2}\right]c^*\right)$;
$\xi_2=\frac{v_2}{u_1}\vartheta\left[(\alpha-1)k^{*\alpha-2}\right]c^*$;
$\xi_3=-\frac{\vartheta k^{*\alpha-1}}{v_2}\mu-\frac{\vartheta}{u_1v_2}\left[(\alpha-1)k^{*\alpha-2}\right]c^*\mu+\frac{\alpha(\vartheta-1)k^{*\alpha-1}}{v_2}$;
$\xi_4=-\frac{\vartheta}{u_1}\left[(\alpha-1)k^{*\alpha-2}\right]c^*\mu+\alpha(\vartheta-1)k^{*\alpha-1}$.

Assume $s=+1$,

Proposition 4. *Let (τ_k,g) be sufficiently close to $(\bar{\tau}_k,\bar{g})$. Then: i) if $\varepsilon_1=0$, there is only one balanced growth path associated to (9); ii) if $\varepsilon_1>0$, there is no equilibrium associated to (9); iii) if $\varepsilon_1<0$, there is a continuum of multiple balanced growth path associated to (9) (i.e., global indeterminacy).*

Proof. See [26] for the mathematical procedure.

For the scope of our analysis, we will concentrate on the case $\varepsilon_1<0$, where the dynamics associated to (9) exhibits a dual steady state. In particular,

Lemma 2. In presence of a dual steady state, one is a saddle (*S*), given $DetJ^*<0$, whereas the other is a non saddle (*NS*), with $DetJ^*>0$. More specifically, under a specific set of parametric restrictions, we may have that the *NS* equilibrium is a sink, and there exists a heteroclinic connection leading from *S* to *NS*. In this case, the basin of attraction of the low-growth steady state can be interpreted as a low-growth (i.e., poverty) trap.

Proof. See [26] for a systematic specification of these results.

In [8], the set of parameters.$\Theta=\{\alpha,\beta,\tau_k,\gamma,\sigma,v,\rho,g\}$ is used to characterize the local stability properties of the steady state in a specific parametric region, where noticeably

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The case with $s=-1$ is very similar, so we leave it out of this demonstration.

Remark 1. If $\hat{\Theta} \subset \Theta : (\alpha, \beta, \varepsilon, \sigma, v, \rho, g, \gamma, \tau) = (0.285, 0.3, -0.19, 1.5, 0.065, 0.05, 0.03, 0.002, 0.3)$, $\kappa_1 = 0.03$, $\kappa_2 = 0.06$, , there is a saddle path stable (unique) equilibrium if $\beta \in [0, 0.3]$ and $\tau_k \in [0, 0.5]$. $u_{low} = 0.5217$ and $u_{high} = 0.6066$ (in line with [5]).

In what follows, we verify our statement in Lemma 2, and provide an example to generalize the results obtained by Ben-Gad (2003), aimed at showing that the economy might be trapped in a basin of attraction (a sink) that surrounds the low-growth steady state, even if for the same set of parameters a unique and determinate equilibrium is locally obtained.

Example 1. Consider $\hat{\Theta}$ as in Remark 1. Let also (τ_k, g) be sufficiently close to $(\bar{\tau}_k, \bar{g}) = (0.349, 0.0328)$. Then, deviations from the saddle path stable trajectory converging to the equilibrium $(c^*, k^*) = (0.4453774926, 1.891310991)$, shall trap the system into a lower growth rate steady state (see, Figure 2).

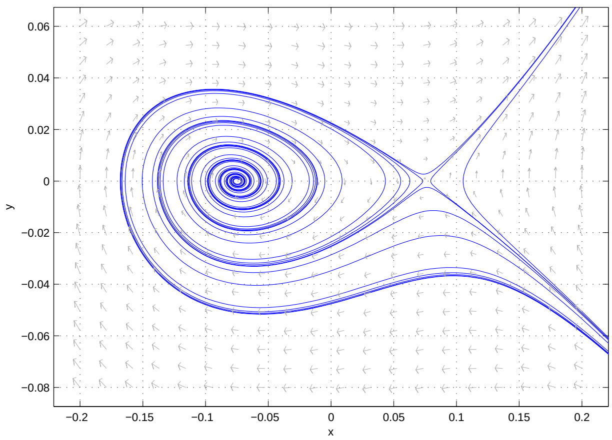

FIGURE 2. Global indeterminacy with $\tau_k = 0.3$

Interestingly, the *BT* bifurcation theorem confirms that a local stability analysis is not sufficient to exactly determine the properties of the transitional dynamics around the long run equilibrium, since we have shown that, under the same parametric region in Remark 1, the equilibrium that is found to be locally unique in [8], can be instead indeterminate in the large. Conclusions drawn from a local analysis are thus misleading. Indeed, policy actions can be therefore not adequate to reach the long run equilibrium, and eventually trap the system in a low growth steady state.

4. Concluding Remarks

We used the principles of global bifurcation theory to gain hints on the global dynamics of the model proposed by Ben-Gad (2003) to study the distributional effects on wealth and income, when changes in tax policies are introduced to finance a rise in government spending. Moreover, we characterized the regions of the parametric space where policy actions can produce undesired fluctuations, and thus become a source of global indeterminacy, even if the local stability analysis might signal the presence of a determinate steady state. In this light, public policies can become a key selection device between uniqueness and indeterminacy of the equilibrium solution. To this end, we have studied the global dynamics of the model in the vicinity of a Bogdanov-Takens singularity, and proved that an economy with distortionary taxes to finance government spending, which exhibits local uniqueness, and a determinate saddle-path equilibrium, may however reveal a dual steady state in the large, one of which can be associated with a low-growth trapping region.

Conflict of Interests

The author declares that there is no conflict of interests.

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